

QUANTUM PROPERTIES OF A MACROSCOPIC OSCILLATOR

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The behavior of a macroscopic mechanical oscillator having a long relaxation time, located in a gas thermostat or in a light beam, is considered. It is shown that under certain conditions such an oscillator behaves like a quantum system. Relations are obtained for the lifetime of the oscillator in the allowed energy levels. It is shown that the action of a directed light beam on the oscillator leads to a "heating" of the vibrational degrees of freedom. A theoretical limit is found for the Q of a macroscopic mechanical oscillator.

1. IN this article the physical conditions under which the behavior of a macroscopic oscillator is essentially determined by Planck's constant \hbar are considered. By convention one can therefore call the effects observed under such conditions macroscopic quantum effects.

At the present time there is interest in experiments which amount to the observation of a small force acting on a mechanical oscillator (verification of the principle of equivalence, searches for gravitational waves, etc.; see, for example, [1]). In order to enhance the resolution in such experiments it is necessary to decrease the coefficient of friction H_{mech} which couples the oscillator to the laboratory, i.e., it is necessary to increase the relaxation time τ^* . Under laboratory conditions relaxation times τ^* of the order of 10^8 to 10^9 sec have been obtained; it may well be possible to attain a level of 10^{11} to 10^{13} sec. Then a reasonable time $\hat{\tau}$ which may be expended on a measurement turns out to be much smaller than the relaxation time τ^* . As shown in [2], under such a condition ($\hat{\tau} \ll \tau^*$) the change of the oscillator's energy due to the action of a random force during the observation time $\hat{\tau}$ does not, with reliability $1 - \alpha$, exceed the value

$$\Delta E \leq \xi(\alpha, \beta) \kappa T \hat{\tau} / \tau^*; \quad E|_{\tau=0} \approx 0, \quad (1)$$

where κ is the Boltzmann constant, T is the temperature, and $\xi(\alpha, \beta)$ is a factor of the order of a few times unity, which depends on the confidence level $1 - \alpha$ and on errors of the second kind β .

Thus, for a classical oscillator it turns out to be possible to measure the "putting-in" by an external force of an amount of energy which is much smaller than the equilibrium value κT , and moreover the amount which can be measured is smaller the smaller the ratio $\hat{\tau} / \tau^*$ is. [1] It is also essential that in such a nonequilibrium case the experimenter has the possibility to reduce artificially (by means of external impulses), during a time interval substantially smaller than τ^* , the amplitude of the oscillator's vibrations to values close to zero. If the time for measuring the amplitude of the oscillations is $\hat{\tau}$, then the minimal oscillator energy which can be achieved is of the order of ΔE from relation (1). See [3] for information about "techniques" for damping an oscillator with a large τ^* .

However, as the energy of the oscillator tends to zero, the presence of discrete allowed energy levels

$E_n = \hbar \omega_0 (n + 1/2)$ may significantly manifest itself if the lifetime of the oscillator in these levels becomes larger than the observation time $\hat{\tau}$. In this case during the time of observation the oscillator's energy will remain unchanged (in contrast to a classical oscillator) in spite of the fact that a random force acts on it, namely, the impacts of the gas molecules in the thermostat.

Let us assume that a one-dimensional oscillator (m, ω_0) is found in a thermostat (a gas with a concentration N_0 at a temperature T). Let us determine under what relationships between $m, \omega_0, N_0,$ and T will the oscillator remain, with reliability close to unity, in the initial energy level (the zero or n-th level) during a given time interval $\hat{\tau}$. If a definite force $f(t)$ acts on the oscillator during the time interval $\hat{\tau}$, then the probability of a transition of the oscillator from the zero level to the n-th level or from the m-th level to the n-th is given by [4]

$$P_{0n} = \frac{e^{-y} y^n}{n!}; \quad P_{mn} = \frac{e^{-y} y^{m+n}}{m!n!} \left| \sum_{k=0}^{\min(m, n)} \frac{m!n!}{k!(m-k)!(n-k)!} (-y)^{-k} \right|^2; \\ y = \frac{1}{2m\hbar\omega_0} \left| \int_0^{\hat{\tau}} f(t) e^{i\omega_0 t} dt \right|^2. \quad (2)$$

In our problem the force $f(t)$ may be specified in the form of a random sequence of δ -shaped impulses

$$f(t) = \sum_{i=1}^A p_i \delta(t - t_i).$$

Here A denotes a random number of collisions of the gas molecules with the oscillator's mass during the time $\hat{\tau}$, this number being distributed according to Poisson's law; $p_i = 2 \mu v_{xi}$ is a random quantity which is distributed according to Maxwell's law; t_i is the arrival time of the i-th impulse.

The desired condition for the noninteraction of the oscillator with the thermostat is evidently: $P_{00} \geq 1 - \alpha, P_{nn} \geq 1 - \alpha$, where $1 - \alpha$ is the chosen level of confidence. One can show that with a level of confidence $1 - \gamma$ these inequalities are equivalent to the following:

$$\kappa T \frac{\tau}{\tau^*} \leq \frac{\alpha}{\ln(1/\gamma)} \hbar \omega_0 \quad (\text{for } n = 0), \\ n \kappa T \frac{\tau}{\tau^*} \leq \frac{\alpha}{\ln(1/\gamma)} \hbar \omega_0 \quad (\text{for } n \approx 1 \text{ or } n \gg 1), \quad (3)$$

where $T = \mu v_{xi}^2$ is the temperature of the thermostat, and $\tau^* = m(2\sqrt{2/\pi} S \mu^{1/2} (\kappa T)^{1/2} N_0)^{-1}$ is the relaxation

time of the oscillator. For $m = 1g$, $\omega_0 = 2\pi \text{sec}^{-1}$, $S = 1 \text{cm}^2$, and $p = 0.5 \times 10^{-12} \text{Torr}$ (the gas in the thermostat is helium) the lifetime of the oscillator in the zero level or in a level close to zero will be of the order of 100 sec with a level of confidence $(1 - \alpha)(1 - \gamma) = 0.9$.

In similar fashion one can solve the problem of the possible noninteraction of an oscillator with a system for the detection of small oscillations which utilizes a source of coherent radiation of power $W = Nh\nu$ (N is the number of photons emitted per second) and a Fabry-Perot resonator (the reflection coefficient of the mirror is denoted by R). In this case the random force acting on the oscillator will be the light pressure $F = 2Wc^{-1} \times (1 - R)^{-1}$, which fluctuates together with W . It turns out that under these conditions the lifetime of the oscillator in the n -th level satisfies the following inequality:

$$n \frac{2W h \nu}{m(1-R)^2 c^2} \hat{\tau} \leq \frac{\alpha}{\ln(1/\gamma)} \hbar \omega_0 \quad (4)$$

(under the assumption that the oscillator is located in a perfect vacuum).

It is clear that one can talk about the presence of discrete energy levels for the oscillator only when the lifetime in these levels, in any event, is not smaller than the period τ_0 of the eigenoscillations. Therefore one can regard the relations

$$n \kappa T \frac{\tau_0}{\tau^*} \leq \frac{\alpha}{\ln(1/\gamma)} \hbar \omega_0, \quad n \frac{2W h \nu}{m(1-R)^2 c^2} \tau_0 \leq \frac{\alpha}{\ln(1/\gamma)} \hbar \omega_0 \quad (5)$$

as the demarcation between classical and quantum behavior of the oscillator. If the conditions of the experiment are such that relations (5) are not fulfilled for any values of n , then one can regard the oscillator as classical for any attainable values of its energy. For example, the oscillator behaves like a classical oscillator if the power flux in the detection system is equal to $W = m\omega_0(1 - R)^2 \lambda_0 c (\pi \hat{\tau})^{-1}$ (see formula (4)). As is well-known, precisely such a value of the power flux is optimal in experiments with test bodies since here the greatest limiting sensitivity of the system "test body + detector" to the influence of a small external force (for more details, see [1]) is guaranteed. For smaller values of W the detector will not respond to vibrations of the oscillator during a time interval $\hat{\tau}$ satisfying relation (4); the light beam will be reflected just like from a fixed mirror with absorption coefficient $1 - R$. After the expiration of this time interval, photons with a Doppler-shifted frequency appear in the reflected beam, and these carry information about the oscillations of the quantum oscillator.

Resuming the discussion, one can conclude that it is possible to set up a macroscopic oscillator having a sufficiently long relaxation time τ^* under conditions such that its behavior will effectively be quantum-mechanical. The relaxation process for such an oscillator will be described by the relations obtained in [5].

2. In order to determine the maximal sensitivity in experiments with test bodies, the influence of the friction H_{mech} (thermal fluctuations) and the fluctuating effect of the detection system are usually considered separately. In an actual experiment both of these factors are present at the same time. This leads to the result that the vibrational degrees of freedom of the oscillator are "warmed up," i.e., in the equilibrium

state the energy of its vibrations becomes greater than κT . In fact, in this case the steady-state random force acting on the oscillator is the sum of the fluctuations of the light pressure and of the thermal fluctuations with mean square deviations given, respectively, by

$$\overline{F_1^2} = 8W h \nu c^{-2} \Delta f, \quad \overline{F_2^2} = 4\kappa T H_{\text{mech}} \Delta f,$$

which also leads to an increase of the average energy of the random oscillations. Comparing the sum of the mean square deviations $\overline{F_1^2} + \overline{F_2^2}$ with the general expression for the mean square deviation of a random force (the Nyquist theorem), one can easily see that the increase of the equilibrium temperature is given by

$$\Delta T = \frac{2W h \nu}{\kappa c^2 H_{\text{mech}}} \left(\text{if } H_{\text{mech}} \gg \frac{2W}{c^2} \right). \quad (6)$$

For $m = 1g$, $S = 1 \text{cm}^2$, $W = 10^7 \text{erg/sec}$, $\tau^* = 2m/H_{\text{mech}} = 10^{10} \text{sec}$ (the oscillator is in helium at a temperature $T = 4.2^\circ \text{K}$, $p = 10^{-8} \text{Torr}$), the increase of the temperature amounts to $\Delta T = 2^\circ \text{K}$.

3. The Q of a mechanical oscillator is higher the weaker the dissipative coupling of the oscillator to the laboratory. Theoretically the only nonremovable source of such coupling is thermal electromagnetic radiation; in the equilibrium case this is blackbody radiation. As is well-known, a mechanical oscillator placed in a light beam of intensity W experiences a friction $H_{\text{EM}} = 2W/c^2$ in this beam. [6] Hence, knowing the spectral density of the black-body radiation energy and its spatial distribution, one can then find the friction and that Q which a macroscopic mechanical oscillator placed in a perfectly evacuated thermostat having a temperature T possesses:

$$H_0 = \frac{2\sigma T^4 S}{c^2} = \frac{4\pi^5 \kappa^4 T^4 S}{15c^4 h^3}. \quad (7)$$

Here T is the temperature of the radiation (and of the oscillator), σ is the Stefan-Boltzmann constant, and S is the area. For $S = 1 \text{cm}^2$, $T = 4.2^\circ \text{K}$, and $\omega_0 = 2\pi \text{sec}^{-1}$, the time constant of the oscillator is equal to $1 \times 10^{21} \text{sec}$, and its Q is approximately 10^{21} . Relation (7) thus determines the theoretical limit for the Q of a macroscopic mechanical oscillator.

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