FORCE OF DRAGGING OF A CRYSTAL LATTICE BY CONDUCTION ELECTRONS IN NON-STATIONARY CURRENTS

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We calculate the force density of dragging of a crystal lattice by conduction electrons under conditions of a rapidly deforming crystal, non-stationary distribution of the electron velocities, and the presence of external time-varying electric and magnetic fields and a concentration gradient is calculated. It is shown that there is a major term in the drag force which is proportional to the external force applied to the electrons, and is not related to the momentum transfer to the lattice from electron scattering. Account of this term makes it possible to generalize and refine the results of previous investigations, and remove their mutual inconsistencies and the errors of calculation obtained in considering trivial examples.

1. INTRODUCTION

In recent years, many investigations have appeared of the amplification on ultrasound by the drift of the current carriers in crystals, and also other works on acousto-electric phenomena. They contain solutions of the equation of motion (oscillation) of a crystal lattice, in which the lattice drag by the electrons is sometimes the important force. For this force, various authors use different expressions that are mutually inconsistent. For example, the expression

$$\mathbf{f}_H = -m\int \mathbf{v}\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} d^3v \qquad (1)$$

is used in[1] for the drag force exerted by the electrons per unit volume of the lattice. Here m is the mass of the free electron, f the velocity distribution function of the conduction electrons, **v** the velocity of the electron, and $(\partial f/\partial t)_{\text{coll}}$ the collision term in the kinetic equation. If the time of free flight of the electron τ does not depend on the velocity, (1) can be rewritten in the form

$$\mathbf{f}_H = \frac{\mathbf{J}_e}{\mu_0}\frac{m}{m^*}, \qquad \mu_0 = \frac{e\tau}{m^*}. \qquad (2)$$

Here $\mathbf{J}_e$ is the electron current density relative to the lattice (the current density in a set of coordinates moving with the lattice), e the algebraic charge of the conduction electron, m* the effective mass and μ_0 the mobility of the electron in the absence of a magnetic field.

In[2] the force density applied to the lattice is, in the same notation,

$$\mathbf{f}_R = \mathbf{J}_e/\mu_0. \qquad (3)$$

In[3], the expression $c^{-1}\mathbf{J}\times\mathbf{H}$ is used for the total ponderomotive Lorentz force applied to the lattice and the drag force by the current carriers, where **H** is the magnetic field intensity, c the velocity of light, and **J** the total current density (electron current and the current of charges attached to the moving lattice); **J** and **H** are in the laboratory system of coordinates. Hence the drag force density due to the carriers is

$$\mathbf{f}_G = \frac{1}{c}[\mathbf{JH}] - \rho_L\left\{\mathbf{E} + \frac{1}{c}[\mathbf{uH}]\right\}. \qquad (4)^*$$

Here ρ_L is the charge density connected with the lattice, **E** the electric field intensity (in the laboratory system of coordinates), **u** the velocity of the lattice.

In[4], the expression

$$\mathbf{f}_K = \mathbf{f}_G - \frac{m}{e}\frac{\partial \mathbf{J}}{\partial t}. \qquad (5)$$

was used for the drag force density.

In[3], Eq. (4) is used for a strong magnetic field, when $eH\tau/m^*c \gg 1$. In[4], Eq. (5) is used for an arbitrary magnetic field. In both[3] and[4], a neutral crystal is considered.

The foregoing four expressions for the drag force densities $\mathbf{f}_H$, $\mathbf{f}_R$, $\mathbf{f}_G$, $\mathbf{f}_K$ are different and are, generally speaking, incompatible with one another. For example, the values of (2) and (3) differ by the factor m/m^*. The values of (4) and (5) differ by the component $me^{-1}\partial\mathbf{J}/\partial t$.

In the experiments of Stuart and Tolman, the drag force is exactly equal to (1) or (2). Equation (5) also gives the true result, inasmuch as in this case $\mathbf{E} = \mathbf{H} = 0$. The same is true for (3) only for $m^* = m$. For constant ohmic current ($\mathbf{H} = 0$), the entire momentum acquired by the conduction electrons in the external electric field is transferred to the lattice. In this case, as is well known, the drag force density is exactly equal to $en\mathbf{E}$, where n is the conduction electron concentration. Consequently, the true result here gives (3) (and in the case of a neutral crystal, (5) also)); on the other hand, Eqs. (1) (2) are now incorrect.

For a stationary diffuse current through the crystal ($\mathbf{E} = \mathbf{H} = 0$), the force (5) is equal to zero. Actually, the drag force is not equal to zero, as will be shown below; forces that differ from zero, and are different in magnitude, are also obtained from (1), (2), (3).

It will be shown below that even for strong magnetic fields Eqs. (4) and (5) are sometimes incorrect. Thus Eqs. (1)–(5) are each true only in a separate particular

*$[\mathbf{JH}] \equiv \mathbf{J}\times\mathbf{H}$.

case. However, in[2-4], these formulas are considered as general, and are applied when the current is nonstationary, when there is accelerated motion and there is diffusion current.

The purpose of the present work is the consideration of the major component in the expression for the drag force, which is omitted in the works mentioned, and also in[5], and the deviation of a drag-force formula that is valid in the more general case. Here we shall clear up the contradictions noted above.

The results obtained below are valid under the following limitations:

1) It is assumed that the external force $\mathbf{F}$ acting on the carriers changes sufficiently smoothly in space (it changes little over a distance of the order of the lattice constant), i.e., we apply the effective-mass method.

2. Part of the results obtained with the help of the kinetic equation is valid when the latter is valid.

3) For simplicity, a simple conduction band is assumed with an isotropic effective mass.

2. GENERAL EXPRESSION FOR THE LATTICE DRAG FORCE DENSITY

We first consider a crystal with immobile fixed nuclei. As is known,[6] the mean quantum mechanical value of the electron velocity, computed in the Bloch state $\psi_{\mathbf{k}}$, is exactly equal to the group velocity of the wave:

$$\langle\psi_{\mathbf{k}}|\hat{\mathbf{v}}|\psi_{\mathbf{k}}\rangle=\frac{1}{\hbar}\nabla_{\mathbf{k}}\mathscr{E}(\mathbf{k}),\quad \hat{\mathbf{v}}=-\frac{i\hbar}{m}\nabla, \tag{6}$$

where $\mathbf{k}$ is the quasi-momentum of the conduction electron and $\mathscr{E}(\mathbf{k})$ is its energy. A similar relation can be obtained in the approximation of the effective-mass method, on the basis of the wave equation with the periodic potential omitted. In this method[7], each Bloch function $\psi_{\mathbf{k}}$ is accompanied by a "smoothed" function $\varphi_{\mathbf{k}} = V^{-1/2}e^{i\mathbf{k}\cdot\mathbf{r}}$ (V is the volume of the principal cyclicity range). As a result, we get in place of (6)

$$\langle\varphi_{\mathbf{k}}|\hat{\mathbf{v}}^*|\varphi_{\mathbf{k}}\rangle=\frac{1}{\hbar}\nabla_{\mathbf{k}}\mathscr{E}(\mathbf{k}),\quad \hat{\mathbf{v}}^*=-\frac{i\hbar}{m^*}\nabla. \tag{7}$$

Inasmuch as $\mathscr{E}(\mathbf{k})$ is the same in both cases, $\langle\psi_{\mathbf{k}}|\hat{\mathbf{v}}|\psi_{\mathbf{k}}\rangle = \langle\varphi_{\mathbf{k}}|\hat{\mathbf{v}}^*|\varphi_{\mathbf{k}}\rangle$.

According to the Ehrenfest quantum-mechanical theorem, we have, for an electron in the conduction band,

$$m\frac{d}{dt}\langle\psi|\hat{\mathbf{v}}|\psi\rangle=\langle\psi|\mathbf{F}-\nabla W|\psi\rangle,\quad \mathbf{F}=e\left\{\mathbf{E}+\frac{1}{c}[\hat{\mathbf{v}}\mathbf{H}]\right\}. \tag{8}$$

Here $\mathbf{F}$ is the force of the external fields and $W(\mathbf{r})$ is the periodic potential of the conduction electron in the crystal. A similar theorem in the effective-mass approximation gives

$$m^*\frac{d}{dt}\langle\varphi|\hat{\mathbf{v}}^*|\varphi\rangle=\langle\varphi|\mathbf{F}|\varphi\rangle. \tag{9}$$

We choose ψ in the form of a wave packet $\psi(\mathbf{r}, t) = \sum_{\mathbf{k}} a_{\mathbf{k}}(t)\psi_{\mathbf{k}}(\mathbf{r})$, in which the $\psi_{\mathbf{k}}$ enter only from the lower part of the conduction band. The corresponding "smoothed" function (in the sense of the effective mass method[6]) is equal to

$$\varphi(\mathbf{r},t)=\sum_{\mathbf{k}}a_{\mathbf{k}}(t)\varphi_{\mathbf{k}}(r),$$

with the same $a_{\mathbf{k}}(t)$. Then

$$\langle\psi|\hat{\mathbf{v}}|\psi\rangle=\sum_{\mathbf{k}\mathbf{k}'}a_{\mathbf{k}}^*a_{\mathbf{k}'}\langle\psi_{\mathbf{k}}|\hat{\mathbf{v}}|\psi_{\mathbf{k}'}\rangle=\sum_{\mathbf{k}}|a_{\mathbf{k}}|^2\langle\psi_{\mathbf{k}}|\hat{\mathbf{v}}|\psi_{\mathbf{k}}\rangle \tag{10}$$
$$=\sum_{\mathbf{k}}|a_{\mathbf{k}}|^2\langle\varphi_{\mathbf{k}}|\hat{\mathbf{v}}^*|\varphi_{\mathbf{k}}\rangle=\langle\varphi|\hat{\mathbf{v}}^*|\varphi\rangle=\langle\mathbf{v}\rangle.$$

Inasmuch as $\mathbf{F}$ is a smooth function of $\mathbf{r}$, we have $\langle\psi|\mathbf{F}|\psi\langle = \langle\varphi|\mathbf{F}|\varphi\rangle = \langle\mathbf{F}\rangle$. Eliminating $d\langle\mathbf{v}\rangle/dt$ from (8) and (9), we obtain

$$\langle\psi|\nabla W|\psi\rangle=(1-m/m^*)\langle\mathbf{F}\rangle. \tag{11}$$

This formula shows that the drag force of a periodic potential of the lattice per electron $-\nabla W$ is not equal to zero in the mean if an external force $\langle\mathbf{F}\rangle$ is present. According to the third law of Newton, the conduction electron should act on the lattice with the force ∇W. If the concentration of conduction electrons is $n(\mathbf{r})$, then the drag force density on the lattice due to the electrons is

$$\mathbf{f}_1=n\left(1-\frac{m}{m^*}\right)\langle\mathbf{F}\rangle. \tag{12}$$

Here $\langle\mathbf{F}\rangle$ can be replaced by the force $\mathbf{F}(\mathbf{r})$ at that macroscopic point $\mathbf{r}$ to which the value of n refers.

The drag force (12) was not considered in previous researches. The meaning of $\mathbf{f}_1$ becomes trivial in the limiting case $m^* \to \infty$, when the conduction electrons become almost bound to the lattice, and in this case it is clear that the lattice drag force density is equal to $\mathbf{f}_1 = n\mathbf{F}$.

The drag force density $\mathbf{f}_1$ has been obtained for immobile nuclei and is not connected with the scattering of conduction electrons by lattice oscillations. If we now consider a lattice executing thermal oscillations, then an additional drag force appears, connected with the transfer of the momentum of the electrons to the lattice upon their scattering (by lattice oscillations, impurities, etc.). For consideration of this force, we introduce the conduction electron velocity distribution function $f(\mathbf{v})$, which is so defined that $f(\mathbf{v})d^3v$ is the number of electrons per unit volume possessing velocities in the ranges dv_x, dv_y, dv_z and any value of spin. Here the Fermi distribution function, i.e., the mean filling factor of a single quantum state, is equal to $4(\pi\hbar/m^*)^3 f(\mathbf{v})$. The kinetic equation has the form

$$\frac{\partial f}{\partial t}+\mathbf{v}\nabla_r f+\frac{e}{m^*}\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\mathbf{H}]\right)\nabla_{\mathbf{v}}f=\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}, \tag{13}$$

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}=\int\{W(\mathbf{v},\mathbf{v}')f(\mathbf{v}')[1-4(\pi\hbar/m^*)^3f(\mathbf{v})]$$
$$-W(\mathbf{v}',\mathbf{v})f(\mathbf{v})[1-4(\pi\hbar/m^*)^3f(\mathbf{v}')]\}d^3v'. \tag{14}$$

Here $W(\mathbf{v}', \mathbf{v})$ is the scattering probability per second, in which an electron with initial velocity $\mathbf{v}$ moves into the unit volume of velocity space located at the point $\mathbf{v}'$.

The additional contribution to the drag force density $\mathbf{f}_2$ is equal to the momentum transferred to the lattice per unit time per unit volume by the scattered electrons:

$$\mathbf{f}_2=m\int\int(\mathbf{v}-\mathbf{v}')W(\mathbf{v}',\mathbf{v})f(\mathbf{v})[1-4(\pi\hbar/m^*)^3f(\mathbf{v}')]d^3v\,d^3v'. \tag{15}$$

Comparing (15) and (14) we readily see that

$$\mathbf{f}_2=-m\int\mathbf{v}\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}d^3v. \tag{16}$$

This is identical with the quantity (I).

If there is a time-varying macroscopic deformation in the crystal, for example, a sound wave, then a co-moving set of Cartesian coordinates can be chosen for the given small element of volume, in which the lattice will be fixed. This system will move in accelerated fashion; therefore, an additional D'Alembert force must be introduced in **F**, equal to $-m\ddot{\mathbf{u}}$. As a result, an additional term appears in the lattice drag force density, equal to $-n(1 - m/m^*)m\ddot{\mathbf{u}}$, where $\mathbf{u}(\mathbf{r}, t)$ is the deformation vector of the crystal. The left side of the equation of motion of the lattice has the form $\gamma\ddot{\mathbf{u}}$, where γ is the density of the crystal. Inasmuch as $\gamma \gg nm(1 - m/m^*)$ always, the D'Alembert force can be neglected. Thus the last term in the left side of (13) will have the same form in the laboratory set of coordinates and in the co-moving coordinates. The first and second terms of the left side of (13), taken separately, are not invariant relative to the Galilean transformation. However, their sum is invariant. The connection between the distribution functions in the laboratory and co-moving coordinates has the form

$$f_{\text{lab}}(\mathbf{r}, \mathbf{v}, t) = f_{\text{com}}(\mathbf{r} - \mathbf{u}, \mathbf{v} - \dot{\mathbf{u}}, t). \tag{17}$$

It is assumed below that Eqs. (13)–(16) refer to the co-moving coordinates, inasmuch as the calculations in them appear to be simpler (one can use the concept of electron mobility, its diffusion coefficient, and so on). However, the same results are obtained even if (13)–(16) refer to the laboratory set of coordinates.

Substituting the left side of (13) in (16) in place of $(\partial f/\partial t)_{\text{coll}}$ and taking it into account that

$$\int \mathbf{v}\frac{\partial f}{\partial t}d^3v = \frac{1}{e}\frac{\partial \mathbf{J}_e}{\partial t} - e\int \mathbf{v}\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}], \nabla_{\mathbf{v}} f\right)d^3v = -en\mathbf{E} - \frac{1}{c}[\mathbf{J}_e\mathbf{H}], \tag{18}$$

where $\mathbf{J}_e$ is the electron current density in the co-moving coordinates, and also introducing the notation

$$R_i \equiv \int v_i(\mathbf{v}, \nabla_r f)d^3v = \sum_{k=1}^{3}\frac{\partial}{\partial x_k}\overline{v_i v_k}n, \qquad \overline{v_i v_k} \equiv \frac{1}{n}\int v_i v_k f d^3v, \tag{19}$$

we can rewrite the force (16) in the form

$$\mathbf{f}_2 = -\frac{m}{e}\frac{\partial \mathbf{J}_e}{\partial t} + \frac{m}{m^*}\left\{en\mathbf{E} + \frac{1}{c}[\mathbf{J}_e\mathbf{H}]\right\} - m\mathbf{R}. \tag{20}$$

The total drag force on the lattice by the conduction electrons, in accord with (12) and (20), is equal to

$$\mathbf{f} = \mathbf{f}_1 + \mathbf{f}_2 = en\mathbf{E} + \frac{1}{c}[\mathbf{J}_e\mathbf{H}] - \frac{m}{e}\frac{\partial \mathbf{J}_e}{\partial t} - m\mathbf{R}. \tag{21}$$

Here it is necessary to emphasize that the component with the factor m/m^* in the force (20) is due to electron scattering. But in the expression for the total drag force, it reduces exactly to the second component of the force (12), which does not have any relation to the electron scattering. This identity of terms cannot be obtained in the theories which take into account only the single force (16), for example, in[1]. In[2], the force (12) is also ignored but, in addition the factor m/m^* is erroneously lost in the calculation of the momentum of the conduction electrons (see[2], Eq. (3)). As a result of random compensation of these two errors, the expression (3) that is obtained turns out to be valid for the stationary ohmic current ($\mathbf{H} = 0$).

In this case, if $f(\mathbf{v})$ differs slightly from the quasi-equilibrium Fermi distribution function with some effective electron temperature T, one can replace $f(\mathbf{v})$ by the quasi-equilibrium distribution function in zeroth approximation in (19). As a result, we obtain

$$v_i v_k = \delta_{ik}\begin{cases} kT/m^* & \text{in the nondegenerate case} \\ \pi^{4/3}\hbar^2(3n)^{2/3}/5m^{*2} & \text{in the degenerate case} \end{cases} \tag{22}$$

If in the presence of a nonquantized magnetic field, we introduce the mobility tensor μ and the diffusion coefficient D, then the Einstein relation will have the form

$$D_{\mathbf{H}} = \mu_{\mathbf{H}}\begin{cases} kT/e & \text{in the nondegenerate case} \\ (n)^{2/3}\pi^{4/3}\hbar^2/3^{1/3}em^* & \text{in the degenerate case.} \end{cases} \tag{23}$$

In both cases, the last term in (21) can be written in the form

$$m\mathbf{R} = \frac{me}{m^*}\mu_H^{-1}D_H\nabla n = \frac{me}{m^*}\frac{D_0}{\mu_0}\nabla n, \tag{24}$$

inasmuch as it is seen from (23) that $\mu_{\mathbf{H}}^{-1}D_{\mathbf{H}}$ does not depend on **H** and consequently it has the same value as in the absence of the magnetic field.

Equation (21) is the general expression for the density of the drag force on the lattice from the conduction electrons. It is obtained without any assumptions on the form of the electron velocity distribution function and without approximations that are usually employed in the solution of the kinetic equation. Equation (24) is also obtained under the assumption that the distribution function is close to the Fermi value.

3. CONSIDERATION OF SPECIAL CASES AND A COMPARISON WITH THE RESULTS OF OTHER AUTHORS

Equation (21) can be simplified in two special cases:

1) If the time of free flight of the conduction electron τ does not depend on its velocity, and the electron scattering can also be regarded as elastic, then, as is known,[8]

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \frac{f_0(v) - f(v)}{\tau}, \tag{25}$$

where $f_0(\mathbf{v})$ is the spherically symmetric part of the distribution function.[1)] Substitution of this expression in (16) gives

$$\mathbf{f}_2 = m\mathbf{J}_e/e\tau, \tag{26}$$

which is identical with the value of (2). As a result, the total drag force, taking (12) into account, is equal to

$$\mathbf{f} = \mathbf{f}_1 + \mathbf{f}_2 = \left(1 - \frac{m}{m^*}\right)\left(en\mathbf{E} + \frac{1}{c}[\mathbf{J}_e\mathbf{H}]\right) + \frac{m}{m^*}\frac{\mathbf{J}_e}{\mu_0}. \tag{27}$$

Formula (21) contains the term $e^{-1}m\partial\mathbf{J}_e/\partial t$, whereas this term is not explicitly separated in (27). Nevertheless, both formulas are equivalent in the case of elastic scattering and when τ does not depend on the velocity. Both formulas, in particular, are valid for a nonstationary velocity distribution when it is necessary to take into account the first term in (13).

2) Equation (21) can be simplified if the external fields change sufficiently slowly with time and for each instantaneous value of these fields, a stationary distri-

[1)] $f_0(\mathbf{v})$ is the zeroth term of the expansion of $f(\mathbf{v})$ in spherical harmonics.

bution of the electron velocities $f(\mathbf{v})$ is established. This is the case when one can neglect the first term in (13), and the corresponding term $e^{-1}m\partial \mathbf{J}_e/\partial t$ in (21). The criterion of this case for periodically changing external fields is the condition $\omega\tau \ll 1$, where ω is the frequency of oscillation of the external field. In this quasistationary case, the current is determined, as is known, from the equation of electrical conductivity and diffusion:

$$\mathbf{J}_e = en\mu_H \left(\mathbf{E} - \frac{D_0}{\mu_0}\nabla \ln n\right). \tag{28}$$

The electron velocity distribution approaches the Maxwellian or Fermi value with an electron temperature T generally different from the lattice temperature. From (21) and (24) we obtain

$$\mathbf{f} = en\mathbf{E} + \frac{1}{c}[\mathbf{J}_e\mathbf{H}] - e\frac{m}{m^*}\frac{D_0}{\mu_0}\nabla n. \tag{29}$$

Expressions (21), (27), (29) for the drag force density on the lattice by the electrons are obtained in the traveling set of coordinates. In nonrelativistic theory, by neglecting terms of order $\dot{u}^2/c^2$ and $\dot{u}v/c^2$, we can assume the force density to be an invariant of a Galilean transformation, i.e., it will be of the same value in the laboratory set of coordinates. It is easy to see this in Eq. (21), where the sum of the first two components of the right-hand side is invariant and the sum of the last two components is invariant.

However, the situation is different with Eqs. (27) and (29), because they were obtained from (21) under the simplifying assumptions of noninvariant character. Thus, if the scattering is elastic in the co-moving set of coordinates, then it will be inelastic in the laboratory system. Therefore, Eqs. (25) and (27) are valid only in the co-moving system, which does not prevent the use of the resultant numerical value of f in any set of coordinates. Furthermore, if there is quasistationarity in the co-moving system, i.e., if $\partial f/\partial t = 0$, then there will not be such in the laboratory system, which can be established by differentiating (17) with respect to time. Therefore, Eqs. (28) and (29) are valid only in the co-moving set of coordinates.

We now compare Eq. (21), obtained by us, with Eqs. (4) and (5). We shall assume for simplicity below that $\mathbf{u} = \dot{\mathbf{u}} = 0$; here the co-moving and laboratory systems are identical. We consider the case of stationary diffusive current through the given region of the crystal in the absence of electric and magnetic fields. According to (4) and (5), $\mathbf{f}_G = \mathbf{f}_K = 0$ in this case. According to (21) and (24), f differs from zero and is equal to

$$\mathbf{f} = -e\frac{m}{m^*}\frac{D_0}{\mu_0}\nabla n = \frac{\mathbf{J}_e}{\mu_0}\frac{m}{m^*},$$

which is identical with $\mathbf{f}_H$ from (2).

In[4], the drag force on the lattice by the electrons is obtained from the law of conservation of momentum of the whole crystal. This momentum is composed of the momenta of the conduction electrons, the electromagnetic field, and the moving lattice (see[4], Eq. (3.7)). The advantage of such an approach is that the interaction of the electrons with the lattice is internal and one can ignore it when considering the change of the total momentum of the crystal with time. In particular, knowledge of the force (11) is not necessary.

However, the inadequacy of this method is that it gives only the lattice drag force, due to the electrons, integrated over the volume of the crystal, and not the density of this force. Knowledge of the integral force makes it possible to determine the force density only with accuracy to within a component of the form $\sum_k \partial\psi_{ik}/\partial x_k$, where $\psi_{ik}(\mathbf{r}, t)$ is an arbitrary tensor. Our component $m\mathbf{R}_i$ in (21), as is seen from (19), has again the same form. Therefore, it is lost in[4].[2)] For a convincing illustration of this, we consider the trivial case of free electrons which do not interact with the lattice ($W(\mathbf{r}) = 0$, $m^* = m$, $\tau = \infty$, $\mathbf{E} = 0$), and which move along circular trajectories in the magnetic field H. This can be, for example, motion along concentric circles which form a ring current, or another stationary motion, which forms a current density different from zero. In this case, $\mathbf{f}_G = \mathbf{f}_K = \mathbf{J}_e \times \mathbf{H}/c \neq 0$ follows from (4) and (5). Actually, however, $\mathbf{f} = 0$, since the conduction electrons, by assumption, do not interact at all with the lattice. Equation (21) in this case gives the true result, since the term $\mathbf{J}_e \times \mathbf{H}/c$ cancels exactly with $-m\mathbf{R}$.

In order to show this, we note that in the case under consideration, Eq. (13) reduces to the form

$$\mathbf{v}\nabla_r f + \frac{e}{m^*c}[\mathbf{v}\boldsymbol{H}]\nabla_v f = 0. \tag{31}$$

Multiplying this equation by v, integrating over velocity space and taking (19) into account, we obtain $\mathbf{R} = \mathbf{J}_e \times \mathbf{H}/m^*c$. Inasmuch as $m^* = m$ in the case considered, we obtain $\mathbf{f} = 0$ from (21).

Turning from the trivial example to real electrons interacting with the lattice, we consider the case of a strong magnetic field where $eH\tau/m^*c \gg 1$. In this case, we can neglect the force (16) in comparison with (12) and obtain

$$\mathbf{f} = \mathbf{f}_1 = \left(1 - \frac{m}{m^*}\right)\left(en\mathbf{E} + \frac{1}{c}[\mathbf{J}_e\mathbf{H}]\right). \tag{32}$$

This formula is valid even when u and $\partial f/\partial t$ are different from zero. It differs significantly from Eq. (4) employed in[3] in the case of a strong magnetic field.

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[2)] An attempt to determine ψ_{ik} from thermodynamic considerations [4] is not justified, since the considered systems are essentially nonequilibrium, and also because the forces should be determined before the thermodynamic functions. But even in the case of a limiting equilibrium of the electrons and electrical neutrality, account of the terms $\partial\psi_{ik}/\partial x_k$ in [4] did not lead to agreement with our results.