OPTICAL BRANCHES OF SPIN WAVES IN FERROMAGNETIC METALS

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By the methods of the microscopic theory of a Fermi fluid, it is shown that in ferromagnetic metals, in addition to the low-frequency branch of spin waves, there is a family of high-frequency spin waves, with a quadratic dispersion law in the long-wavelength region and with a relative damping $\sim\sqrt{\Theta/\epsilon_F}$ (Θ is the Curie temperature, ϵ_F the Fermi energy of the conduction electrons).

1. The usual low-frequency branch of spin waves in ferromagnets, in the long-wave limit, corresponds to oscillations of the density of resultant magnetic moment of the system. A characteristic property of its spectrum, with neglect of relativistic interactions, is the absence of a gap; this is due to the property of conservation of total magnetic moment.

But in ferromagnetic metals the total moment, as is well-known, consists of the moment of the electronic groups with a large density of states, or the d-electrons, and the moment of the conduction electrons (s-electrons). The latter corresponds to the fact that as a result of exchange interaction, the s-electrons are magnetized; the effective field determining the size of their moment, under the condition that the s-d interaction is not anomalously small, has the order of magnitude $\sqrt{\Theta\epsilon_F/2\mu_0}$ [1] (Θ is the Curie temperature, ϵ_F the Fermi energy of the conduction electrons, and μ_0 the Bohr magneton). If the subsystem of conduction electrons were isolated, then in an external magnetic field of such magnitude it could possess characteristic frequencies of magnetic oscillations of order $\sqrt{\Theta\epsilon_F}$ [2]. On the other hand, the characteristic frequencies of the oscillations of the magnetic moment of the d-electrons do not exceed Θ; that is, they are smaller by a factor $\sqrt{\epsilon_F/\Theta}$ than the frequencies of the s-electrons. It will be shown below, within the framework of the microscopic theory of a Fermi fluid, that because of the circumstance mentioned, a ferromagnetic metal has an additional family of spin-wave branches, whose frequencies in the long-wave limit approach a finite value $\sim\sqrt{\Theta\epsilon_F}$. For small wave vectors, their relative damping is small, and the function $\omega(k)$ is quadratic.

The calculations will be carried out for the case of zero temperature. We shall use the isotropic model of the s-electrons; this has no fundamental significance as regards existence of the phenomena considered.

2. We consider the properties of the component perpendicular to the spin of the two-particle vertex part of the s-electrons $\Gamma(p_1, p_2; k)$ (we have in mind the component $\Gamma_{\alpha\beta\gamma\delta}$ with spin structure $\Gamma_{\uparrow\downarrow\downarrow\uparrow}$) in the region of small transmitted momenta $k = (\omega, \mathbf{k})$: $\Theta \ll \omega \ll \epsilon_F$, $|\mathbf{k}| \ll p_0$, where p_0 is the Fermi momentum). As is well-known, its singularities with respect to k determine the spin-wave spectrum. We shall use the fact that in the range of transmitted frequencies indicated, the loops of the Green functions of the d-electrons are not singular; and we shall introduce a function $\widetilde{\Gamma}^{(1)}$, determining the set of graphs for Γ which it is impossible to cut along the two lines of s-electrons carrying transmitted momentum k. Then the vertex part Γ can be expressed, obviously, in terms of $\Gamma^{(1)}(p_1, p_2; k)$ by means of the following equation:

$$\Gamma(p_1, p_2; k) = \widetilde{\Gamma}^{(1)}(p_1, p_2; k) - i\int\frac{d^4p'}{(2\pi)^4}\widetilde{\Gamma}^{(1)}(p_1, p'; k)G_-(p'+k)G_+(p')\Gamma(p', p_2; k). \quad (1)$$

Here $G_\pm(p)$ are the Green functions of the s-electrons, corresponding to the two possible values of the projection of the electron spin on the direction of the total angular momentum of the system. Because of the relative proximity of the poles of $G_-(p' + k)$ and of $G_+(p')$ when $\Theta \ll \omega \ll \epsilon_F$, the product $G_-(p' + k)G_+(p')$ depends strongly on k. In connection with this, we shall make the transformation of equation (1) that is usual in the theory of a Fermi fluid[3]. Namely, we separate from $G_-(p' + k)G_+(p')$ in explicit form the product of the polar parts of the Green function:

$$G_-(p'+k)G_+(p') = \frac{a}{\varepsilon'+\omega-v(|\mathbf{p}'+\mathbf{k}|-p_-)+i\delta\operatorname{sign}(\varepsilon'+\omega)} \times\frac{a}{\varepsilon'-v(|\mathbf{p}'|-p_+)+i\delta\operatorname{sign}\varepsilon'}+\overline{G_-(p'+k)G_+(p')}. \quad (2)$$

Here $\overline{G_-(p' + k)G_+(p')}$ is a regular term, p_+ and p_- are the Fermi momenta of s-electrons with opposite values of the spin projection, v is the Fermi velocity, a is the residue of the Green function at the pole, and $\delta \to +0$. We neglect the difference in speeds of excitations on the Fermi surfaces of electrons with opposite values of the spin projection. Allowance for this difference in the energy interval $\sim v(p_+ - p_-)$ that is important in the integration of the polar term in (1) would give a correction to the one-particle energies of the excitations of order $v(p_+ - p_-)\sqrt{\Theta/\epsilon_F}$. This, however, is devoid of meaning, because, as was shown in[4], the damping of the Fermi excitations is of the same order of magnitude when $\epsilon \sim v(p_+ - p_-)$.

We introduce the function $\widetilde{\Gamma}(p_1, p_2; k)$ in the following form:

$$\widetilde{\Gamma}(p_1, p_2; k) = \widetilde{\Gamma}^{(1)}(p_1, p_2; k) - i\int\frac{d^4p'}{(2\pi)^4}\widetilde{\Gamma}^{(1)}(p_1, p'; k)\overline{G_-(p'+k)G_+(p')}\widetilde{\Gamma}(p', p_2; k). \quad (3)$$

Using this equation, we can exclude from the integral equation (1) the region of integration far from the Fermi surface, going over from the kernel $\widetilde{\Gamma}^{(1)}$ to $\widetilde{\Gamma}$:

$$\Gamma(p_1, p_2; k) = \widetilde{\Gamma}(p_1, p_2) + \int\frac{d\mathbf{p}'}{(2\pi)^3}\widetilde{\Gamma}(p_1, p')\cdot$$

$$\times \frac{n_+(\mathbf{p}') - n_-(\mathbf{p}' + \mathbf{k})}{\omega - v(\Delta + \mathbf{kn}) + i\delta \operatorname{sign} \omega} \Gamma(p', p_2; k),$$
$$\Delta = p_+ - p_-, \quad n' = \frac{\mathbf{p}'}{|\mathbf{p}'|}, \quad n_\pm(\mathbf{p}) = \begin{cases} 1, & |\mathbf{p}| < p_\pm \\ 0, & |\mathbf{p}| > p_\pm \end{cases}. \tag{4}$$

Having in mind the regularity of $\tilde{\Gamma}$ as a function of the transmitted momentum (which follows from its definition), we shall neglect in it the dependence on k, since for investigation of the singularities of the vertex part $\Gamma(p_1, p_2; k)$ we are actually interested in a relatively narrow region of variation of the variables ω and $\mathbf{k}$ (ω and $v|\mathbf{k}| \ll \epsilon_F$). In addition, the electron energies are assumed to be small in comparison with the Fermi energy, and the absolute values of the three-dimensional electron momenta are assumed to be close to their Fermi values.

A method of solution of equations analogous to Eq. (4) was described in a paper by Blank and the author[5]. We shall therefore present here without derivation its solution for $k = 0$:

$$\Gamma(p, p'; k) = \sum_{l,m} \Gamma_l(\omega) \cdot Y_{lm}(\mathbf{n}) Y_{lm}^*(\mathbf{n}'), \tag{5}$$

$$\Gamma_l(\omega) = \frac{1}{a^2 v} \frac{\tilde{B}_l(\omega - v\Delta)}{\omega - v\Delta(1 + \tilde{B}_l)}. \tag{6}$$

Here $\tilde{B}_l = a^2 \tilde{\Gamma}_l \nu/(2l + 1)$, $\tilde{\Gamma}_l$ is the l-th spherical harmonic of the function $\tilde{\Gamma}(p, p')|_{\epsilon = \epsilon' = 0}$, $Y_{lm}(\mathbf{n})$ is a normalized spherical function, and $\nu = p_0^2/2\pi^2 v$ is the density of states of electrons of one spin direction on the Fermi surface, $p_0 \approx p_+, p_-$.

As is evident from equation (6), the vertex part at $k = 0$ has poles at frequency values $\omega = \omega_l$,

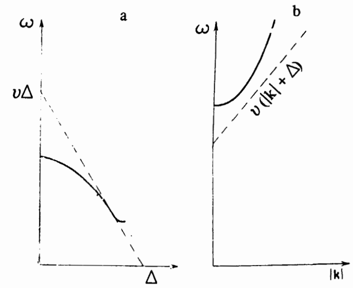
$$\omega_l = v\Delta(1 + \tilde{B}_l). \tag{7}$$

These frequencies correspond to uniform modes of magnetic excitations. The existence of a family of magnetic excitations in addition to the low-frequency spin waves, with limiting frequencies ω_l, is a consequence of the two-component nature of a metallic ferromagnet (s- and d-bands), and therefore the excitations under consideration may be called optical spin waves.

A characteristic peculiarity of the resonance frequencies ω_l is that their magnitude is practically independent of the intensity of the external field. Actually such a dependence, as is clear from the equations for Γ, should occur because of a change in separation of the Fermi surface on application of the external field. But because the frequency $v\Delta$ corresponds to magnetic fields of order $\sqrt{\Theta \epsilon_F}/2\mu_0 \sim 10^8$ Oe, at any fields at present experimentally attainable Δ, and hence also the frequencies ω_l, may be considered constant.

For the same reason, the optical magnons are practically uninfluenced by the orbital motion of the electrons in a magnetic field. This justifies the fact that in the equations for the vertex part, we neglected the interaction of the electronic charge with the magnetic field, in consequence of which there was a degeneracy of the resonance frequencies ω_l with respect to the quantum number m.

It should be mentioned that, in contrast to the low-frequency spin-wave branch, the optical magnons possess damping even at zero wave vector and with neglect of interactions that do not conserve the magnetic moment. The damping is caused principally by decay



processes whose ultimate product is a low-frequency spin wave and a pair of Fermi excitations. An estimate of its order of magnitude can be obtained if, in the derivation of formula (6), allowance is made for the damping of Fermi excitations; this, as has already been mentioned above, at energies of order $\omega_l \sim v\Delta$ has a value $\sim \sqrt{(\Theta/\epsilon_F)} \cdot \Theta \epsilon_F$. This leads to the result that the imaginary part of the pole of the vertex part also has a value $\sim \sqrt{\Theta/\epsilon_F}\,\omega_l$; that is, the relative damping of the optical spin waves has the order $\sqrt{\Theta/\epsilon_F}$.

3. We shall not present here the general formulas for the spectrum with $\mathbf{k} \neq 0$. In the approximation quadratic in k, for $|\mathbf{k}| \ll \Delta$, their derivation is quite analogous to that given in[5]. We mention only that the degeneracy with respect to the quantum number m that occurs in the case of uniform resonance is removed when $\mathbf{k} \neq 0$.

As an example we consider a simple model in which the function $\tilde{\Gamma}(p, p')|_{\epsilon = \epsilon' = 0}$ is isotropic ($\tilde{B}_0 = 0$, $\tilde{B}_l = 0$ for $l \geq 1$). In this case the solution of equation (4) for arbitrary values of the wave vector takes the form

$$\Gamma(p_1, p_2; k) = \frac{\tilde{B}_0}{a^2 v}\left[1 + \tilde{B}_0 - \tilde{B}_0 \frac{\omega}{2v|\mathbf{k}|} \ln \frac{v(\Delta - |\mathbf{k}|) - \omega}{v(\Delta + |\mathbf{k}|) - \omega}\right]^{-1}. \tag{8}$$

The spectrum of the optical spin waves is determined by the equation

$$1 + \tilde{B}_0 - \tilde{B}_0 \frac{\omega}{2v|\mathbf{k}|} \ln \frac{v(\Delta - |\mathbf{k}|) - \omega}{v(\Delta + |\mathbf{k}|) - \omega} = 0. \tag{9}$$

The general form of this solution depends significantly on the sign of $\tilde{B}_0$. When $\tilde{B}_0 < 0$ (case a in the figure), the frequency decreases with increase of $|\mathbf{k}|$. In the range $|\mathbf{k}| \lesssim \Delta$ the curve of the spectrum has an abrupt drop with a slope $\partial\omega/\partial|\mathbf{k}| \approx -v$ and reaches a minimum[1)] $\omega_{min} \sim v\Delta/\ln(\epsilon_F/\Theta)$. For $|\mathbf{k}| > \Delta$, the imaginary part of the solution (9) is comparable with the real; that is, in this k-range optical spin waves are absent when $\tilde{B}_0 < 0$.

If $\tilde{B}_0 > 0$ (case b in the figure), $\omega(\mathbf{k})$ increases with increase of $|\mathbf{k}|$ and asymptotically approaches a linear law of dispersion. In fact, in this case when $|\mathbf{k}| > 0$ the optical magnons correspond to spin waves of zero-sound type, which occur in the absence of a magnetic field in nonferromagnetic metals.

4. In the preceding treatment, it was assumed for definiteness that $p_+ > p_-$; that is, that the resultant magnetic moment of the conduction electrons is directed along the total moment of the ferromagnet. Then the polarizations of the optical and acoustic spin waves are

1)From Eq. (9) it follows that $\omega_{min} = 0$. Allowance for damping of the Fermi excitations, however, leads to the indicated estimate of the value of ω_{min}.

the same. It is easy to see that in the case $p_+ < p_-$, $\Delta < 0$ the singularities of the vertex part $\Gamma_{\downarrow\uparrow\uparrow\downarrow}$ lie in the negative frequency range. This means that in the case of antiparallel orientation of the magnetic moment of the s-electrons and of the total moment, the polarization systems of the optical and acoustic spin waves are opposite.

5. A specific peculiarity of the optical branches of the spin waves is the fact that in consequence of the large value of the separation of the Fermi surface of the conduction electrons, they are well separated in frequency from the low-frequency branch of the spin waves. Therefore it is chiefly the conduction electrons that participate in the spin-density fluctuations that correspond to optical magnons.

Because of this last circumstance, there is a significant formal analogy between the excitations that we have considered and spin waves in nonmagnetic metals in the presence of an external magnetic field, first investigated by Silin[2]. In both phenomena, Fermi-fluid interaction plays a determining role. The effective Landau function in the case of optical spin waves is our function $a^2\widetilde{\Gamma}(p, p')|_{\epsilon = \epsilon' = 0}$. At the same time, there is an important difference between optical magnons and spin waves in nonferromagnetic metals. Not only is an external magnetic field not necessary for existence of the optical modes, but their spectrum is practically independent of the value of the field intensity. In addition, the d-electrons, which provide the effective exchange field for the conduction electrons, lead also to a "viscosity" that accompanies the oscillations of the spin density of the s-electrons, and as a consequence the optical spin waves have damping over the whole spectral range.

We note that the isotropic branch of the optical spin waves was studied earlier, on particular models of ferromagnets, by various authors (see, for example,[6]). There, however, there was no consideration of Fermi-fluid effects and of damping of the spin waves.

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