

STIMULATED MANDEL'SHTAM-BRILLOUIN AND STIMULATED ENTROPY BACKSCATTERING OF LIGHT PULSES

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A theory of stimulated Mandel'shtam-Brillouin and entropy backscattering of light is developed in the prescribed pumping field approximation, taking into account fluctuation processes in the medium. The intensity and spectral distribution of light scattered by laser pulses, whose duration is either greater than or smaller than the time of flight of the scattering volume by the light, are determined. Accumulative effects arising in the scattering of a set of ultrashort pulses are discussed. In particular, narrowing of the scattered light spectrum with increase in the number of the exciting pulse is considered.

1. INTRODUCTION

IN the first theoretical papers devoted to stimulated light scattering,^[1,4] it was assumed that the scattering time greatly exceeds the time of the transient process. The results obtained by such an approach permit us to determine the minimum power of the incident radiation (pumping), for which the scattered light differs appreciably from the spontaneous. However, sufficiently high intensity of the excited coherent light is obtained in most cases only from lasers generating pulses of length about 10^{-8} sec and shorter, i.e., of the order of or smaller than the relaxation time of the entropy and hypersound, respectively, for stimulated entropy (temperature) scattering (SES) and stimulated Mandel'shtam-Brillouin scattering (SMBS). In the SES and SMBS of such pulses, the effect of the nonstationarity of the process on the characteristics of the scattered light become important. Furthermore, in the scattering of picosecond pulses,^[5] an additional feature appears, determined by the fact that the spatial dimension of the pulse $l_{\text{pul}} = t_{\text{pul}} v_{\text{gr}}$ becomes less than the dimensions of the scattered volume. In this case, the effect of the boundaries on the course of nonlinear phenomena can be insignificant and the corresponding processes can be regarded as processes in an unbounded medium. In addition to some difference in the mathematical approach to the analysis of phenomena in bounded ($l_{\text{pul}} \gg L$ corresponds to "short" pulses) and unbounded ($l_{\text{pul}} \ll L$ corresponds to "ultrashort" pulses) media, the corresponding processes have a number of physical peculiarities. Thus, for example, the stimulated backscattering of pulses in unbounded media begins with the leading edge of the pulse, while the scattering in bounded media begins with the boundary of the nonlinear volume. The theoretical study of the amplification in the field of an intense plane wave of the prescribed pumping field of spontaneous scattering^[6,7] makes it possible to estimate the time of the transient process in stimulated scattering, and to explain a series of experimental results which pertain to the scattering of short pulses. However, a more precise and complete characteristic of scattered light, both here and in other cases, can be obtained in the analysis

of the development of fluctuations of parameters directly in the field of the laser beam. Such an approach is also used in the present research in the theoretical study of nonstationary SMBS and SES of short pulses, and also SMBS of ultrashort pulses.¹⁾

The results pertaining to scattering of ultrashort pulses are easily generalized to other types of stimulated scattering, SES, stimulated Raman scattering (SRS) and stimulated Rayleigh wing scattering (SRWS). The difference lies only in the size of the relaxation time and in the order of the material equations.

2. INITIAL EQUATIONS

Statement of the problem. We assume that a monochromatic beam of light of amplitude that is constant in time is incident perpendicularly (along the z axis) on a plane-parallel layer of a nonlinear medium. We shall neglect reflection from the boundaries. The transverse cross section of the beam of light S_{\perp} is assumed to be sufficiently great that we can use the quasi-optical approximation in the calculations. We shall carry out the analysis in the prescribed pumping field approximation.

The temporal and spatial change in the dielectric permittivity ϵ , which leads to the light scattering, can be due to the change in a number of parameters which characterize the thermodynamic state of the medium. Consequently, in the general case, in the consideration of stimulated light scattering in the prescribed field approximation, simultaneous account of all processes taking place in the medium is necessary. The corresponding relations for the increments were obtained in^[4,9]. However, the systematic procedure of simultaneous nonstationary consideration, associated with the account of fluctuations of thermodynamic parameters of the medium, is associated with a number of difficulties. Taking it into account that conditions are frequently realized in the experiments for which a single process dominates the others, independent consideration is given below of nonstationary SMBS and SES.

Spontaneous Mandel'shtam-Brillouin and entropy

¹⁾Preliminary results of this research were reported at the IV All-union Symposium on Nonlinear Optics.^[8]

scattering is due, as is known, to fluctuating deviations of the pressure \mathcal{P} and the entropy S in a unit volume from their equilibrium values \mathcal{P}_0 and S_0 :

$$\alpha_p = \mathcal{P} - \mathcal{P}_0, \quad \alpha_s = S - S_0, \quad |\alpha_p| \ll \mathcal{P}_0, \quad |\alpha_s| \ll S_0.$$

SMBS and SES can be treated as scattering on the fluctuations α_p and α_s , which develop in the field of an intense laser beam.

The change in the local values of the pressures in the medium is connected with the action on the material of the electrostrictive force, the density of which $f_{st} = (\frac{1}{8}\pi)(\rho\partial\epsilon/\partial\rho)_S \nabla|\mathbf{E}|^2$ ($\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_p$ is the intensity of the total electric field, due to the incident and scattered (by the pressure fluctuations) waves, ρ the density of the medium). This force, together with the external fluctuation force producing thermal Debye waves, with density f_{ex} ; enters into the right hand side of the Navier-Stokes equation^[10]

$$\frac{\partial^2}{\partial t^2} \alpha_p - v_p^2 \Delta \alpha_p - \Gamma \Delta \frac{\partial \alpha_p}{\partial t} = -v_p^2 \operatorname{div}(f_{ex} + f_{stim}), \quad (1)$$

where

$$\Gamma = \frac{1}{\rho} \left[\frac{4}{3} \eta + \eta' + \frac{\kappa_T}{c_p} (\gamma - 1) \right],$$

η and η' are the coefficients of the shear and bulk viscosities, κ_T the coefficient of thermal conductivity, $\gamma = c_p/c_v$ is the ratio of specific heats at constant pressure and volume, v_p is the velocity of hypersound.

In SES the stimulated entropy change of the medium is due both to the absorption of light and to the electrocaloric effect.^[11,12] In what follows, we have in mind only a liquid with a sufficiently large amplitude coefficient of optical damping ($\delta \approx 10^{-1} - 10^{-2} \text{ cm}^{-1}$), we neglect the effect of the electrocaloric effect.^[11]

Taking into account the fact that the power absorbed in a unit volume, $p_{stim} = (\frac{1}{2}\pi)\delta v_{gr} |\mathbf{E}|^2$ ($\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_s$, \mathbf{E}_s is the electric field of the wave scattered by the entropy fluctuations), we write down the equation for the departure of the entropy α_s from the position of equilibrium in the form

$$\frac{\partial \alpha_s}{\partial t} - \chi \Delta \alpha_s = \frac{\delta v_{gr} \epsilon |\mathbf{E}|^2}{2\pi T} - \frac{\partial S_{ex}}{\partial t} \quad (2)$$

$\chi = \kappa_T/\rho c_p$ is the coefficient of temperature conductivity, T the temperature of the medium, S_{ex} the intensity of the external source, due to random local changes in the heat in a unit volume, $p_{ex} = -T(\partial S_{ex}/\partial t)$, and consequently, the random fluctuations of the entropy. The intensity of the sources can easily be found from the fluctuation-dissipation theorem (FDT) for linearly distributed systems.^[13] Assuming $f_{ex}(\mathbf{r}, t)$ and $S_{ex}(\mathbf{r}, t)$ to be homogeneous stationary processes,^[13] we write down the correlation function of their space-time Fourier components, on the basis of the FDT, in the form

$$\langle f_{ex}(\boldsymbol{\kappa}, \Omega) f_{ex}^*(\boldsymbol{\kappa}', \Omega') \rangle = (\frac{1}{8}\pi^4) k_B T \Gamma \chi^2 \delta^3(\boldsymbol{\kappa} - \boldsymbol{\kappa}') \delta(\Omega - \Omega'),$$

$$\langle S_{ex}(\boldsymbol{\kappa}, \Omega) S_{ex}^*(\boldsymbol{\kappa}', \Omega') \rangle = (\frac{1}{8}\pi^4 \Omega^2) k_B \rho c_p \chi^2 \delta^3(\boldsymbol{\kappa} - \boldsymbol{\kappa}') \delta(\Omega - \Omega'),$$

k_B is Boltzmann's constant.

Abbreviated equations for the scattering of short pulses. We shall represent the electromagnetic pumping field in the scattering region in the form

$$\mathbf{E}_0 = \int_{-\infty}^{+\infty} \int \vec{\mathcal{E}}_0(k_x^{(0)}, k_y^{(0)}) \cdot$$

$$\times \exp \left[ik_x^{(0)} x + ik_y^{(0)} y + i \left(k^{(0)} - \frac{[k_x^{(0)}]^2 + [k_y^{(0)}]^2}{2k^{(0)}} \right) z - i\omega^{(0)} t \right] dk_x^{(0)} dk_y^{(0)} + \text{c.c.} \quad (3)$$

Such a representation is suitable if the condition $\lambda^3 L/a^4 \ll 1$, where a $\sim S_{tr}^{1/2}$ is the characteristic dimension of change in the field in the transverse direction, $\lambda = 2\pi/k^{(0)}$, $k^{(0)} = \omega^{(0)}/v_{gp}$. The fact that the pumping field is not constant in time and has the shape of a rectangular pulse is taken into account in the following in the form of initial conditions for the scattered field and the fluctuations of the medium. The pulse length is automatically accounted for by the choice of the observation interval. It is evident that such an approach is justified only if the condition $t_{pul} \gg L/v_{gr}$. The case $t_{pul} \ll L/v_{gr}$, which occurs for scattering of ultrashort pulses, will be considered below. Since the length of the exciting beam in the nonlinear medium is usually much greater than its transverse dimension, the most effective light scattering takes place in directions close to the backward direction.²⁾ We shall seek the backscattering field in the form of a set of plane waves each of which is propagated at some angle to the longitudinal axis and has a wave vector and a frequency defined by the Bragg condition^[10] for the given angle. Assuming that the amplitude of each of the plane waves of the specified set is a slowly changing function of the longitudinal coordinate z and the time t , we write down the field of the scattered light in the form

$$\mathbf{E}_{p,s}(\mathbf{r}, t) = \int_{-\infty}^{+\infty} \int dk_x^{(p,s)} dk_y^{(p,s)} \vec{\mathcal{E}}_{p,s}(k_x^{(p,s)}, k_y^{(p,s)}, z, t) \times \exp(i\mathbf{k}^{(p,s)} \mathbf{r} - i\omega^{(p,s)} t) + \text{c.c.} \vec{\mathcal{E}}_{p,s} \perp \mathbf{k}^{(p,s)}, \quad (4)$$

where $\mathbf{k}^{(p,s)} = \omega^{(p,s)}/v_{gr}$, $\omega^{(p,s)}$ is the frequency of each of the plane waves of the set. In the case of SMBS,³⁾ $\omega^{(p)} = \omega^{(0)} - q^{(p)} v_p$, where $q^{(p)}$ is determined by the well-known relation^[10] $q^{(p,s)} = 2k^{(0)} \sin(\theta/2)$. The angle of back scattering $\varphi = \pi - \theta$ is connected with the components $k_x^{(p,s)}$ and $k_y^{(p,s)}$ of the vector $\mathbf{k}^{(p,s)}$ of the corresponding plane wave by the equality

$$\sin^2 \varphi = [k_x^{(p,s)}/k^{(0)}]^2 + [k_y^{(p,s)}/k^{(0)}]^2, \quad \varphi \ll 1.$$

In the case of SES, it is convenient to choose $\omega^{(s)} = \omega^{(0)}$.

We represent the fluctuations of the parameters of the medium α_p and α_s in a form analogous to (4):

$$\alpha_{p,s}(\mathbf{r}, t) = \int_{-\infty}^{+\infty} \int dq_x^{(p,s)} dq_y^{(p,s)} \alpha'_{p,s}(q_x^{(p,s)}, q_y^{(p,s)}, z, t) \times \exp[iq^{(p,s)} \mathbf{r} - i(\omega^{(0)} - \omega^{(p,s)}) t] + \text{c.c.} \quad (5)$$

Substituting (3)–(5) in Eqs. (1) and (2) and also in the Maxwell equation

$$\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \operatorname{rot} \operatorname{rot} \mathbf{E} = -\frac{\partial \epsilon}{\partial \alpha_{p,s}} \frac{\partial^2}{\partial t^2} (\alpha_{p,s} \mathbf{E}), \quad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}_{p,s} \quad (6)$$

and equating the Fourier components over the transverse coordinates, we get (after averaging), equations for $\vec{\mathcal{E}}_{p,s}$ and $\alpha'_{p,s}$:

²⁾The forward scattering which takes place without any appreciable shift in frequency^[11] and which is related to the problem of the self-action of the light, is not considered here.

³⁾SMBS for the stimulated process is negligibly small at the anti-Stokes frequency.

$$\begin{aligned}
& \left(\frac{1}{v_{gr}} \frac{\partial}{\partial t} - \frac{\partial}{\partial z} \right) \mathcal{E}_{p,s}(k_x^{(p,s)}, k_y^{(p,s)}, z, t) \\
&= \frac{i}{2\epsilon} \left(\frac{\partial \epsilon}{\partial \alpha_{p,s}} \right)_{s,p} k^{(0)} \int_{-\infty}^{+\infty} dk_x^{(0)} dk_y^{(0)} \mathcal{E}_0(k_x^{(0)}, k_y^{(0)}) \\
&\times \alpha_{p,s}'(q_x^{(p,s)} - k_x^{(0)}, q_y^{(p,s)} - k_y^{(0)}, z, t) \exp[iq_z^{(p,s)}(q_x^{(p,s)}, q_y^{(p,s)})z \\
&\quad - iq_z^{(p,s)}(q_x^{(p,s)} - k_x^{(0)}, q_y^{(p,s)} - k_y^{(0)})z], \quad (7) \\
&\left(\frac{\partial}{\partial t} + v_{p,s} \frac{\partial}{\partial z} + t_{p,s}^{-1} \right) \alpha_{p,s}'(q_x^{(p,s)}, q_y^{(p,s)}, z, t) = \\
&+ \int_{-\infty}^{+\infty} dk_x^{(0)} dk_y^{(0)} \mathcal{E}_0'(k_x^{(0)}, k_y^{(0)}) \mathcal{E}_{p,s}(k_x^{(p,s)} - k_x^{(0)}, k_y^{(p,s)} - k_y^{(0)}, z, t) \\
&\times \exp[ik_z^{(p,s)}(k_x^{(p,s)}, k_y^{(p,s)})z - ik_z^{(p,s)}(k_x^{(p,s)} - k_x^{(0)}, k_y^{(p,s)} - k_y^{(0)})z] \\
&+ \int_{-\infty}^{+\infty} d\mathcal{M}_z d\Omega \Psi_{p,s}^*(\mathcal{M}, \Omega) \exp[-i(\mathcal{M}_z - q_z^{(p,s)})z - i(\Omega + \omega^{(0)} - \omega^{(p,s)})t].
\end{aligned}$$

For convenience of common recording in (7), we introduce $v_s = 0$ and use the notation

$$\begin{aligned}
A_p &= -\frac{i}{4\pi} \left(\frac{\partial \epsilon}{\partial \rho} \right)_s k^{(0)} v_p, \quad A_s = \frac{v_{gr} \delta \epsilon}{\pi T}, \quad t_{p,s}^{-1} = \frac{\Gamma}{2} [q^{(p)}]^2, \\
t_{s,s}^{-1} &= \chi [q^{(s)}]^2, \quad \Psi_p = \frac{\chi v_p}{2q^{(p)}} f_{ex}(\mathcal{M}, \Omega), \quad \Psi_s = -i\Omega S_{ex}(\mathcal{M}, \Omega), \\
&\quad q_{x,y} = \mathcal{M}_x, v,
\end{aligned}$$

$t_{p,s}$ are the relaxation times of hypersound and entropy, respectively.

If the conditions for the applicability of the geometric optics are satisfied ($\lambda/La^2 \ll 1$) for components scattered at angles $\varphi \ll 1$, Eqs. (7) remain valid even for $a/L \geq 1$. This allows us, for example, to consider the scattering of a plane wave for $\varphi \ll 1$.

An important feature of the scattering of bounded beams is that the components $\mathcal{E}_{p,s}(k_x^{(p,s)}, k_y^{(p,s)}, z, t)$ of the scattered field are not correlated over the transverse wave vectors, in contrast with the scattering of a plane wave. This is connected with the fact that the various components of $\mathcal{E}_0(k_x^{(0)}, k_y^{(0)})$, scattered by the same wave $\alpha_{p,s}'(q_x^{(p,s)}, q_y^{(p,s)}, z, t)$ lead to correlation of the components $\mathcal{E}_{p,s}(k_x^{(p,s)}, k_y^{(p,s)}, z, t)$ in the region $\sim a^{-1}$. The latter circumstance generally makes the solution of Eqs. (7) difficult, since all the functions in the integrand in these equations have a characteristic scale $\Delta k_{x,y}^{(p,s)} \sim a^{-1}$. However, in this case, when the change in $k_{x,y}^{(p,s)}$ takes place in limits appreciably exceeding $a^{-1}((k_{x,y}^{(p,s)})_{\max} \approx ak^{(0)}/L \gg a^{-1})$ is the

geometric-optical approximation), the correlation of $\mathcal{E}_{p,s}(k_x^{(p,s)}, k_y^{(p,s)}, z, t)$ in the region a^{-1} has no significant effect on the quadratic characteristics, since the latter are already averaged over a scale exceeding a^{-1} , but at the same time less than $(k_{x,y}^{(p,s)})_{\max} \approx ak^{(0)}/L$.

Therefore it is sufficient to consider the problem of the scattering of a plane wave ($a = \infty$, $a^{-1} = 0$) and then to identify the quadratic characteristics obtained in this approach in a unit solid angle for $\varphi \ll 1$ with the corresponding quantities in the case of scattering of bounded beams with uniform amplitude distribution over the cross section.⁴⁾ Equations (7) for the plane pumping

⁴⁾This conclusion is supported, in particular, by the results of a consideration of spontaneous scattering, when Eqs. (7) are decoupled ($A_{p,s} = 0$) and are solved with comparative ease.

wave $\overline{\mathcal{E}}_0(k_x^{(0)}, k_y^{(0)}) = \overline{\mathcal{E}}_0 \delta(k_x^{(0)}) \delta(k_y^{(0)})$, take the form

$$\begin{aligned}
\frac{\partial \mathcal{E}_{p,s}}{\partial z} &= -\frac{i}{2\epsilon} \left(\frac{\partial \epsilon}{\partial \alpha_{p,s}} \right)_{s,p} k^{(0)} \mathcal{E}_0' \alpha_{p,s}'^* \\
\left(\frac{\partial}{\partial t} + v_{p,s} \frac{\partial}{\partial z} + t_{p,s}^{-1} \right) \alpha_{p,s}' &= A_{p,s} \mathcal{E}_0' \mathcal{E}_{p,s} \\
+ \int_{-\infty}^{+\infty} d\mathcal{M}_z d\Omega \Psi_{p,s}^*(\mathcal{M}, \Omega) \exp[-i(\mathcal{M}_z - q_z^{(p,s)})z - i(\Omega + \omega^{(0)} - \omega^{(p,s)})t]. \quad (7a)
\end{aligned}$$

Change in the scattered wave during time of passage of the light through the medium,^[7,14] is neglected in (7a), which can be valid for $t_p \gg L/v_{gr}$.

3. STATIONARY SCATTERING

The scattered wave in an established stationary process is the Fourier transform of the system (7a). For zero boundary conditions $\mathcal{E}_{p,s}|_{z=L} = 0$, Eq. (7a) has the form

$$\begin{aligned}
\mathcal{E}_{p,s} &= i \frac{(\partial \epsilon / \partial \alpha_{p,s})_{s,p} k^{(0)} \mathcal{E}_0}{2\epsilon} \int_{-\infty}^{+\infty} d\mathcal{M}_z d\Omega \Psi_{p,s}^* e^{i(\Omega + \omega^{(0)} - \omega^{(p,s)})t} \\
&\times \left\{ \exp \left[\frac{iM_{p,s} t_{p,s}^{-1} L}{t_{p,s}^{-1} - i(\Omega + \omega^{(0)} - \omega^{(p,s)})} - i(\mathcal{M}_z - q_z)L \right] \right. \\
&\left. - 1 \right\} \{ iM_{p,s} t_{p,s}^{-1} - i(\mathcal{M}_z - q_z) [-i(\Omega + \omega^{(0)} - \omega^{(p,s)}) + t_{p,s}^{-1}] \}^{-1}, \quad (8)
\end{aligned}$$

where $M_{p,s} = (\frac{1}{2}\epsilon)(\partial \epsilon / \partial \alpha_{p,s})_{p,s} k^{(0)} |\mathcal{E}_0|^2 A_{p,s} t_{p,s}$. For SMBS, the conditions $M_p \ll \frac{1}{4} v_p t_p$ are used, for which generation on the opposed wave is excluded,^[1,2,6] and only amplification of the fluctuations takes place.^[3] For most experiments, this condition is satisfied at room temperature. Thus, for example, for scattering in CS₂ ($(\rho \partial \epsilon / \partial \rho)_s = 2.39$, $\epsilon = 2.8$, $\rho = 1.26$ g/cm³, $v_p = 1.26 \times 10^5$ cm/sec^[10]), ruby laser light ($\omega^{(0)} = 2.7 \times 10^{15}$ sec⁻¹, $t_p = 5 \times 10^{-9}$ sec), having an intensity I_0 (megawatt/cm²), the value of $M_p = 0.075 I_0$ [cm⁻¹] while $\frac{1}{4} v_p t_p \approx 400$ cm⁻¹, i.e., $M_p < \frac{1}{4} v_p t_p$ for $I_0 < 53 \times 10^4$ megawatt/cm².

In obtaining (8), we also used the condition of the smallness of the region of correlation of the sound in comparison with the length of the scattering volume, $L/v_p t_p \gg 1$.⁵⁾ Equation (8) makes it possible to find the correlation function of the electric field $G_{p,s} = (\frac{1}{4}\pi) v_{gr} \epsilon < \mathbf{E}_{p,s}(\mathbf{r}_1, t_1) \cdot \mathbf{E}_{p,s}^*(\mathbf{r}_2, t_2)$. The Fourier integral for the correlation function of the density angular spectrum of the scattered light (Ω_T is the solid angle) has the form

$$\frac{dG_{p,s}}{d\Omega_T} = \int_{-\infty}^{+\infty} d\Omega \cos(\Omega \Delta t) G_{p,s}(\Omega), \quad \Delta t = t_1 - t_2.$$

The functions $G_{p,s}(\Omega)$, which determine the spectral intensity of the scattered light for the region outside the scattering volume ($z \leq 0$) are expressed in the following way:⁶⁾

⁵⁾At low temperatures, this condition cannot be satisfied. In this case the intensity of the spontaneous scattering depends nonlinearly on L , since the spatial part of the correlation of acoustic photons is comparable with the dimensions of the scattering medium.

⁶⁾As the limiting case for $|M_{p,s}| \rightarrow 0$, these formulas give the well-known spectra of spontaneous scattering.^[10]

$$\begin{aligned}
 G_p(\Omega) &= K_p t_p \left\{ \exp \left[\frac{2M_p L}{1 + t_p^2 (\Omega + \omega^{(0)} - \omega^{(p)})^2} \right] - 1 \right\}, \\
 K_p &= \frac{v_{gr} [k^{(0)}]^2 k_B T}{16\pi^3 v_p t_p}, \\
 G_s(\Omega) &= \frac{K_s}{\sqrt{2}\Omega} \left\{ \exp \left[\frac{2M_s L \Omega t_s}{1 + t_s^2 \Omega^2} \right] - 1 \right\}, \\
 K_s &= \frac{\sqrt{2} (\partial \epsilon / \partial T)_p [k^{(0)}]^3 k_B T^2}{32\pi^3 \delta \epsilon t_s}. \quad (9)
 \end{aligned}$$

The bounded character of the interpretation of stimulated scattering as the stimulated amplification of the ordinary thermal scattering considered as an external field is seen from (9) (see, for example, [3]). However, in two limiting cases $M_{p,s}L \ll 1$ and $M_{p,s}L \gg 1$, both interpretations lead to the same results for the integrated light intensity scattered in a unit solid angle at the angle $\varphi \ll 1$. For sufficiently large power of the exciting radiation ($M_{p,s}L \gg 1$)

$$\begin{aligned}
 \frac{dG_{p,s}}{d\Omega_\tau} \Big|_{\Delta t=0} &= I_{p,s} = K_{p,s} \sqrt{\frac{\pi}{\Gamma_{p,s}}} \exp \Gamma_{p,s}, \\
 \Gamma_{p,s} &= (\text{Re} \sqrt{2iM_{p,s}L})^2 = \begin{cases} 2|M_p|L & \text{for SMBS,} \\ |M_s|L & \text{for SES,} \end{cases}
 \end{aligned}$$

$\Gamma_{p,s}$ is the increment of stationary scattering. At the optimal frequency, the halfwidth of the spectrum of stimulated scattering here is determined by the relation

$$\delta\omega^{(p)} = \Gamma_p^{-1/2} t_p^{-1} = \frac{\delta\omega_{sp}^{(p)}}{\Gamma_p^{1/2}}, \quad \delta\omega^{(s)} = \sqrt{2} \Gamma_s^{-1/2} t_s^{-1} = \sqrt{2} \frac{\delta\omega_{sp}^{(s)}}{\Gamma_s^{1/2}},$$

i.e., for $\Gamma_{p,s} \gg 1$, the spectrum of stimulated scattering is much narrower than the spectrum of spontaneous scattering $\delta\omega = t_p^{-1}$.

The most typical estimates for the backscattering of ruby laser light in CS_2 : $|M_s|L = 0.05I_0L$, $|M_p|L = 0.075I_0L$, $K_s = 12 \text{ W/cm}^2$, $K_p = 5.5 \text{ W/cm}^2$. For estimates other than the quantities given above, we used the values: $(\partial \epsilon / \partial T) = -2.75 \times 10^{-3} \text{ deg}^{-1}$, $c_p = 10^7 \text{ ergs/g-deg}$, $t_s \approx 10^{-8} \text{ sec}$, $^{[10]} \delta \approx 0.05 \text{ cm}^{-1}$, $T \approx 300^\circ$. It follows from (9) that the SMBS spectrum is symmetric with respect to the frequency $\omega^{(p)}$; the spectral distribution of SES has an asymmetric character with a maximum displaced in the antistokes region (for $\partial \epsilon / \partial T)_p < 0$) by a value $\Delta\omega^{(s)} \approx t_s^{-1}$. As a consequence of the narrowing of the spectrum in stimulated scattering, there is an increase in the correlation times t_s^k and t_p^k in comparison with the corresponding spontaneous values⁷⁾

$$\begin{aligned}
 dG_s / d\Omega_\tau &= I_s \exp [-(\Delta t)^2 / (t_s^k)^2] \cos (\Delta t / t_s), \quad t_s^k = (2\Gamma_s)^{1/2} t_s, \\
 dG_p / d\Omega_\tau &= I_p \exp [-(\Delta t)^2 / (t_p^k)^2], \quad t_p^k = 2\Gamma_p^{1/2} t_p, \quad (10) \\
 dG_{p,s}^{\text{sp}} / d\Omega_\tau &= I_{p,s}^{\text{sp}} \exp (-|\Delta t| / t_{p,s}).
 \end{aligned}$$

All these formulas, by virtue of what has been said above, refer both to the scattering of a plane wave and also to bounded beams for $\varphi \ll a/L \ll 1$. Since the scattering intensity in the stimulated process ($|M_{p,s}|L \gg 1$) of components with $\varphi > a/L$ is negligibly small relative to the scattering intensity of components with $\varphi < a/L \ll 1$, then the approximate expression for the

⁷⁾The appearance of $\cos (\Delta t / t_s)$ in the first formula of (10) is connected with the fact that the spectral expression for G_s in (9) is written relative to the unshifted frequency.

total intensity of light scattered in the backward direction will be $G_{p,s} |_{\Delta t=0, z=0} \approx I_{p,s} (S_{\text{tot}} / L^2)$.

4. NONSTATIONARY SCATTERING OF SHORT PULSES

The solution of Eqs. (7a) in the nonstationary case for zero initial and boundary conditions, for an electric field $\mathcal{E}_{p,s} |_{t=0} = \mathcal{E}_{p,s} |_{z=L} = 0$, obtained by a Laplace transformation, has the form

$$\begin{aligned}
 \mathcal{E}_{p,s} &= i \frac{(\partial \epsilon / \partial \alpha_{p,s})_s \rho^{k^{(0)}} \mathcal{E}_0}{2e} \int_{-\infty}^{\infty} d\kappa_z d\Omega \Psi_{p,s}^* \int_0^L dz_1 \exp \{ -i(\kappa_z - q_z) z_1 \} \\
 &\times \left[\frac{e^{-t/t_{p,s}} J_0 (2 \sqrt{-iM_{p,s} t t_{p,s} z_1})}{t_{p,s}^{-1} - i(\Omega + \omega^{(0)} - \omega^{(p,s)})} + \int_0^t \exp \left\{ -\frac{t-t_1}{t_{p,s}} \right. \right. \\
 &\left. \left. - i(\Omega + \omega^{(0)} - \omega^{(p,s)}) t_1 \right\} J_0 (2 \sqrt{-iM_{p,s} t_{p,s} z_1 (t-t_1)}) dt_1 \right]. \quad (11)
 \end{aligned}$$

The integrated light intensity, scattered in nonstationary fashion in unit solid angle ($\varphi \ll 1$) can be represented in the form of the sum of two components $I_{p,s}^{\text{inst}} = I_{p,s}^{\text{init}} + I_{p,s}^{\text{source}}$, one of which is determined by the initial fluctuations ($I_{p,s}^{\text{init}}$), and the other by the development of fluctuations as the result of a fluctuating source in the time of the pulse ($I_{p,s}^{\text{source}}$):

$$I_{p,s}^{\text{init}} = \sqrt{\Gamma_{p,s} / 16\pi} I_{p,s} \tau^{-1} e^{-\nu}, \quad I_{p,s}^{\text{source}} = \frac{1}{2} I_{p,s} \begin{cases} 1 + \Phi(\nu), & \nu > 0 \\ 1 - \Phi(-\nu), & \nu < 0 \end{cases} \quad (12)$$

where

$$\Phi(\nu) = (2/\sqrt{\pi}) \int_0^\nu e^{-u^2} du$$

is the error integral, $\nu = \sqrt{2\tau} - \sqrt{\Gamma_{p,s}}$, $\tau = t/t_{p,s} \gg \Gamma_{p,s}^{-1}$. For $\tau \ll \Gamma_{p,s}/2$, we have

$$I_{p,s}^{\text{init}} / I_{p,s}^{\text{source}} \approx \sqrt{\Gamma_{p,s} / 2\tau} > 1,$$

i.e., the scattering on the initial fluctuations dominates the scattering on the fluctuations due to the action of the fluctuating source in the time of the pulse. However, even for $\tau = \Gamma_{p,s}/2$, $I_{p,s}^{\text{init}} / I_{p,s}^{\text{source}} \approx (\pi \Gamma_{p,s})^{-1} \ll 1$ and $I_{p,s}^{\text{init}} \rightarrow 0$ as $\tau \rightarrow \infty$. It follows from (12) that the time of the transition process to the stationary scattering⁸⁾ $t_{p,s}^{\text{tr}} = t_{p,s} (\Gamma_{p,s}/2) \cdot (1 + 2/\sqrt{\Gamma_{p,s}})$, $\Gamma_{p,s} \gg 1$. For the quantity $\Gamma_{p,s} \lesssim 10$, $t_{p,s}^{\text{tr}}$ is approximately twice the corresponding value obtained by Kroll^[6] for SMBS under the assumption of the specified light source.

Figure 1 (curve a) shows the development of stimulated scattering ($\Gamma_{p,s} = 8$) with time. In the same figure

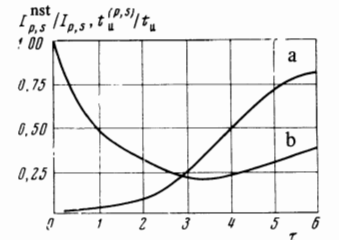


FIG. 1. Development of stimulated scattering ($\Gamma_{p,s} = 8$) in time — curve a, ratio of length of scattered pulse to length of incident, curve b.

⁸⁾In the presence of saturation (inverse reaction of the scattering field on the pumping), the time of the transition process is reduced and for sufficiently intense pumping, can evidently be less than the relaxation time.

is shown (curve b) the dependence on time of the ratio of the length of the scattered pulse to the incidence pulse for the level 0.5. Estimates show that for propagation of megawatt pulses of length 20–30 nsec in liquids at room temperature, SMBS can occur at the stationary level. SES under these conditions is essentially nonstationary. In this connection, the problem of the spectrum of nonstationary SES is of interest.

Analysis of Eq. (11) shows that for $t_{\text{pul}} \ll \Gamma_s t_s$ the maximum in the spectrum of light scattering is shifted relative to $\omega^{(0)}$ into the antistokes region (for $(\partial \epsilon / \partial T)_p < 0$) by an amount

$$\Delta \omega_t^{(s)} = \sqrt{\Gamma_s / 2 t_{\text{pul}} t_s} \approx (\Gamma_s^{(t)} / 2) t_{\text{pul}}^{-1}, \quad (13)$$

where $\Gamma_s^{(t)} = \sqrt{2 \Gamma_s t_{\text{pul}} / t_s}$ is the total spatial increment of amplitude at the end of the pulse for SES, t_{pul}^{-1} is the width of the spectrum of the incident pumping pulse with sinusoidal carrier. With increase in the pumping pulse length, the maximum in the scattered light spectrum is shifted to the side of $\omega^{(0)}$ right up to the transition to the stationary process. The half-width of the spectrum of nonstationary scattered light for $t_{\text{pul}} \ll \Gamma_s t_s$ is equal to $\delta \omega_t^{(s)} = \Delta \omega_t^{(s)}$. If the threshold of stimulated scattering is exceeded by several fold, then $\Gamma_s^{(t)}$ is a quantity of the order of several units and the shift in the scattered light spectrum should exceed the width of the spectrum of the incident pulse with sinusoidal carrier. It is evident that for sufficiently high intensity of the incident light, a more rapid progress to the stationary regime is possible, a consequence of which can be a decrease in the value of the shift and the width of the scattered light spectrum.

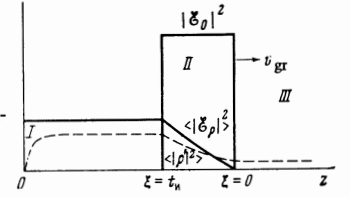
The interpretation of the experiments, from the viewpoint of the given calculations, is very complicated at first glance, since under real conditions the width of the frequency spectrum of the incident pulse $\delta \omega^{(0)}$ is much greater than the value determined from the pulse length, $\delta \omega^{(0)} \gg t_{\text{pul}}^{-1}$. However, one can show that the value of the shift in the spectrum of the nonstationary scattered light, apart from the pulse length, is determined only by the rate of growth of the stimulated process. The latter depends basically on the mean of the value of the pumping intensity and is practically unconnected with the degree of modulation of the exciting radiation. Therefore, the shift in the spectrum of stimulated scattering should not depend on the width of the frequency spectrum of the incident pulse, and Eq. (13) can be applied for the estimate of the frequency in real experiments. Some results of the experiment applicable to this case, and the corresponding estimates are given in [14].

Estimates for nonstationary SMBS in a quasi-parallel laser beam show a rather good agreement of the calculations with experiment (see, for example, [15]), especially close to the threshold, when the effects of saturation are unimportant. It should be noted that nonstationarity for SMBS leads not to a shift in the maximum of the scattered light spectrum, but only to a broadening of the spectrum.

5. SCATTERING OF ULTRASHORT PULSES

General relations. As has been noted in the introduction, the features of stimulated scattering of ultrashort

FIG. 2. Spatial distribution of the exciting radiation, density fluctuations of the medium and scattered light in the scattering of ultrashort pulses ($\rho' \equiv \alpha$).



pulses of light will be considered below in the example of SMBS. The condition $t_{\text{pul}} < L/v_{\text{gr}}$ corresponding to this is satisfied in most cases simultaneously with the condition $t_{\text{pul}} \lesssim T_p = 2\pi/\Omega_p$ ($\Omega_p = q^{(p)}v_p$ is the frequency of hypersound), for which in place of the pressure fluctuations α_p it is more convenient to consider the density fluctuations α_ρ . The wave equation for α_ρ for $t_{\text{pul}} \lesssim T_p$ can be averaged only over the spatial coordinates. The backscattering electric field E_ρ will be sought at the unshifted frequency $\omega^{(0)}$ (similar to E_s in (4)), while the fluctuations α_ρ will be described similar to α_s (see (5)). Neglecting the dispersion spreading and the inverse effect of the scattered pulse on the pumping, we divide the scattering medium into three regions (Fig. 2): I—the region in front of the pulse ($\xi = t - z/v_{\text{gr}} < 0$), II—the region inside the pulse ($0 < \xi < t$), III—the region behind the pulse ($t_{\text{pul}} < \xi < t$). The equations describing the process of scattering of a plane wave in all three regions, for $\varphi \ll 1$, have the form (the index ρ is omitted for q, q_x, y, k_x, y)

$$\begin{aligned} \left(\frac{2}{v_{\text{gr}}} \frac{\partial}{\partial \xi} - \frac{\partial}{\partial z} \right) \mathcal{E}_\rho(k_x, k_y, z, \xi) &= \frac{i}{2\epsilon} \left(\frac{\partial \epsilon}{\partial \rho} \right)_s k^{(0)} \mathcal{E}_0(\xi) \alpha_\rho''(q_x, q_y, z, \xi), \\ \left(\frac{\partial^2}{\partial \xi^2} + v_p^2 q^2 + 2t_p^{-1} \frac{\partial}{\partial \xi} \right) \alpha_\rho''(q_x, q_y, z, \xi) &= \frac{1}{8\pi} \left(\frac{\rho \partial \epsilon}{\partial \rho} \right)_s \\ &\times q^2 \mathcal{E}_0^*(\xi) \mathcal{E}_\rho(k_x, k_y, z, \xi) + \int_{-\infty}^{+\infty} d\alpha_z d\Omega (i\alpha f_{\text{ex}}^*(\alpha, \Omega)) \\ &\times \exp \left[-i \left(\alpha_z - q_z - \frac{\Omega}{v_{\text{gr}}} \right) z - i\Omega \xi \right]. \end{aligned} \quad (14)$$

In the case considered, when the pumping amplitude has the form of a rectangle (Fig. 2), the equations are materially simplified. Their solution in region I makes it possible to determine the following quantities:

$$\begin{aligned} \alpha_\rho''(q_x, q_y, z, \xi) \Big|_{\xi=0} &= \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{d\Omega d\alpha_z (i\alpha f_{\text{ex}}^*(\alpha, \Omega)) \exp[-i(\alpha_z - q_z - \Omega/v_{\text{gr}})z]}{-\Omega^2 + v_p^2 q^2 - 2i\Omega t_p^{-1}}, \\ \frac{\partial \alpha_\rho''}{\partial \xi}(q_x, q_y, z, \xi) \Big|_{\xi=0} &= \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{d\Omega d\alpha_z (\Omega \alpha f_{\text{ex}}^*(\alpha, \Omega)) \exp[-i(\alpha_z - q_z - \Omega/v_{\text{gr}})z]}{-\Omega^2 + v_p^2 q^2 - 2i\Omega t_p^{-1}} \end{aligned}$$

which, together with $\mathcal{E}(k_x, k_y, z, \xi)|_{\xi=0} = 0$ serve as the boundary conditions for the solution of Eqs. (14) in region II. Solving the system (14) in the region II by the Laplace transformation, we find

$$\begin{aligned} \mathcal{E}_\rho &= \frac{i}{4\epsilon} \left(\frac{\partial \epsilon}{\partial \rho} \right)_s k^{(0)} v_{\text{gr}} \mathcal{E}_0 \iint_{-\infty}^{+\infty} d\mu_1 d\Omega (i\alpha f_{\text{ex}}^*(\alpha, \Omega)) e^{i\mu_1 z} \left(\sum_{j=1}^4 a_j e^{p_j \xi} \right), \\ \alpha_\rho &= \iint_{-\infty}^{+\infty} d\mu_1 d\Omega (i\alpha f_{\text{ex}}^*(\alpha, \Omega)) e^{i\mu_1 z} \left[\sum_{j=1}^4 a_j e^{p_j \xi} \left(p_j - i \frac{\mu_1 v_{\text{gr}}}{2} \right) \right], \end{aligned}$$

$$\mu_1 = \kappa_2 - q_2 - \frac{\Omega}{v_{gr}}$$

Here $p_4 = -i\Omega$, $p_{1,2,3} = -i\Omega p s_{1,2,3}$; $s_{1,2,3}$ are the roots of the cubic equation

$$\text{where } (\mu - s)(s^2 - 1 - i\gamma_1 s) = \sigma, \quad (15)$$

$$\mu = \mu_1 v_{gr} / 2\Omega p, \quad \sigma = A^3 / \Omega p^3, \quad \gamma_1 = T_p / \pi t_p \ll 1, \\ A^3 = \rho(\partial \epsilon / \partial \rho) s^2 v_{gr} [k^{(0)}]^3 |\mathcal{E}_0|^2 / 4\pi \epsilon.$$

The coefficients a_j are determined by the expression

$$a_j = \frac{p_j^2 + v_p^2 q^2 - 2i\Omega t_p^{-1}}{-\Omega^2 + v_p^2 q^2 - 2i\Omega t_p^{-1}} \prod_{k \neq j} \frac{1}{p_j - p_k}; \quad j, k = 1, 2, 3, 4.$$

In the region III, relaxation of the hypersound and linear propagation of the scattered light take place independently (in the approximation used).

Since the solution of Eq. (15) is complicated in its general form, we shall limit ourselves here only to two limiting cases: 1) $\gamma_1^2 \ll \sigma \ll 1$ and 2) $\sigma \gg 1$, $t_{pul} < T_p$. For example, for ruby laser light scattering in CS_2 ($\gamma_1 = 10^{-2}$) we have $\sigma = 0.2 I_0$, I_0 in gigawatts/cm². Consequently, the first case corresponds to scattering of giant (0.5 megawatt/cm² $\ll T_0 \ll 5$ gigawatt/cm²) pulses of duration of the order of nanoseconds ($t_{pul} < t_p$) and shorter in sufficiently large volumes ($L \geq 1$ m). The second case refers to scattering of picosecond pulses with a power of several tens of gigawatts.

In both cases the contribution to the value of the increment associated with the relaxation of sound is relatively small and can be neglected in the zero approximation. Then the cubic equation (15) is written in the form

$$f_\mu(s) = (\mu - s)(s - 1)(s + 1) = \sigma. \quad (15a)$$

Since the function $f_\mu(s)$ has three points of intersection with the s axis, it is evident that the complex roots of Eq. (15a) and consequently the stimulated scattering ($\text{Re } p_j = \pm |\text{Re } p_j| \neq 0$) exist only for those harmonics for which $\sigma_\mu < \sigma$, where σ is the maximum of the function $f_\mu(s)$. The smallest threshold (in the zeroth approximation considered) has a harmonic width $\mu = 1$. We now consider the two cases mentioned above in greater detail.

Scattering of giant pulses ($\gamma_1^2 \ll \sigma \ll 1$). For $\sigma \ll 1$ the threshold conditions are satisfied only for harmonics with $|\bar{\mu}| \ll 1$ ($\bar{\mu} = \mu - 1$). The approximate roots of Eq. (15a) are

$$s_{1,2} = 1 \pm (i/2)\sqrt{2\sigma - \bar{\mu}^2}, \quad s_3 = -1,$$

and the increment of the amplitude is

$$\text{Re } p_{1,2} = -\Omega p I_m s_{1,2} = \begin{cases} \pm 1/2 \Omega p \sqrt{2\sigma - \bar{\mu}^2}, & \bar{\mu} < 2\sigma \\ 0, & \bar{\mu} > 2\sigma \end{cases}$$

The spectral distribution for $z \leq 0$ of the intensity of scattered light in a unit solid angle in the backward direction for angles $\varphi \ll 1$ for plane wave pumping has the form

$$G_p^{(4)}(\Omega) = I_p^{(4)} \left(\frac{t_{pul}}{2\pi\beta} \right)^{1/2} \exp \left[-\frac{t_{pul}}{2\beta} (\Omega - \Omega_p)^2 \right], \quad \beta = \sqrt{2\sigma\Omega_p}. \quad (16)$$

In this case the integrated intensity of light scattered in a unit solid angle is determined by the expression

$$I_p^{(4)} = \frac{v_{gr} [k^{(0)}]^2}{64\pi^2 v_p} (k_B T) \sqrt{\frac{\beta}{2\pi t_{in}}} e^{\beta t_{in}}. \quad (16a)$$

For bounded beams of pumping with uniform amplitude distribution over the cross section S_b , the total light intensity scattered in the backward direction, for $S_b/L_p^2 \ll 1$ is equal to $G_p^{(4)}|_{\Delta t=0, z=0} \approx I_p^{(4)}(S_b/L_p^2)$. Since $\sigma \ll 1$, the stimulated scattering takes place only for $t_{pul} \gg T_p$. A comparison of the obtained increment with the corresponding value in the case of bounded media shows that the stimulated scattering, for $L/v_{gr} \ll t_{pul}$, is less intense, as is to be expected, than for $L/v_{gr} \ll t_{pul}$. The maximum in the spectrum of scattered light (16) is displaced by the standard Mandel'shtam-Brillouin shift in the Stokes region and the halfwidth of the spectrum $\delta\omega^{(p)} = \sqrt{2\beta}/t_{pul} = \sqrt{2\beta} t_{pul}^{-1} \delta\omega_p$ for stimulated scattering exceeds several fold the width of the spectrum of the exciting pulse with sinusoidal occupation, $\delta\omega_p \approx t_{pul}^{-1}$. We note that for $S_b \gtrsim L_p^2$, Eqs. (16) and (16a) are valid for directions close to the axis of the beam ($\varphi \ll 1$).

Scattering of homogeneous picosecond pulses ($\sigma \gg 1$). For $\sigma \gg 1$ the greatest contribution to stimulated scattering is made by harmonics with $\mu \ll \sigma$ (as has been shown, the maximum increment is for harmonics with $\mu = 1$). The roots of the cubic equation (15a) corresponding to these harmonics, $s(\mu)$, are such that $|s(\mu)| \gg 1$. The very equation (15a) can in this approximation be written in the form $s^2(\mu - s) = \sigma$. Actually, making this substitution, we neglect effects connected with the propagation of acoustic excitations in the time of the linear interaction. This approximation is valid for scattering of powerful pulses, whose duration is less than the period of the generated hypersound $t_{pul} < T_p$. In this case the spectral distribution of the light radiation scattered in a unit solid angle for $\varphi \ll 1$ (both for φ plane wave and for bounded beams for $\varphi < \sqrt{S_b/L_p^2}$) is represented in the form

$$G_p^{(2)}(\Omega) = I_p^{(2)} (t_{pul}/A)^{1/2} \exp(-\Omega^2/\Omega_1^2), \quad \Omega_1 = 2.3(A/t_{pul})^{1/2}. \quad (17)$$

The integrated intensity in a unit solid angle is

$$I_p^{(2)} \approx 2 \cdot 10^{-4} \frac{[k^{(0)}]^2 v_{gr} \epsilon A^2}{v_p^2} \frac{\exp(\sqrt{3} A t_{pul})}{(A t_{pul})^{1/2}} \quad (17a)$$

Equations (17) and (17a) are valid for $A t_{pul} \gg 1$ (stimulated process).

The light spectrum scattered by the picosecond pulse has a maximum at the unshifted frequency in the approximation considered ($\Omega_p/A \ll 1$). The width of the spectrum of stimulated scattering ($A t_{pul} \gg 1$) is $\Omega_1 = 2.3\sqrt{A t_{pul}} \delta\omega$ and exceeds by several fold the width of the spectrum of the incident pulse with sinusoidal filling, $\delta\omega \approx t_{pul}^{-1}$. Analysis of the expression for the increment shows that the integrated intensity of scattered light sharply increases with increase in the length of the picosecond pulse, even if the total energy of the exciting pulse remains constant.

Numerical estimates show that the scattering in liquids takes on a stimulated character ($A t_{pul} \gg 1$) for pumping powers $I_0 \gtrsim 10^3$ gigawatts/cm² and picosecond pulse length exceeding 10^{-11} sec. In particular, for scattering of the pulse of a ruby laser in CS_2 , the value of

$\sqrt{3} A t_{\text{pul}} \approx 3 \times 10^{-2} I_0^{1/3} t_{\text{pul}}$, where I_0 is in gigawatts/cm² and t_{pul} in nanoseconds.

6. SCATTERING OF SUCCESSIVE PICOSECOND PULSES

A succession of picosecond pulses generated by solid-state lasers in the regime of Q modulation ordinarily has a power of the order of several tens of gigawatts with a repetition interval of $t = 10^{-8} - 10^{-10}$ sec and a corresponding number of pulses $n \approx 100 - 1000$. In the scattering of such a sequence, certain features appear connected with the fact that in some cases the acoustic excitation induced by the previous pulse does not succeed in being materially damped (relaxation time of several nanoseconds) before the arrival of the next pulse. If the succeeding picosecond pulse is incident on the nonlinear medium after the light radiated by the previous pulse emerges from the scattering volume, then the interaction of the light waves will not take place. Together with this, the acoustic excitation induced by the previous pulses remains in the scattering medium ($v_p \ll v_{gr}$) up to the moment of arrival of the next pulse; therefore, generally speaking, "storage" of the acoustic excitations from pulse to pulse is possible, up to the appearance of a saturation effect.

Analytic consideration of the scattering of a sequence of picosecond pulses presents no difficulties in principle in comparison with the scattering of a homogeneous pulse, but is very involved. The field scattered by the n th pulse consists of a series of components connected with acoustic excitations induced by each of the previous pulses and amplified by the latter. Each component has its own growth increment. We shall write down here only the maximum increment of growth of the light amplitude scattered after passage of the n th picosecond pulse:

$$g_n = (n-1) \left(\frac{\sqrt{3}}{2} A t_{\text{pul}} \frac{\Delta t}{t_p} \right) + \frac{\sqrt{3}}{2} A t_{\text{pul}}.$$

The threshold condition of scattering growth from pulse to pulse ("accumulation" of sound) has the form

$$I_0 > I_0^{\text{thr}} = 4.9 \cdot 10^{-16} \frac{e^2 (\Delta t / t_p)^3}{\rho (\partial \epsilon / \partial \rho)^2 [k^{(0)}]^3 t_{\text{pul}}^3}, \quad I_0 \text{ in GW/cm}^2,$$

It follows from the estimates that in light scattering with $\lambda = 0.69$ micron in CS₂ the value of $I_0^{\text{thr}} = 2.3 (\Delta t) 100 t_{\text{pul}}^3$ gigawatt/cm². "Accumulation" can occur independently of the regularity in the succession of picosecond pulses. Experimental realization of this effect for a succession of gigawatt pulses ($n \approx 20 - 40$) with a length of several picoseconds each is possible only if the relaxation time of the sound is several times greater than the distance between pulses.

We note a characteristic feature connected with the narrowing of the spectrum of scattered light. The incident pulse, scattered by the acoustic "lattice" formed by the previous picosecond pulse affects the density distribution such that the acoustic "lattice" becomes more regularly established (only components with resonant wave vectors are amplified). For sufficiently large n , the acoustic profile becomes so regular that scattering takes place with a spectrum much narrower than the spectrum of the incident pulse. In this case, if the intensity of the scattering of the pulses in the suc-

cession is much greater than that of the spontaneous and the intervals between pulses is such that the acoustic excitation induced by the first pulse at the moment of arrival of the second pulse becomes much greater than the unexcited value, then the width of the spectrum of the scattered light decreases with increase in n as $\Omega_n = \Omega_1 / \sqrt{n}$.⁹⁾ The latter expression can easily be obtained if we proceed from the spectral light distribution scattered by a single picosecond pulse, $G_\rho^{(2)}(\Omega) \sim \exp(-\Omega^2 / \Omega_1^2)$, in the given approximation, to the spectrum of light scattered by the n th pulse, $G_{\rho,n}^{(2)}(\Omega) \sim \exp(-n\Omega^2 / \Omega_1^2)$. A succession of less powerful and longer pulses in unbounded media ($I_0 \approx 100 - 1000$ megawatts/cm², $t_{\text{pul}} \approx 1 - 10$ nanosec) also for definite conditions (growth increment of a single scattering act exceeds the relaxation within the time between pulses) can produce accumulation of acoustic excitations in the medium.

In conclusion, we point out the possibility of a similar process of accumulation of temperature excitations in the medium for SES ($t_s \approx 10^{-8}$ sec). For stimulated combination scattering and stimulated Raman scattering, a similar process can take place only for very high intensity in the succession of pulses, since the corresponding relaxation times are very small.

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⁹⁾However, it is evident that as $n \rightarrow \infty$, Ω_n is bounded by a quantity equal to the reciprocal of the length of the scattered radiation.