THE DE HAAS–VAN ALPHEN EFFECT IN THIN METALLIC FILMS

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Formulas defining the magnetic-film energy levels of a conduction electron in an oblique field are derived for weak magnetic field strengths H, when the Larmor radius is much greater than the film thickness. The oscillating part of the thermodynamic potential Ω and the magnetic moment in weak oblique fields are calculated. It is shown that the oscillation period is defined by the area of the Fermi surface projections onto a plane parallel to the film. In contrast with oscillations in the bulky metal, a polynomial $P_n(H)$ appears in the oscillation phase in films. The coefficients of the polynomial are determined by the shape of the Fermi surface near the line with zero projection of the velocity on the normal to the film. The region of sharp transition from magnetic-film oscillations to the ordinary de Haas—van Alphen oscillations is analyzed.

INTRODUCTION

THE de Haas-van Alphen effect in metallic films has been studied by Kosevich and I. Lifshitz for an arbitrary dispersion law.[1] For the case of a parallel magnetic field, they computed the quasiclassical energy levels of the conduction electrons, and investigated the dependence of the oscillating part of the magnetic moment on the magnetic field and on the film dimensions. The oscillations of the thermodynamic quantities were studied in[2] in the range of weak fields, when the observation of the quantum effect of the dimensions is possible without changing the film thickness.

In the present work, the de Haas–van Alphen effect is studied in films in an oblique magnetic field. Formulas are obtained which define the magnetic-film energy levels of an electron with an arbitrary dispersion law $\mathscr{E}(\mathbf{p})$ in the weak-field region

$$H \ll H_L, \tag{1}$$

where $H_L = cp_F/eL$ is the field intensity at which the Larmor radius is equal to the film thickness L.

It is shown that in the range of fields (1), the oscillating part of the magnetic moment is perpendicular to the film for any angle of inclination of H, excluding parallel fields. The period of the oscillations is determined by the area of the Fermi surface projection onto a plane parallel to the film. In the oscillation phase, along with a term inversely proportional to H, there is, in contrast to oscillations in the bulky metal, a polynomial $P_n(H)$ whose coefficients are determined by the shape of the Fermi surface near the line with zero projection of the velocity on the normal to the film.

In the range of fields $H \sim H_L$, the formulas obtained for the energy levels lose their applicability, and further analysis is carried out for the special case of a quadratic isotropic dispersion law and a perpendicular magnetic field. This case was previously considered by Gurevich and Shik.[3] It is shown in the present paper that in fields $H < \pi H_L$, in addition to the ordinary de Haas–van Alphen oscillations, oscillations appear with different periods, and this guarantees a sharp transition in the vicinity of $H = \pi H_L$ at not very low temperatures, when the first harmonics are significant. In the considered region of magnetic fields and temperatures, the inhomogeneity of the magnetic field can be neglected.

In the experimental study of oscillations in weak fields, rather thin films are required. For films of thickness $L \sim 10^{-6}$ cm, fields at which the de Haas-van Alphen effect is observed satisfy condition (1) with $H_L \sim 10^5$ Oe. For thicker films, smaller fields and correspondingly lower temperatures are required. Measurements of the period and phase of the oscillations in the given case allow us to obtain additional information on the Fermi surface. One can find the area of the Fermi surface projection and certain integral quantities which characterize the shape of the Fermi surface in the vicinity of the line with $v_z = 0$. Moreover, by determining the area of the Fermi surface projections and the corresponding effective mass from measurements of the period and amplitude of the oscillations according to the formulas in[4], one can find the shift in the Fermi level caused by the boundaries of the specimen, and the surface part of the number and density of states with energy equal to the Fermi energy.

MAGNETIC-FILM ENERGY LEVELS OF THE CONDUCTION ELECTRONS IN WEAK FIELDS

The quantum energy levels can be found from a solution of the effective Schrödinger equation for the conduction electrons[5] in the film with a magnetic field

$$\hat{\mathscr{H}}\Psi(x, y, z) = \varepsilon\Psi(x, y, z), \quad \Psi(x, y, 0) = \Psi(x, y, L) = 0 \tag{2}$$

with the Hamiltonian $\hat{\mathscr{H}}$, which is symmetrized[1)] with accuracy up to terms of order $(\hbar eH/c)^2$

$$\begin{aligned}\hat{\mathscr{H}} = \mathscr{E}\Big(\hat{p}_x, \hat{p}_y - x\frac{eH}{c}\cos\theta, \hat{p}_z + x\frac{eH}{c}\sin\theta\Big) \\ + i\frac{\hbar eH}{2c}\Big(\cos\theta\frac{\partial^2}{\partial p_x\partial p_y} - \sin\theta\frac{\partial^2}{\partial p_x\partial p_z}\Big) \\ \times\mathscr{E}\Big(\hat{p}_x, \hat{p}_y - x\frac{eH}{c}\cos\theta, \hat{p}_z + x\frac{eH}{c}\sin\theta\Big),\end{aligned} \tag{3}$$

[1)]The necessity of symmetrization was shown in [6].

where $\hat{\mathbf{p}} = i\hbar\partial/\partial\mathbf{r}$ and the operator $\hat{p}_x$ no longer acts on x in the argument of the function $\mathscr{E}$, acting at the same time as the ordinary differentiation operator relative to the functions to which the Hamiltonian $\hat{\mathscr{H}}$ is applied; θ is the angle between the field **H** and the z axis, which is directed along the normal to the film; the x axis is perpendicular to **H**. It follows from (2) and (3) that p_y = const.

Limiting ourselves to the quasiclassical approximation in the magnetic field, we seek the solution of Eq. (2) in the form

$$\Psi(\mathbf{r}) = \exp(i\hbar^{-1}p_y y + i\hbar^{-1}\sigma(x))\varphi(z, x) \qquad (3')$$

with the boundary condition

$$\varphi(0, x) = \varphi(L, x) = 0. \qquad (3'')$$

To obtain the energy levels, it is sufficient to find the first two terms in the expansion

$$\sigma(x) = \int \mathscr{P}_x(x)\,dx + \frac{\hbar}{i}\sigma^{(1)}(x) + \ldots \qquad (4)$$

In first approximation in quasi-classical form, we have, from (2), (3) and (3′),

$$\left[\mathscr{E}\left(\mathscr{P}_x(x), \mathscr{P}_y, \hat{p}_z + x\frac{eH}{c}\sin\theta\right) - \varepsilon\right]\varphi(z, x) = 0, \qquad (5)$$

where $\mathscr{P}_y = p_y - x(eH/c)\cos\theta$.

A solution of Eq. (5) satisfying the boundary condition (3″) is

$$\varphi = \exp(i\hbar^{-1}p_z^{(1)}z) - \exp(i\hbar^{-1}p_z^{(2)}z), \qquad (5')$$

with $p_z^{(k)} = \mathscr{P}_z^{(k)} - x(eH/c)\sin\theta$, $k = 1, 2$, where the functions $\mathscr{P}_z^{(k)}$ satisfy the relations

$$\begin{gathered} |\mathscr{P}_z^{(1)}(\mathscr{P}_x, \mathscr{P}_y, \varepsilon) - \mathscr{P}_z^{(2)}(\mathscr{P}_x, \mathscr{P}_y, \varepsilon)| = 2\pi\hbar n/L, \\ \mathscr{E}(\mathscr{P}_x, \mathscr{P}_y, \mathscr{P}_z^{(1)}) = \mathscr{E}(\mathscr{P}_x, \mathscr{P}_y, \mathscr{P}_z^{(2)}) = \varepsilon, \\ n = 1, 2, 3, \ldots \end{gathered} \qquad (6)$$

Equations (6) determine the double-valued function $\mathscr{P}_x(\mathscr{P}_y)$ which describes a certain closed curve in the $(\mathscr{P}_x, \mathscr{P}_y)$ plane, a curve which is isochordic,[2] at the points of which the given constant energy surface has a chord equal to $2\pi\hbar n/L$. We denote the upper part of the curve (Fig. 1) by $\mathscr{P}_{x1}(\mathscr{P}_y)$ and the lower by $\mathscr{P}_{x2}(\mathscr{P}_y)$.

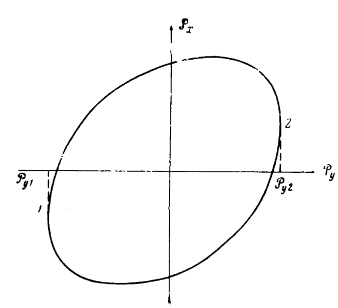

FIG. 1.

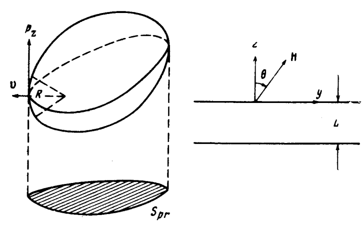

FIG. 2.

In the second quasiclassical approximation, we succeeded in obtaining an explicit equation for $\sigma^{(1)}$, without making any assumptions on the dispersion law, only in the vicinity of the line on the constant energy surface with $v_z = 0$ (Fig. 2):

$$\sigma^{(1)}(x) = -\frac{1}{2}\ln\left|\frac{\partial}{\partial\mathscr{P}_x}\mathscr{E}(\mathscr{P}_x, \mathscr{P}_y, \mathscr{P}_z)\right| + \ln C, \qquad (7)$$

where C is the constant of integration. In the next section, it will be shown that the principal contribution to the oscillations in the weak field region (1) is made by the energy levels in the neighborhood of the line with $v_z = 0$.

Joining together the solution in the classically allowed region $\mathscr{P}_{y1} < \mathscr{P}_y < \mathscr{P}_{y2}$ (see Fig. 1) with a solution in the classically forbidden region $\mathscr{P}_y < \mathscr{P}_{y1}$ and $\mathscr{P}_y > \mathscr{P}_{y2}$, we obtain

$$\begin{aligned} \Psi(\mathbf{r}) = C\exp\left(\frac{i}{\hbar}p_y y + \frac{i}{2\hbar}(p_z^{(1)} + p_z^{(2)})z\right)\sin\frac{\pi n z}{L} \\ \times\left[|v_{x1}|^{-1/2}\exp\left(-\frac{ic}{\hbar eH\cos\theta}\int_{\mathscr{P}_{y1}}^{\mathscr{P}_y}\mathscr{P}_{x1}(\xi)\,d\xi + \frac{i\pi}{4}\right)\right. \\ \left. + |v_{x2}|^{-1/2}\exp\left(-\frac{ic}{\hbar eH\cos\theta}\int_{\mathscr{P}_{y1}}^{\mathscr{P}_y}\mathscr{P}_{x2}(\xi)\,d\xi - \frac{i\pi}{4}\right)\right], \end{aligned} \qquad (8)$$

where

$$\begin{gathered} v_{xk} = \frac{\partial}{\partial p_x}\mathscr{E}(\mathscr{P}_{xk}, \mathscr{P}_y, \mathscr{P}_z), \quad k = 1, 2; \\ \exp\left[\frac{ic}{\hbar eH\cos\theta}\int_{\mathscr{P}_{y1}}^{\mathscr{P}_{y2}}(\mathscr{P}_{x1} - \mathscr{P}_{x2})\,d\mathscr{P}_y - i\pi\right] = 1. \end{gathered} \qquad (9)$$

The wave function Ψ determined by Eq. (8) is a solution of Eq. (2) for condition (1) since in this case, one can neglect the derivative of φ with respect to x.

The quantum energy levels are determined by the implicit relation

$$S\left(\varepsilon, \frac{2\pi\hbar n}{L}\right) \equiv \int_{\mathscr{P}_{y1}}^{\mathscr{P}_{y2}}(\mathscr{P}_{x1} - \mathscr{P}_{x2})\,d\mathscr{P}_y = \frac{2\pi\hbar\,eH\cos\theta}{c}(\nu + \gamma), \qquad (10)$$

where $S(\epsilon, d)$ is the area bounded by the isochord corresponding to chord length d.

Near the line of points with $v_z = 0$ on the constant energy surface, $\gamma = 1/2$ and Eq. (10) is identical with (9). In the general case, in the determination of an explicit equation for γ, it is necessary to compute $\sigma^{(1)}$ in the expansion (4).

In the case of several pairs of roots in Eq. (6), the roots that satisfy the condition of reflection of the quasiparticle from the potential wall[7,4] correspond to the given isochords, so that the roots of the corresponding pair are divided by a region in which the kinetic energy of the quasiparticle is less than some fixed value. Far from the singular points (such as points of self-intersection) the areas S_i, bounded by the corresponding isochords, are quantized according to Eq. (10).

The quantization condition (10) determines the energy levels $\epsilon_{n,\nu+\gamma}$ in the quasiclassical approximation in the magnetic field $cS/2\pi\hbar eH\cos\theta \gg 1$ in the weak-field region $H \ll H_L$ for an arbitrary anisotropic constant energy surface.[2)] The quasiclassical condition is violated near the chord maximum d_{max} at which $S(\epsilon, d_{max}) = 0$.

In the case of a quadratic isotropic dispersion law, $\gamma = 1/2$, $S(\epsilon, d) = \pi(2m\epsilon - d^2/4)$, and Eq. (10) is identical with the well-known expression

$$\varepsilon_{n,\nu+1/2} = \frac{\hbar eH}{mc}\left(\nu + \frac{1}{2}\right) + \frac{\pi^2\hbar^2 n^2}{2mL^2}, \tag{11}$$

which determines the energy levels of an electron in a magnetic field perpendicular to the film.

In a parallel field ($H_z = 0$), the quantization is determined by the Kosevich-Lifshitz formula.[1] In this case the energy spectrum is quasi-discrete and is characterized by a magnetic-film quantum number n and two continuously changing parameters p_x and p_y. In the transition from parallel to oblique fields ($H_z \neq 0$), the character of the energy spectrum changes materially. As soon as H_z becomes different from zero, "compression" of the levels takes place and the quasi-discrete spectrum is converted to a discrete one with energy levels that are degenerate in p_y. The distance between the levels is of the order of $\Delta\epsilon \sim \hbar eH_z/mc$, and for $T \lesssim \Delta\epsilon$ the resulting discreteness of the spectrum appears essentially as oscillations of thermodynamic quantities.

A sharp change in the spectrum leads to a very significant dependence of the oscillating terms on the angle θ, which defines the direction of the field **H**. In the angular range $\theta \lesssim \theta_0$, it is necessary to take into account the discrete character of the spectrum, while for $\pi/2 - \theta \ll \theta_0$ the discrete character of the spectrum can be neglected. The angle θ_0 satisfies the condition $\cos\theta_0 \sim 2\pi^2 cmT/e\hbar H$. In taking account of scattering by impurities, the temperature T should be replaced by the effective temperature[10] $T + \hbar/\tau$, where τ is a quantity of the order of the time of free flight of the electron.

A characteristic example of such an angular dependence is the sharp change in the direction of the oscillating part of the magnetic moment M_{osc} in the transition from parallel fields (in this case, M_{osc} is perpendicular to the film; see the next Section).

OSCILLATIONS OF THE MAGNETIC MOMENT

According to the formulas of statistical physics, we have the following expression for the thermodynamic potential Ω:

$$\Omega = -T\frac{VeH\cos\theta}{2\pi\hbar cL}\sum_{s=1}^{2}\sum_{n=1}^{\infty}\sum_{\nu=0}^{\infty}\ln\left(1 + \exp\frac{\zeta_s - \varepsilon_{n,\nu+\gamma}}{T}\right), \tag{12}$$

$$\zeta_s = \zeta + (-1)^s\frac{e\hbar H}{2cm_0}, \tag{12'}$$

where the energy levels $\epsilon_{n,\nu+\gamma}$ are determined by Eq. (10); m_0 is the mass of the free electron.

For calculation of the oscillations, we transform the sum in (12) by the Poisson summation formula.

Using (10) and setting $\gamma = 1/2$ for simplicity, we obtain from (12)

$$\Omega = -\frac{TV}{(2\pi\hbar)^3}\sum_{s=1}^{2}\int_0^{\infty}d\rho\int_{S(\varepsilon,\rho)\geqslant 0}d\varepsilon\ln\left(1 + \exp\frac{\zeta_s - \varepsilon}{T}\right)\frac{\partial S(\varepsilon,\rho)}{\partial\varepsilon}$$
$$\times\left[1 - \frac{\pi\hbar}{L}\delta(\rho) + 2\sum_{k=1}^{\infty}\cos\left(\frac{Lk}{\hbar}\rho\right)\right]\left[1 + 2\sum_{l=1}^{\infty}(-1)^l\cos\left(\frac{lcS(\varepsilon,\rho)}{\hbar eH\cos\theta}\right)\right] \tag{13}$$

Integrating (13) over ϵ by parts and then computing the integrals asymptotically for $cS/\hbar eH\cos\theta \gg 1$, we get as a result $\Omega = \Omega_0 + \Omega_{osc}$, where Ω_0 is a function changing smoothly with changing magnetic field, $\Omega_{osc} = \Omega_{osc}^{(1)} + \Omega_{osc}^{(2)}$ is the oscillating part of the thermodynamic potential. For $\Omega_{osc}^{(1)}$, we have the following expression

$$\Omega^{(1)} = \frac{TV}{2\pi^2\hbar^3\sqrt{|\alpha|}}\left(\frac{\hbar eH\cos\theta}{c}\right)^{3/2}\sum_{l=1}^{\infty}\frac{(-1)^l}{l^{3/2}}\cos\left(\frac{\pi l m_{pr}}{m_0\cos\theta}\right)\mathrm{sh}^{-1}\left(\frac{2\pi^2 lcm_{pr}T}{\hbar eH\cos\theta}\right)$$
$$\times\left[\cos\left(\frac{lcS_{pr}(\zeta)}{\hbar eH\cos\theta} \mp \frac{\pi}{4}\right) - \left(\frac{l\pi^2|\alpha|c\hbar}{L^2 eH\cos\theta}\right)^{1/2}\cos\left(\frac{lcS_{pr}(\zeta)}{\hbar eH\cos\theta}\right)\right], \tag{14}$$

where $m_{pr} = (2\pi)^{-1}\partial S_{pr}/\partial\zeta$ is the effective mass, corresponding to the isochord $d = 0$, which limits the area $S_{pr}(\zeta)$.

In the argument of the cosine, the upper sign is chosen when S_{pr} is equal to the maximum area ($\alpha > 0$), and the lower sign in the case of the minimum area ($\alpha < 0$). The value of α is determined by the formula

$$|\alpha| = \frac{1}{2\pi}\oint\frac{d\mathscr{L}}{R}, \tag{15}$$

where the integration is carried out over the contour $\mathscr{L}$, which is the isochord $d = 0$, and R is the radius of curvature at points $v_z = 0$ of the cross section of the Fermi surface parallel to the film normal.

In many and, evidently, typical cases (see, e.g., Fig. 2), the value of S_{pr} is equal to the area of the Fermi surface projection on a plane parallel to the film, i.e., it has a simple geometric meaning.

The quantity $\Omega_{osc}^{(1)}$ oscillates with changing magnetic field, with a period in the reciprocal field

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi\hbar e\cos\theta}{cS_{pr}(\zeta)}. \tag{16}$$

In the general case, S_{pr} differs substantially from the extremal cross section of the Fermi surface which defines the oscillation period in the bulky metal,[9] with the exception of certain special cases (for example, the case of a spherical surface), when S_{pr} is identical in value with the area of the extremal cross section.

Furthermore, angular oscillations occur that are associated with a change in the direction of **H**.[3)] For fixed H, the period of the angular oscillations $\Delta\theta$ is determined by the following simple formulas:

$$\Delta\theta \cong \frac{2\pi\hbar eH\cos^2\theta}{cS_{pr}\sin\theta}, \quad \theta \neq 0,$$
$$\Delta\theta \cong 2\sqrt{\frac{\pi\hbar He}{cS_{pr}}}, \quad \theta \approx 0. \tag{16'}$$

[2)]In the recently published work of Lutskiĭ and Fesenko, [8] the authors generalized the quantization of energy of the free electron in a film in a perpendicular magnetic field to the more general case of a dispersion law replacing, the momentum component p_z in the condition of Lifshitz-Kosevich [9] by its quantized value $\pi\hbar n/L$. Such a generalization is not valid for an anisotropic law.

[3)]The attention of the author has been turned to the possibility of the existence of angular oscillations by M. I. Kaganov.

An exact but more complicated formula for $\Delta\theta$ can be found from (14).

Together with $\Omega^{(1)}_{osc}$, characteristic oscillations $\Omega^{(2)}_{osc}$ arise in the film with a phase depending materially on H:

$$\Omega^{(2)}_{osc} = \frac{TV}{\pi^2\hbar^3}\left(\frac{\hbar eH\cos\theta}{c}\right)^{3/2}\sum_{l=1}^{\infty}\sum_{k=1}^{k_l}\sum_{j}|\bar{\alpha}|^{-1/2}\frac{(-1)^l}{l^{3/2}}\cos\left(\frac{\pi l\tilde{m}_{pr}}{m_0\cos\theta}\right)$$
$$\times\cos\left[\Phi_{lk}(\zeta, H, L)\mp\frac{\pi}{4}\right]\operatorname{sh}^{-1}\left[2\pi^2T\left(\frac{l}{\Delta\varepsilon_H}+\frac{k}{\Delta\varepsilon_L}\right)\right]. \quad (17)$$

The phase Φ_{lk} is determined by the formula

$$\Phi_{lk}(\zeta, H, L) = \frac{lcS(\zeta, \rho_j(H, L))}{\hbar eH\cos\theta}\pm\frac{kL}{\hbar}\rho_j(H, L), \quad (18)$$

where $\rho_j(H, L)$ satisfies the condition

$$\frac{\partial}{\partial\rho_j}S(\zeta, \rho_j)\pm\frac{k}{l}\frac{LeH\cos\theta}{c} = 0, \quad (18')$$

while the signs in (18) and (18′) are chosen to be identical, either the upper (plus) or lower (minus); k_l is the maximum value of k for which Eq. (18′) has positive roots ρ_j (j numbers the roots for fixed values of k and l). The upper sign in the argument of the cosine is chosen at $\bar{\alpha} = -(2/\pi)\partial^2S/\partial\rho_0^2 > 0$, and the lower sign when $\bar{\alpha} < 0$; $\tilde{m}_{pr}$ is the effective mass corresponding to the isochord $d = \rho_j$.

$$\frac{l}{\Delta\varepsilon_H}+\frac{k}{\Delta\varepsilon_L} = \frac{1}{2\pi}\frac{\partial}{\partial\zeta}\Phi_{lk}.$$

In the considered field range (1), one can obtain an explicit expression for $\Phi_{lk}(\zeta, H, L)$, since $\rho_j/p_F \sim H/H_L \ll 1$ for not too large k can be used in the neighborhood of $d = 0$:

$$S(\zeta, d) = S_{pr}(\zeta) - \frac{\pi}{4}\pi\alpha d^2 + C_1d^3 + C_2d^4 + \ldots, \quad (19)$$

where α is obtained from Eq. (15); the remaining coefficients have the order of magnitude $|C_n| \sim p_F^{-n}$ and are determined by the shape of the Fermi surface in the vicinity of the line of points with $v_z = 0$.

Substituting (19) in (18′) and solving the equation by successive approximations, we get for Φ_{lk} from (18):

$$\Phi_{lk} = \frac{lcS_{pr}(\zeta)}{\hbar eH\cos\theta}+\frac{k^2}{\pi l\alpha}\frac{L}{\hbar}\left[\frac{eHL\cos\theta}{c}+\frac{8k}{(\pi\alpha)^2l}C_1\left(\frac{eHL\cos\theta}{c}\right)^2\right.$$
$$\left.+\frac{16k^2}{(\pi\alpha)^3l^2}\left(\frac{9}{\pi\alpha}C_1^2+C_2\right)\left(\frac{eHL\cos\theta}{c}\right)^3+\ldots\right]+o(1), \quad (20)$$

where the expansion is carried out up to terms $(L/\lambda_F)(H/H_L)^n \sim 1$, inclusively; λ_F is the Fermi wavelength.

For sufficiently large k, the expansion (20) for Φ_{lk} is not suitable, but such values of k are unimportant in the range of fields and temperatures where the first harmonics play a principal role.

By differentiating Ω_{osc}, we obtain corresponding expressions for the oscillations of the magnetic moment $M_{osc} = -\partial\Omega_{osc}/\partial H$. Inasmuch as Ω_{osc} depends only on one component of the field intensity H_z, as a result the oscillating part of the moment M_{osc} is perpendicular to the film. At not very low temperatures $2\pi^2T/\Delta\epsilon_H \gg 1$, one can retain a single component in the sum over l with $l = 1$, and for $M_{osc} = M^{(1)}_{z\,osc} + M^{(2)}_{z\,osc}$, we have

$$M^{(1)}_{z\,osc} = \frac{TVS_{pr}(\zeta)}{\pi^2\hbar^3\sqrt{|\alpha|}}\left(\frac{\hbar e}{cH\cos\theta}\right)^{1/2}\cos\left(\frac{\pi m_{pr}}{m_0\cos\theta}\right)\left[\sin\left(\frac{cS_{pr}(\zeta)}{\hbar eH\cos\theta}\mp\frac{\pi}{4}\right)\right.$$
$$\left.-\left(\frac{\pi^2|\alpha|c\hbar}{L^2eH\cos\theta}\right)^{1/2}\sin\left(\frac{cS_{pr}(\zeta)}{\hbar eH\cos\theta}\right)\right]\exp\left(\frac{-2\pi^2cm_{pr}T}{\hbar eH\cos\theta}\right), \quad (21)$$

$$M^{(2)}_{z\,osc} = 2\frac{TVS_{pr}(\zeta)}{\pi^2\hbar^3\sqrt{|\alpha|}}\left(\frac{\hbar e}{cH\cos\theta}\right)^{1/2}\cos\left(\frac{\pi m_{pr}}{m_0\cos\theta}\right)$$
$$\times\sum_{k=1}^{k_1}\sin\left[\Phi_{1k}(\zeta, H, L)\mp\frac{\pi}{4}\right]\exp\left(-\pi T\left|\frac{\partial}{\partial\zeta}\Phi_{1k}\right|\right), \quad (22)$$

where Φ_{1k} is determined by Eq. (20) for $l = 1$.

It follows from (17) and (20) that $\Omega^{(2)}_{osc}$ can be approximately regarded in fields (1) as an oscillating function with periods in the reciprocal fields $\Delta(1/H)$ and in the angle $\Delta\theta$ determined by Eqs. (16) and (16′). The characteristic feature of these oscillations is the presence of a factor $P_n(H)$ of n-th degree in the phase where n satisfies the condition $(L/\lambda_F)(H/H_L)^n \sim 1$. The coefficients for powers of H in (20) are expressed in terms of the expansion coefficients (19) which characterize the dispersion law of the electron near the isochord $d = 0$. The presence in the phase of the factor $P_n(H)$ leads to a corresponding change in the periods of oscillations of $\Omega^{(2)}_{osc}$ in comparison with the periods (16) and (16′) of the function $\Omega^{(1)}_{osc}$. The effect of the factor $P_n(H)$ on the period and the character of the oscillations increases with increase in the field.

Inasmuch as the formulas obtained earlier lose their applicability upon reaching fields $H \sim H_L$ due to the approximate character of the energy levels determined by Eq. (10), further analysis is carried out for the case of a quadratic isotropic dispersion law of the electron in the film in a perpendicular magnetic field, where the energy levels are determined by Eq. (11) for any H.

OSCILLATIONS OF THE MAGNETIC SUSCEPTIBILITY

By computing Ω from Eq. (13) in the case of quantum energy levels (11), we get for the oscillating part of the thermodynamic potential $\Omega_{osc} = \Omega^{(1)}_{osc} + \Omega^{(2)}_{osc}$,

$$\Omega^{(1)}_{osc} = \frac{VT}{2\pi^2\hbar^3}\left(\frac{\hbar eH}{c}\right)^{3/2}\sum_{l=1}^{\infty}\frac{(-1)^l}{l^{3/2}}\cos\left(\pi l\frac{m}{m_0}\right)\operatorname{sh}^{-1}\left(2\pi^2l\frac{mcT}{\hbar eH}\right)$$
$$\times\left[\cos\left(\frac{2\pi lmc\zeta}{\hbar eH}-\frac{\pi}{4}\right)-\frac{\pi}{L}\sqrt{l\frac{c\hbar}{eH}}\cos\left(\frac{2\pi lmc\zeta}{\hbar eH}\right)\right], \quad (23)$$

where $\Omega^{(1)}_{osc}$ is identical with the known expression[11] for the oscillating part of the potential in the case of the bulky metal ($m = m_0$) with the exception of the second component, which is proportional to the area of the film and oscillating with the same period

$$\Delta_0\left(\frac{1}{H}\right) = \frac{\hbar e}{cm\zeta}, \quad (23')$$

$$\Omega^{(2)}_{osc} = \frac{VT}{2\pi^2\hbar^3}\left(\frac{\hbar eH}{c}\right)^{3/2}\sum_{l=1}^{\infty}\frac{(-1)^l}{l^{3/2}}\cos\left(\pi l\frac{m}{m_0}\right)\operatorname{sh}^{-1}\left(2\pi^2l\frac{mcT}{\hbar eH}\right)$$
$$\times\sum_{k=1}^{k_l}\left\{\cos\left(\frac{2\pi lmc\zeta}{\hbar eH}+\frac{k^2L^2eH}{\pi lc\hbar}-\frac{\pi}{4}\right)+\sqrt{2}\left[\cos\left(\frac{2\pi lmc\zeta}{\hbar eH}\right.\right.\right.$$
$$\left.+\frac{k^2L^2eH}{\pi lc\hbar}\right)C\left(\sqrt{\frac{2\pi lmc\zeta}{\hbar eH}}-\sqrt{\frac{k^2L^2eH}{\pi lc\hbar}}\right)$$
$$\left.\left.+\sin\left(\frac{2\pi lmc\zeta}{\hbar eH}+\frac{k^2L^2eH}{\pi lc\hbar}\right)S\left(\sqrt{\frac{2\pi lmc\zeta}{\hbar eH}}-\sqrt{\frac{k^2L^2eH}{\pi lc\hbar}}\right)\right]\right\}, \quad (24)$$

where C(x) and S(x) are the Fresnel integrals and

k_l satisfies the condition

$$\frac{\pi lc\sqrt{2m\zeta}}{LeH}+\frac{\hbar}{L}\sqrt{\frac{\pi lc}{\hbar eH}}-1\ll k_l\ll 2\frac{\pi lc\sqrt{2m\zeta}}{LeH}. \tag{24'}$$

In the region of fields

$$\lambda_F\ll\sqrt{c\hbar/eH}\ll L \tag{25}$$

the condition (24′) is simplified, and we have for k_l

$$k_l=\left[l\frac{\pi c}{LeH}\sqrt{2m\zeta}\right], \tag{25'}$$

where [x] is the integral part of the number x.

For not very low temperatures $2\pi^2 mcT \gg \hbar eH$, only a single component will be retained in Eqs. (23) and (24) with $l = 1$.

We have $\chi_{osc}^{(1)}+\chi_{osc}^{(2)}$ for the oscillating part of the magnetic susceptibility $\chi_{osc} = -\partial^2\Omega_{osc}/\partial H^2$ from (23) and (24), where

$$\chi_{osc}^{(1)}=-4\frac{VT}{\hbar}\left(\frac{em\zeta}{c}\right)^2\left(\frac{c}{\hbar eH}\right)^{5/2}\cos\left(\pi\frac{m}{m_0}\right)\exp\left(-\frac{2\pi^2mcT}{\hbar eH}\right)$$
$$\times\left[\cos\left(\frac{2\pi mc\zeta}{\hbar eH}-\frac{\pi}{4}\right)-\frac{\pi}{L}\sqrt{\frac{c\hbar}{eH}}\cos\left(\frac{2\pi mc\zeta}{\hbar eH}\right)\right]. \tag{26}$$

In the range of fields

$$\tfrac{1}{2}\pi H_L < H < \pi H_L, \tag{27}$$

where $H_L = c\sqrt{2m\zeta}/eL$, one component remains ($k_1 = 1$) in the sum over k in (24) and, excluding, for simplification of the formulas, the immediate vicinity of the point $H = \pi H_L$, outside of which one can set $C = S = 1/2$ in (24), we obtain the following expression for $\chi_{osc}^{(2)}$:

$$\chi_{osc}^{(2)}=-8\frac{VT}{\hbar}\left(\frac{em\zeta}{c}\right)^2\left(\frac{c}{\hbar eH}\right)^{5/2}\cos\left(\pi\frac{m}{m_0}\right)\exp\left(-\frac{2\pi^2mcT}{\hbar eH}\right)$$
$$\times\left[1-\left(\frac{H}{\pi H_L}\right)^2\right]\cos\left(\frac{2\pi mc\zeta}{\hbar eH}+\frac{L^2eH}{\pi c\hbar}-\frac{\pi}{4}\right). \tag{28}$$

The period of oscillations Δ_i of the function $\chi_{osc}^{(2)}$ differs appreciably from the period Δ_0 of the oscillations of $\chi_{osc}^{(1)}$, and when $1-(H/\pi H_L)^2 \gg \hbar eH/cm\zeta$ it is determined by the following simple expression when k = 1:

$$\Delta_k\left(\frac{1}{H}\right)=\Delta_0\Big/\left[1-\left(\frac{kH}{\pi H_L}\right)^2\right]. \tag{29}$$

In the region of fields $\tfrac{1}{4}\pi H_L < H < \tfrac{1}{2}\pi H_L$, another oscillating function arises along with (28), with period Δ_2 (see Eq. (29) for k = 2) and so forth. With decrease of the field, the number of oscillations increases jumpwise at the points[4)] $H = \pi H_L/n$, n = 1, 2, 3, ...

The interference of these oscillations leads to the appearance at the points

$$H=nH_0,\quad n=1,2,3,\ldots, \tag{30}$$

where $H_0 = 2\pi^2 c\hbar/eL^2$, of peaks with an amplitude of the order of $\pi H_L/nH_0$, as follows from Eq. (24) if we limit our consideration to a single harmonic with $l = 1$ ($S = C = 1/2$). The amplitude of the peaks decreases linearly with increase in field H.

In the region of fields $H > \pi H_L$, the oscillations of $\chi_{osc}^{(2)}$ disappear, and the magnetic susceptibility has the usual de Haas-van Alphen form (26) with the oscillation period the same as the period for the bulky metal.

The physical meaning of the condition $H < \pi H_L$ for the appearance of oscillations with different periods can be made clear by noting that it is equivalent to the condition

$$\tau_L < \tau_H, \tag{31}$$

where $\tau_L = L/\bar{v}_z$ is the mean time of flight of the electron from one side of the film to the other, $\bar{v}_z = \tfrac{1}{2}v_F$, v_F is the Fermi velocity and $\tau_H = 2\pi mc/eH$ is the period of revolution of the electron in the magnetic field. Under the condition (31), an electron moving along a helical trajectory in the mean does not have time to make a complete loop before collision with the film boundaries. Such electrons make the same contribution to the oscillations as electrons on the extremal cross section with $v_z = 0$, but, in contrast to the latter, the role of the boundaries is essential here, and this also leads, as follows from the calculations made above, to the generation of oscillations with different periods. In the opposite case $\tau_L > \tau_H$, the electron manages to make one or more loops, and such electrons, as also in the case of the bulky metal, make a smaller contribution to the oscillations in comparison with the contribution of electrons with $v_z = 0$.

Thus, in films in perpendicular magnetic fields, at not very low temperatures $2\pi^2 mcT \gg \hbar eH$, where only the first harmonic with $l = 1$ is important, a unique transition takes place in the vicinity of the point $H = \pi H_L$ from the ordinary de Haas-van Alphen oscillations to magnetic-film oscillations with different periods. At sufficiently low temperatures, when higher harmonics with $l \gg 1$ also become important, such a sharp transition does not take place. The transition considered in this case differs from the transition in a parallel magnetic field,[1] when the amplitude changes materially upon reaching a field $H = 2H_L$, and the period of the oscillations is temperature independent.

In conclusion, I consider it my pleasant duty to thank M. I. Kaganov for useful discussions.

[4)]The presence of singularities in the thermodynamic quantities at the points $\pi H_L/n$ was noted by Gogadze and Kulik.[12] However, the numerical analysis performed by them, based on the use of the equations of the research of Gurevich and Shik [3] does not allow us to clarify the analytic character of these singularities.

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