QUASILINEAR RELAXATION OF AN ELECTRON BEAM IN AN INHOMOGENEOUS BOUNDED PLASMA

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Submitted April 28, 1969

Zh. Eksp. Teor. Fiz. 57, 1401–1410 (October, 1969)

The influence of plasma inhomogeneities on the solution of the stationary boundary problem concerning the interaction between an electron beam and a plasma is investigated. It is shown that the inhomogeneity leads to two important effects. Firstly, if the plasma contains regions in which the density increases in the direction of motion of the beam, then quasilinear relaxation causes the appearance of accelerated electrons. Secondly, the quasilinear relaxation in an inhomogeneous plasma proceeds at a much slower rate than in a homogeneous one (in the absence of "trapped" oscillations).

INTRODUCTION

FOR a theoretical description of experiments on the stationary injection of weak electron beams into a plasma it is necessary to solve a boundary quasilinear problem. In the homogeneous plasma approximation such a problem was investigated in detail in the papers by Faĭnberg and Shapiro[1] and Tsytovich[2]. On the other hand, in[3] it was established that in a quasilinear problem with initial conditions if even very insignificant inhomogeneities of the density are taken into account this leads to a cardinal change in the nature of the quasilinear relaxation. Therefore, the problem arises of the influence of plasma inhomogeneities on the solution of the boundary problem. In the present communication we show that under definite conditions this influence turns out to be very significant.

For the sake of simplicity we assume that the parameters of the plasma and of the beam depend only on the one coordinate x directed along a strong constant magnetic field, and that the characteristic wavelength of the Langmuir oscillations induced by the beam is small compared to the scale of the plasma inhomogeneities and to the size of the region of quasilinear relaxation.

Under these conditions it is possible, firstly, to restrict oneself to the one-dimensional variant of the quasilinear theory and, secondly, to utilize the quasiclassical approximation for the Langmuir oscillations.

With respect to the properties of the beam we assume that its density n_b is small compared to the plasma density n, while its initial velocity v_0 considerably exceeds the thermal velocity of the plasma electrons v_T. We neglect the initial spread of the velocities in the beam.

Before we proceed to solve the problem formulated above we obtain some simple estimates which refer to the relaxation of an electron beam in a homogeneous plasma. In particular, we find the spread of the velocities in the beam when it leaves the plasma as a function of the density of the beam. The corresponding results will contain nothing that is in principle new compared with[1], but the qualitative derivation utilized by us will enable us to understand in a simpler manner the special features of relaxation in an inhomogeneous plasma.

It is clear that for sufficiently small values of the density of the beam the spread in the velocities Δv when the beam leaves the plasma is small in comparison with v_0, so that the characteristic increment in the beam instability γ can be estimated by means of the formula

$$\gamma \sim \frac{n_b}{n}\left(\frac{v_0}{\Delta v}\right)^2 \omega_p,$$

where $\omega_p = (4\pi ne^2/m)^{1/2}$ is the electronic plasma frequency.

The oscillations to which the beam gives rise propagate into the interior of the plasma with the group velocity $v_g \sim v_T^2/v_0$. If we denote by L the length of the region occupied by the plasma, then the time τ during which the oscillations are amplified by the beam[1)] can be estimated as L/v_g. During the time τ the energy of the oscillations must grow from the initial (thermal) level to a level which significantly exceeds the initial one (otherwise the oscillations could not react back on the beam). This condition can be written in the following manner:

$$\gamma\tau \sim \Lambda, \tag{1}$$

where Λ is the logarithm of the ratio of the final energy density of the oscillations to the initial one. Since this ratio is very large, then Λ depends weakly on the final level of oscillations, and usually one can assume that $\Lambda \sim 10$–20.

The spread of the velocities when the beam leaves the plasma is established just at such a level at which relation (1) is satisfied:

$$\gamma\tau \sim \frac{n_b}{n}\frac{\omega_p L}{v_0}\frac{v_0^2}{v_T^2}\frac{v_0^2}{\Delta v^2} \sim \Lambda,$$

whence we have

$$\frac{\Delta v}{v_0} \sim \left(\frac{\omega_p L}{v_0}\frac{n_b}{n}\frac{\mathscr{E}}{T}\frac{1}{\Lambda}\right)^{1/2}. \tag{2}$$

Here we have denoted by $\mathscr{E}$ and T respectively the energy of the electrons in the beam and the temperature

1) We make the natural assumption that at the boundary of the plasma the oscillations are absorbed and, therefore, we do not take into account the possibility of repeated traversal by the oscillation of the plasma interval. In the case when the boundaries of the plasma reflect oscillations perfectly, and the plasma density is homogeneous the problem concerning the relaxation was solved in [4].

of the electronics of the plasma. Of course the estimate so obtained is valid only under the condition $\Delta v/v_0 \ll 1$.

The relaxation process in the case under consideration has one characteristic property: it does not lead to the appearance of electrons with velocities $v > v_0$. Therefore, the estimate (2) obtained above for the spread of the velocities in the beam must be interpreted in the sense that the distribution function for the electron beam as it leaves the plasma differs from zero over a range of velocities $v_0 - \Delta v < v < v_0$.

From formula (2) it may be seen that as the density of the beam n_b is increased the spread of the velocities Δv increases and for

$$n_b \sim n_{b_0} = \Lambda n \frac{v_0}{\omega_p L}\frac{T}{\mathscr{E}} \tag{3}$$

becomes of order of v_0, i.e., the beam has time over the length L to relax to a plateau state:

$$f = \begin{cases} 2n_b/v_0, & v < v_0 \\ 0, & v > v_0 \end{cases} \tag{4}$$

where f is the distribution function for the electrons of the beam. But if the concentration of the beam becomes greater than the critical value n_{b_0} which is determined by relation (3), then the beam has time to go over into the plateau state (4) over a certain distance $l < L$, after which the relaxation comes to an end[2)]. In order to find l one should in formula (2) replace L by l, and set $\Delta v/v_0$ equal to unity. As a result of this we find that

$$l \sim \Lambda \frac{n}{n_b}\frac{T}{\mathscr{E}}\frac{v_0}{\omega_p} \sim L\frac{n_{b_0}}{n_b} < L. \tag{5}$$

We now proceed to investigate the relaxation of the beam in an inhomogeneous plasma. We consider three density profiles typical for experiments (cf., Fig. 1, a-c) where for concreteness we assume that the characteristic scale of the inhomogeneity is in order of magnitude equal to L—the size of the plasma interval.

In the case of a homogeneous plasma the stationary nature of the solution of the boundary problem is guaranteed by the fact that the generation of oscillations in the case of beam instability is compensated by their being carried in the direction of the beam[1]. A stationary solution is also possible in the cases shown in Fig. 1 a—c since in the case of such density profiles the oscillations are not "trapped" within the volume of the plasma. But if the function n(x) has sufficiently deep minima (Fig. 1 d), then the oscillations are "trapped" in the corresponding potential wells and the instability is not compensated by the displacement of the oscillations[3)]. In such a case the boundary problem has no stationary quasilinear solution.

If the characteristic dimension and the characteristic amplitude of the small-scale inhomogeneities in the density are respectively equal to a and Δn, then the condition for the absence of minima superimposed on a background of smoothly varying functions n(x) shown in Fig. 1 a—c will be given by $\Delta n/n \ll a/L$. In what follows we shall consider that this condition is satisfied.

[2)]Naturally, for $n_b > n_{b_0}$ the quantity Δv can no longer be determined by means of formula (2).

[3)]In particular this refers to those potential wells which are situated close to the point x = 0, where the beam has not yet been spread out in energy and the increment is maximal.

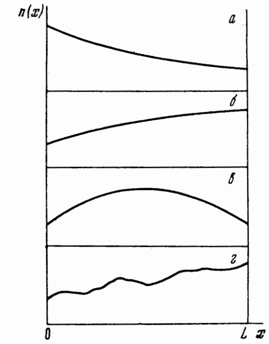

FIG. 1. Density profiles considered in this paper.

1. THE PLASMA DENSITY DECREASES MONOTONICALLY IN THE DIRECTION OF INJECTION OF THE BEAM

In this case as the oscillations generated by the beam are propagated in the direction $x > 0$ they enter a region of lower values of the density, and as a result of this their phase velocity diminishes. We find the change in the phase velocity of the oscillations v_{ph} over a distance Δx. In order to do this we make use of the dispersion relation for the Langmuir oscillations which has the form

$$\omega = \omega_p(x)\,[1 + {}^3/_2 (v_T/v_{ph})^2].$$

Since the frequency of a wave propagating in a medium with stationary parameters does not vary we can write

$$\Delta\omega = \left(1 + \frac{3}{2}\frac{v_T^2}{v_{ph}^2}\right)\frac{d\omega_p}{dx}\Delta x - 3\omega_p \frac{v_T^2}{v_{ph}^2}\frac{\Delta v_{ph}}{v_{ph}} = 0.$$

From this we can first of all see that $\Delta v_{ph} < 0$ for $d\omega_p/dx < 0$. Further, since $v_{ph} \sim v_0 \gg v_T$, $|d\omega_p/dx| \sim \omega_p/L$, then we have

$$|\Delta v_{ph}| \sim v_0 \frac{\Delta x}{L}\frac{v_0^2}{v_T^2}. \tag{6}$$

Let the velocity spread in the beam be Δv. A definite Langmuir oscillation will be excited by a beam with such a spread if the phase velocity of the oscillation lies in the range $v_0 - \Delta v < v_{ph} < v_0$. As this oscillation propagates into the region of lower values of the density its phase velocity will finally leave the velocity range indicated above, and the oscillation will stop growing. This will occur over a distance

$$\Delta x \sim L \frac{\Delta v}{v_0}\frac{v_T^2}{v_0^2} \tag{7}$$

and during a time

$$\tau \sim \frac{\Delta x}{v_g} \sim \frac{L}{v_0}\frac{\Delta v}{v_0}$$

(in order to obtain the estimate (7) one should substitute into formula (6) Δv in place of Δv_{ph}. From this it can be seen that the product $\gamma\tau$ which determines the effectiveness of the growth of oscillations is equal to

$$\frac{n_b}{n}\frac{v_0}{\Delta v}\frac{\omega_p L}{v_0}$$

and decreases as Δv is increased. It is clear that relaxation ceases for such values of Δv when this product becomes of order Λ i.e., when

$$\frac{\Delta v}{v_0} \sim \frac{1}{\Lambda}\frac{n_b}{n}\frac{\omega_p L}{v_0}. \tag{8}$$

A rough estimate of the size of the domain of relaxation l can be obtained by substituting this value of Δv into formula (7):

$$l \sim L\frac{\Delta v}{v_0}\frac{v_T^2}{v_0^2} \sim \frac{L}{\Lambda}\frac{n_b}{n}\frac{T}{\mathscr{E}}\frac{\omega_p L}{v_0}.$$

In this case, just as in a homogeneous plasma, relaxation does not lead to the appearance of electrons with velocities $v > v_0$ since as the oscillations propagate in the direction of lower values of the density their phase velocity can only decrease. However, there also exists an essentially new circumstance: in an inhomogeneous plasma relaxation proceeds much less effectively than in a homogeneous plasma. Indeed, even for $n_b \sim n_{b_0}$ when the relaxation in a homogeneous plasma already leads to the formation of a plateau, in an inhomogeneous plasma only a small velocity spread appears: $\Delta v/v_0 \sim T/E \ll 1$. A plateau is formed when n_b exceeds the new critical value:

$$n_{b_1} = \Lambda n\frac{v_0}{\omega_p L} \sim n_{b_0}\frac{\mathscr{E}}{T} \gg n_{b_0}.$$

As n_b increases further the form of the distribution function at the point where the beam leaves the plasma is not altered.

We note that for $n_b \gg n_{b_1}$ inhomogeneity exerts no influence on the relaxation process since in this case the length of the region of relaxation which is evaluated according to the homogeneous plasma model $l \sim Ln_{b_0}/n_b \ll LT/\mathscr{E}$ is so small that the phase velocity of the oscillations does not have time to be altered significantly over this length (cf. (6)).

From the estimates for l given above it follows that both for $n_b < n_{b_1}$ and for $n_b > n_{b_1}$ the dimension of the relaxation region (i.e., the dimension over which the distribution function attains its final state) is small compared to L. Oscillations generated in this region propagate freely through the plasma until they are damped either due to collisions or due to the Cerenkov absorption by plasma electrons.

2. THE PLASMA DENSITY INCREASES MONOTONICALLY IN THE DIRECTION OF THE INJECTION OF THE BEAM

In this case the phase velocity of the Langmuir oscillations generated by the beam increases as they propagate in the direction $x > 0$. The change in the phase velocity over a distance Δx can be determined as before by formula (6). In accordance with this the estimate (8) for the final spread of the velocities in the beam remains valid (for $n_b \ll n_{b_1}$). Only the meaning of the quantity Δv is altered: now when the beam leaves the plasma there appear in it not only decelerated ($v < v_0$), but also accelerated ($v > v_0$) electrons, and approximately in equal numbers. In other words, the distribution function for the beam at the point $x = L$ differs from zero in the velocity range $v_0 - \Delta v/2 < v < v_0 + \Delta v/2$.

The mechanism for the production of accelerated electrons manifests itself particularly clearly for $n_b \gg n_{b_1}$, and we consider this case in greater detail. As has been noted in the preceding section, under the condition $n_b \gg n_{b_1}$ the inhomogeneity of the plasma has no influence on the establishment of a plateau in the distribution function for the electrons of the beam since relaxation towards the plateau state (4) occurs over a very small distance $l \sim Ln_{b_0}/n_b \ll LT/\mathscr{E}$. In a homogeneous plasma or in a plasma with a density which decreases monotonically in the direction of the injection of the beam relaxation ceases at this point. But in the case of increasing density the oscillations generated by the beam in the region $x < l$ and having initially phase velocities $v_{ph} < v_0$ in propagating into the interior of the plasma increase their phase velocity and lead to an acceleration of the electrons.

The phase velocity of the Langmuir oscillations changes by an amount of the order of magnitude of itself over a distance $s \sim Lv_T^2/v_0^2$ (cf., (6)). It is just over this distance that the effect of acceleration begins to play a significant role. On the other hand, for $n_b \gg n_{b_1}$ we have the inequality $s \gg l$. Therefore, the whole relaxation process can be divided into two stages: at first, over a distance l, the establishment of a plateau occurs in the velocity range $0 < v < v_0$, and the distribution function acquires the form (4); then over a distance s, the phase velocity of the oscillations increases and acceleration of electrons takes place.

In order to investigate the relaxation process quantitatively we utilize the following system of quasilinear equations:

$$v\frac{\partial f}{\partial x} = \frac{\partial}{\partial v}D\frac{\partial f}{\partial v}, \tag{9}$$

$$\frac{v^2}{L^*}\frac{\partial W}{\partial v} + 3\frac{v_T^2}{v}\frac{\partial W}{\partial x} = \pi\omega_{p_0}\frac{v^2}{n_0}\frac{\partial f}{\partial v}W, \tag{10}$$

$$D = 4\pi^2e^2W/m^2v, \quad L^* = |d\ln\omega_p/dx|^{-1} \sim L,$$

$$\omega_{p_0} = \omega_p(0),\ n_0 = n(0),$$

where $f \equiv f(v, x)$ is the distribution function for the electrons of the beam, normalized to the total number of particles in the beam, while $W \equiv W(v, x)$ is the spectral density of the energy of the oscillations[4]. In writing Eq. (9) we have utilized the fact that the size of the domain of relaxation is small compared to the scale of the inhomogeneity.

As has been pointed out above, the first stage of the relaxation can be described in the approximation of a homogeneous plasma. In other words, in considering this stage one can omit the first term on the left hand side of (10). The resultant system of equations has a quasilinear integral:

$$vf - 3\frac{\omega_{p_0}}{m}v_{Te}^2\frac{\partial}{\partial v}\frac{W}{v^4} = n_b\delta(v - v_0). \tag{11}$$

We have taken into account the fact that at the left-hand boundary of the plasma the distribution function for the electrons has the form $f = n_b\delta(v - v_0)$ and that the noise level at the end of the first stage of relaxation is significantly higher than the thermal noise level. Substituting into Eq. (11) the function (4) for the function f(v) we obtain the spectrum of oscillations at the end of the first phase of relaxation:

[4]The quantity $W(v,x)\,\omega_p v^{-2}dvdx$ represents the energy of oscillations contained within the range of phase velocities $(v, v + dv)$ in a layer of plasma of thickness dx.

$$W = W_1(v) = \begin{cases} \frac{1}{3}\frac{n_b}{v_0}\frac{m}{\omega_{p_0}}\frac{v^6}{v_T^2}, & v < v_0 \\ 0, & v > v_0 \end{cases}. \quad (12)$$

The further evolution of the functions f and W is described by the system (9) and (10) with the boundary conditions

$$f|_{x=0} = f_1(v) = \begin{cases} 2n_b/v_0 & \text{for } v < v_0 \\ 0 & \text{for } v > v_0 \end{cases}, \quad (13)$$

$$W|_{x=0} = W_1(v). \quad (14)$$

We specify the boundary conditions at the point x = 0, and not at the point $x \sim l$, since l is much smaller than s—the characteristic scale for the second stage of the relaxation.

We make estimates of the left hand side and the right hand side of (9):

$$v\frac{\partial f}{\partial x} \sim v_0\frac{f}{s}, \quad \frac{\partial}{\partial v}D\frac{\partial f}{\partial v} \sim D\frac{f}{v_0^2} \sim \omega_p\frac{n_b}{n}\frac{v_0^2}{v_T^2}f.$$

We have taken into account Eq. (12) in accordance with which $W \sim mn_b v_0^5/\omega_p v_T^2$. From the estimates given above it seems to follow that for $n_b \gg n_{b_1}$ the left hand side of (9) is much smaller than the right hand side. But in actual fact this means that the distribution function is near to the plateau state (i.e., $|\partial f/\partial v| \ll f/v_0$) over the whole range of velocities where the diffusion coefficient D(v, x) differs from zero (in other words, in the domain $0 < v < u(x)$, where by u(x) we have denoted the maximum phase velocity of oscillations at the point x):

$$f(v, x) = \begin{cases} 2n_b v_0/u^2(x), & v < u(x) \\ 0, & v > u(x) \end{cases} \quad (15)$$

The normalization of the distribution function is obtained from the condition of conservation of the flux of electrons.

Since at the point x = 0 the maximum phase velocity of the oscillations is equal to v_0, then we have for u(x)

$$u(x) = v_0\left(1 - \frac{2}{3}\frac{x}{L^*}\frac{v_0^2}{v_T^2}\right)^{-1/2}. \quad (16)$$

We shall see below that this formula breaks down for

$$x > x_0 = \frac{3}{2}L^*\frac{v_T^2}{v_0^2}\left(1 - \frac{1}{\sqrt{3}}\right), \quad (16')$$

so that the problem of the divergence of u(x) at the point $x = 3L^* v_T^2/2v_0^2$ does not arise.

Since the coefficient quasilinear diffusion is great albeit not infinite, the distribution function in the domain $0 < v < u(x)$ will have a nonvanishing derivative with respect to v, i.e., it will deviate somewhat from the plateau. The value of $\partial f/\partial v$ can be obtained by substituting (15) into the left hand side of (9). As a result we obtain that

$$D\frac{\partial f}{\partial v} = \frac{4\pi^2 e^2}{m^2 v}W\frac{\partial f}{\partial v} = -\frac{2n_b v_0 v^2}{u^3}\frac{du}{dx}. \quad (17)$$

We see that $\partial f/\partial v < 0$, i.e., the oscillations are damped as x increases. Determining from (17) the quantity $W\partial f/\partial v$ and substituting it into the right hand side of (10), we obtain the equation for the spectra function W(v, x):

$$\frac{v^2}{L^*}\frac{\partial W}{\partial v} + \frac{3v_T^2}{v}\frac{\partial W}{\partial x} = -2\frac{mn_b v_0^2}{\omega_{p_0}}\frac{v^5}{u^3}\frac{du}{\partial x}, \; v < u(x). \quad (18)$$

In order to solve (17) it is convenient to go over from the variable v to the variable

$$V = v\left(1 + \frac{2}{3}\frac{x}{L^*}\frac{v^2}{v_T^2}\right)^{-1/2},$$

which represents the phase velocity of a definite Langmuir oscillation at the point x = 0. In this case (18) assumes the form

$$\frac{\partial W}{\partial x} = \frac{mn_b v_0}{3\omega_{p_0} v_T^2}v^6(V, x)\frac{d}{dx}\frac{1}{u^2(x)}, \quad (19)$$

where

$$v(V, x) = V\left(1 - \frac{2}{3}\frac{x}{L^*}\frac{V^2}{v_T^2}\right)^{-1/2},$$

while W is now regarded as a function of V and x. The solution of (19) which satisfies the boundary condition (14) can be obtained in an elementary matter:

$$W(V, x) = \frac{mn_b V^6}{3v_T^2\omega_{p_0}v_0}\left[1 + \frac{v_0^6}{V^6}\int_0^x dx' v^6(V, x')\frac{d}{dx'}\frac{1}{u^2(x')}\right]$$
$$= \frac{mn_b V^6}{3v_T^2\omega_{p_0}v_0}\left[1 + v_0^2\int_0^x dx'\left(1 - \frac{2}{3}\frac{x'}{L^*}\frac{V^2}{v_T^2}\right)^{-3}\frac{d}{dx'}\frac{1}{u^2(x')}\right]. \quad (20)$$

Under the condition (16) it follows from (20) that

$$W(V, x) = \frac{mn_b V^6}{3v_T^2\omega_{p_0}v_0}\left[1 + \frac{v_0^2}{2V^2} - \frac{v_0^2}{2V^2}\left(1 - \frac{2}{3}\frac{x}{L^*}\frac{V^2}{v_T^2}\right)^{-2}\right].$$

It can be easily seen that the energy of those Langmuir oscillations which at the point x = 0 had the phase velocity $V = v_0$,

$$W(V, x)|_{V=v_0} = \frac{mn_b v_0^5}{2v_T\omega_{p_0}}\left[1 - \frac{1}{3}\left(1 - \frac{2}{3}\frac{x}{L^*}\frac{v_0^2}{v_T^2}\right)\right]^{-2},$$

decreases, and at $x = x_0$ (cf., (16′)) vanishes. With increasing penetration into the interior of the plasma there is a progressive damping to zero of oscillations with smaller and smaller values of the initial phase velocity V, and at each point $x > x_0$ only those oscillations are present the initial phase velocity of which satisfies the inequality $V < V(x)$, where the function V(x) is determined by the equation

$$1 + v_0^2\int_0^x dx'\left(1 - \frac{2}{3}\frac{x'}{L^*}\frac{V^2(x)}{v_T^2}\right)^{-3}\frac{d}{dx'}\frac{1}{u^2(x')} = 0. \quad (21)$$

(This equation is obtained from (20)).

The position of the boundary of the plateau u = u(x) in the region $x > x_0$ is expressed in terms of V(x):

$$u(x) = V(x)\left(1 - \frac{2}{3}\frac{x}{L^*}\frac{V^2(x)}{v_T^2}\right)^{-1/2}. \quad (22)$$

Formula (16) is now, naturally, inapplicable: it could be used only as long as the energy of the oscillations with the initial phase velocity $V = v_0$ was different from zero.

From (21) and (22) we can obtain the equation for the function u(x) which in terms of the dimensionless variables $y = v_0^2/u^2(x)$, $\xi = 2xv_0^2/3L^* v_T^2$ has the form

$$1 + [\xi + y(\xi)]^3\int_0^\xi \frac{d\xi'}{[\xi - \xi' + y(\xi)]^3}\frac{dy(\xi')}{d\xi'} = 0. \quad (23)$$

We are unable to obtain an exact solution of this integral equation, but the asymptotic value of u(x) for $x \to \infty$

$$u_\infty = \lim_{x\to\infty} u(x),$$

can be found very simply, even without making use of (23). In order to do this we must utilize the fact that the sum of the energy fluxes for the particles and the oscillations does not depend upon x and is equal to $mn_b v_0^3/2$. The end of the relaxation process corresponds to a complete damping of the oscillations and the vanishing of their energy flux. On the other hand, the energy flux for the particles is given by

$$\frac{m}{2}\int_0^{u(x)} v^3 f(v,x)\,dv = \frac{mn_b v_0 u^2(x)}{4}$$

(cf., (15)). Therefore we can write that

$$\tfrac{1}{4} mn_b v_0 u_\infty^2 = \tfrac{1}{2} mn_b v_0^3,$$

i.e., $u_\infty = v_0\sqrt{2}$. Naturally, this result can also be obtained from (23) but by means of a lengthier argument.

It should be noted here that in actual fact the decrease in the energy of the oscillations finally leads to the fact that the left hand side and the right hand side of (9), formerly estimated as $v_0 f/s$ and Df/v_0^2 become of the same order of magnitude and the "plateau approximation" breaks down. This occurs for

$$W \sim mn_b v_0^2 \frac{n}{n_b}\frac{v_0^2}{v_T^2}\frac{v_0^2}{L\omega_p^2}.$$

The energy flux density corresponding to this value of W which in order of magnitude is equal to $mn_b v_0^3 n v_0/n_b L\omega_p$ is small compared to the energy flux density for the particles. Thus, the "plateau approximation" breaks down when the energy of oscillations becomes so small that the oscillations in practice cease to exert an influence on the distribution function for the electrons. Consequently, one can use the "plateau approximation" to the very end of the second stage of the relaxation.

Summarizing the above one can assert that for $n_b \gg n_{b_1}$ relaxation proceeds in two stages: at first the beam excites oscillations, transfers its energy to them and at the same time relaxes towards the state (4); then the oscillations are again absorbed by the beam and it arrives at the state

$$f = \begin{cases} n_b/v_0, & v < v_0\sqrt{2} \\ 0\quad, & v > v_0\sqrt{2} \end{cases}. \qquad (24)$$

3. THE DENSITY HAS A MAXIMUM WITHIN THE PLASMA INTERVAL

In this case for low densities of the beam relaxation occurs near the position of the maximum since here the plasma density varies most slowly. In exactly the same way as it was done above one can show that for sufficiently small values of n_b the spread of the velocities is determined by the formula

$$\frac{\Delta v}{v_0} \sim \left(\frac{1}{\Lambda}\frac{n_b}{n}\frac{\omega_p L}{v_T}\right)^{2/3}. \qquad (25)$$

As should have been expected, the spread of the velocities increases with increasing n_b more rapidly than in a plasma with a monotonically varying density, but slower than in a plasma with a homogeneous density. Since to the left of the maximum the plasma density is an increasing function of x relaxation leads to the appearance both of decelerated as well as accelerated electrons, i.e., the distribution function for the electron beam when it leaves the plasma turns out to be different from zero in the velocity interval $v_0 - \Delta v/2 \lesssim v \lesssim v_0 + \Delta v/2$.

Relaxation occurs within a region of width

$$l \sim L\left(\frac{1}{\Lambda}\frac{\omega_p L}{v_0}\frac{n_b}{n}\frac{v_T^2}{v_0^2}\right)^{1/3} \qquad (26)$$

near the point where the plasma density has a maximum.

Formulas (25) and (26) are valid for such values of n_b when the spread Δv determined by means of the first of these two formulas is small compared to v_0, i.e., for

$$n_b \lesssim n_{b_2} = \Lambda n \frac{v_T}{\omega_p L} = \frac{v_T}{v_0} n_{b_1} = \frac{v_0}{v_T} n_{b_0}.$$

As the density increases further the velocity spread increases, and the domain of relaxation gradually "slips" towards the left hand boundary of the plasma. Finally, when n_b becomes of the order of n_{b_1} relaxation is completed in a region of width of order LT/E near the origin. The form of the distribution function in this case is determined by the relation (24).

4. DISCUSSION OF RESULTS

The estimates obtained in the present paper are qualitatively illustrated in Fig. 2. From the figure it can be seen that the inhomogeneity of the plasma leads to two significant effects. Firstly, when there exist in the plasma regions with a positive derivative of the density relaxation gives rise to the appearance of accelerated electrons. Secondly, the quasilinear relaxation in an inhomogeneous plasma proceeds at a considerably slower rate than in a homogeneous plasma ($n_{b_1} \gg n_{b_2} \gg n_{b_0}$). The last assertion refers, however, only to the case when "trapped" oscillations are absent.

The role played by the scattering of the Langmuir oscillations by the electrons of the plasma is not a significant one in the problem under consideration. Indeed, the oscillations are scattered only by electrons with very low velocities: $|v| \le v_T^2/v_0 \sim v_T(T/E)^{1/2}$. In the case when the spectrum of oscillations is one-dimensional the effect of scattering leads to the establishment of a plateau in the distribution function for the electrons of the plasma (in the domain of the velocities $|v| \lesssim v_T(T/\mathscr{E})^{1/2}$). This requires a very small energy (in a unit volume $nT(T/\mathscr{E})^{5/2}$). Therefore, after a certain time after the beam is switched on a plateau is established in the distribution function of the electrons of the plasma, and after this scattering stops completely (if we do not take pair collisions into account).

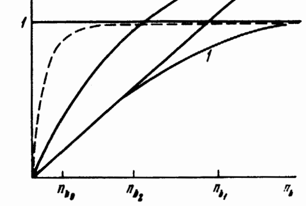

FIG. 2. Dependence of the velocity spread in the beam when it leaves the plasma on the beam density: 1 – the density decreases monotonically, 2 – the density increases monotonically, 3 – the density has a maximum in the central portion of the plasma interval. The dotted line represents the curve corresponding to a homogeneous plasma.

The polarization electric field E ~ T/eL which exists in an inhomogeneous plasma is also not significant, since a change in the phase velocity of the oscillations due to the inhomogeneity is much greater than the change of the velocity of the electrons of the beam in this field.

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Translated by G. Volkoff
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