POSITRON PRODUCTION DURING THE MUTUAL APPROACH OF HEAVY NUCLEI AND THE POLARIZATION OF THE VACUUM

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The spontaneous production of electron-positron pairs during the gradual transition of the potential through the critical value corresponding to the removal of the energy barrier for pair production, is considered. A possible example is the assumed pair production which takes place when bare subcritical nuclei approach each other. It is suggested that the charge density distribution of the electron bound in a level with $\epsilon_1 = -mc^2$, together with the density distribution of the polarization charge is localized near the nucleus (whereas the charge density of only the bound electron is delocalized). It is pointed out that the polarization of the vacuum for heavy nuclei differs from that for light nuclei. This difference corresponds to the adiabatic escape to infinity of the positrons when the critical nuclear charge is attained.

THE problem of the discrete energy levels of an electron which reach the boundary of the continuous spectrum $\epsilon_1 < -mc^2$ has come up 40 years ago.[1] However, it has never been settled completely. Today, this problem has attained additional urgency in connection with the progress made in the synthesis of superheavy nuclei: it is not excluded that nuclei with $Z > 137$ will soon be synthesized.[2] But such a situation can also be brought about in a different way: when two nuclei with charge Z smaller than the critical value approach each other more closely than the distance $r < \hbar/mc = 3.7 \times 10^{-11}$ cm, a field is seen by the electron which differs little from the field of a point-like charge 2Z, although the nuclei are still far from coalescing, since the nuclear radius is $R \sim 1.1 \times 10^{-13} A^{1/3} = 10^{-12}$ cm for $A \sim 700$.

We assume that, if the nuclei have come so close to each other that the lowest electron level attains the energy $\epsilon_1(R) < -mc^2$ (including the rest mass), spontaneous production of a pair sets in, the electron occupying this level and the positron flying off as a real particle. Since the level is doubly degenerate on account of the spin, two electrons and two positrons appear. Further production of pairs and positrons is not prevented by the screening of the nuclear charge by the electrons, but on account of the fact that the lowest (1 S) level is filled (Pauli principle). Additional pairs can be produced only when the 2P levels (which are split owing to the nonsphericity of the potential) in their turn reach the energy $\epsilon_2(R) < -m_e c^2$, which however requires a much larger value of Z.

Thus a new type of positron production (more precisely, of pair production with a bound electron) becomes possible, which is distinguished by the circumstance that it occurs also during a slow, quasi-static approach of the nuclei.

The value of the critical charge Z_c at which pair production becomes possible depends strongly on the finite dimensions of the nucleus: according to the calculations of Pomeranchuk and Smorodinskiĭ,[3] for $R = 1.1 \times 10^{-13} A^{1/3}$ cm, $A = 2.5$ Z, one must have $Z_c = 200$ to obtain $\epsilon_1 = -mc^2$. Therefore, it is entirely possible that an experiment with two uranium or plutonium nuclei (which is difficult, but more or less realistic) does not suffice to produce pairs. Nevertheless, even the consideration of a gedanken experiment is of interest to clarify the subtle questions of the theory of positrons and of the polarization of the vacuum.

Let us first convince ourselves that there is no intrinsic contradiction or ambiguity in the interpretation of the experiment proposed.

Actually, the approach of the nuclei must not be too slow; the kinetic energy of the nuclei must be sufficient to overcome their repulsion. However, this condition is completely consistent with our quasistatic consideration, owing to the large mass of the nuclei. For example, for Z = 92, A = 238, and a distance $\hbar/mc$, one needs a velocity of 0.012c in the system of the center of inertia. Thus, in principle, the new process can take place under conditions when the pair production is vanishingly small on account of the Fourier components of the field with frequency $2mc^2/\hbar$.

Okun' has noted that pair production processes are possible via the excitation of nuclei during collisions. However, even for the excitation of nuclei with $\Delta E > 2mc^2$ the same criterium of the adiabaticity of the collision is decisive. The experimental distinction of the process going through a nuclear excitation is also that here the probability w_2 for the production of two pairs is of the order of the square of the probabilities for the production of one pair or less, $w_2 \lesssim w_1^2$. For the process proposed by us, one may expect $w_2 > w_1$—either two positrons are created together or none is, which allows one easily to distinguish the character of the process.

Evidently, the electrons remain, as a rule, bound to the outgoing nuclei with $Z < Z_c$. In these nuclei the binding energy is smaller,[1)] $E_1 > -mc^2$. However, the overall energy balance is not disturbed; the energy is

[1)]Nevertheless, it is rather improbable that, as the nuclei recede from each other, the process goes adiabatically in the direction of pair annihilation, which becomes energetically advantageous after the nuclei have receded beyond a certain distance. The light positrons are repelled by the approaching nuclei and manage to fly away earlier than the nuclei move apart.

taken from the kinetic energy of the nuclei. Because of the lower charge $(Z - 1)$ of the outgoing nuclei, they fly apart with an energy which is smaller than the initial one. We recall that it is necessary for this process that the nuclei in the initial state be bare, i.e., without electrons.

In the discussion of this work theoretical doubts have come up as to the character of the bound state whose energy lies at the boundary of the continuous spectrum $E = -mc^2$ of the filled background. What is the relation of the process to the polarization of the vacuum? Will screening set in for $Z > Z_C$ until $Z_{eff} = Z - n(e^-) = Z_C$ is reached? Does not the proposed idea contradict the fact that in the exact solution of the Dirac equation for the Coulomb potential

$$E_1 = mc^2\sqrt{1 - (Ze^2/\hbar c)^2},$$

i.e., bound levels exist only for $Z < 137$, and here $E_1 > 0$?

It is convenient to begin with the last question. The expression for E_1 can also be obtained by the Bohr theory from the relations $pr = \hbar$ and $pv/r = Ze^2/r^2$ with $p = mv/\sqrt{1 - v^2/c^2}$. Here $Ze^2/\hbar c \to 1$ corresponds to $r \to 0$ and $E = mc^2/\sqrt{1 - v^2/c^2} - Ze^2/r \to 0$. The absence of bound S states for $Ze^2/\hbar c \geq 1$ corresponds to the "collapse into the center" in the classical relativistic problem of the motion of a particle in the Coulomb field of a point-like charge. In the case of an "extended" source there is no collapse into the center, and the bound levels may reach the energy $E_1 = -mc^2$.

However, even in the case of a nuclear charge extending over a finite volume, for example, a sphere with radius r_0, the wave function behaves pathologically as $Z \to Z_C$, where Z_C depends on r_0.[4]

Indeed, the asymptotic form for $r \gg r_0$ must be $\psi \sim e^{-\kappa r}$, where $\kappa = \hbar^{-1}\sqrt{m^2c^2 - E_1^2/c^2}$. Hence, for $E_1 = 0$ the quantity κ has a maximum; as E_1 is decreased further, κ also decreases, and $\kappa \to 0$ for $E_1 \to -mc^2$. This means physically that $E_1 \to -mc^2$ for $Z \to Z_C$, and the wave function approaches in form the functions of the boundary of the continuous spectrum with E $E = -mc^2 - \epsilon$.

At first glance, a delocalization takes place, the electrons bound in the lowest level spread apart, which contradicts the picture described above. We propose that actually no delocalization of the charge takes place, and the correct answer should be obtained when the polarization of the vacuum or more precisely, the readjustment of the polarization of the vacuum for $Z \to Z_C$ is taken into account. Such a complete and consistent calculation has not been carried out, but below we give some qualitative arguments in favor of our hypothesis.

These arguments are based on the analogy with the problem of the distribution of a Fermi gas in a potential field.[5] In[5] a space was considered which is filled by a Fermi gas with a given Fermi surface and with the corresponding particle density ρ_0 in the region where $V = 0$. If somewhere $V(x) \neq 0$, the density is disturbed. In particular, it is possible that in the potential there exists a discrete level with $E < 0$ in nonrelativistic language. The disturbance of the density $\delta\rho = \rho(x) - \rho_0$, in which also the density of the particles in the discrete level is included, has no singularities near the value of the potential V_C corresponding to the appearance of the level.

The reason for this is that in the transition from $V_1 < V_C$ [the notation is symbolic since the existence and the energy of the level are functionals of $V(x)$] to $V_2 > V_C$, the appearance of the level is accompanied by a resonant readjustment of the scattering phase of the particles belonging to the continuum, with $E = \epsilon$. The depletion of the density of these particles in the region of the potential well compensates precisely the additional density of bound particles. The wave function of the bound particles is delocalized in the limit of $V = V_C + \epsilon$, but the total increase of the density $\delta\rho$ remains localized in the region defined by the dimensions of the well and the wave length at the Fermi surface. (Here ϵ is taken as a "small quantity.")

One may assume that the situation is analogous in the relativistic theory of the transition through $Z = Z_C$. The polarization of the vacuum is the analog of the change in the density distribution of the Fermi gas. As is known, in the linear approximation

$$\delta\rho(x) \backsim -\frac{e^2}{\hbar c}\Delta\varphi \ln\frac{k}{mc},$$

where k is the cut-off momentum of the relativistic theory and Δ is the Laplacian. This formulas yields

$$\delta Z = -Z\cdot\text{const}\cdot\frac{e^2}{\hbar c}\ln\frac{k}{mc},$$

which is a result of wide applicability, since a universal proportionality exists between δZ and Z, not at all only in the linear approximation! If $Z < Z_C$ and there are two electrons in some energy level with $mc^2 > E - mc^2$, then

$$\delta Z = -2 - Z\cdot\text{const}\cdot\frac{e^2}{\hbar c}\ln\frac{k}{mc}.$$

This relations holds up to $Z = Z_C$ and even for $Z > Z_C$.

When Z_C is approached, a rearrangement (delocalization) of the wave function of the bound electrons occurs, but at the same time the charge distribution which causes the polarization of the vacuum is rearranged (breaking the proportionality $\delta\rho \backsim \Delta\varphi$). We assume that the summed charge density remains localized. The rearrangement of the distribution describing the polarization of the vacuum is completely natural. In the exact (nonlinear in φ) theory, of course, one deals with creation and annihilation operators for electrons and positrons in states which are eigenstates in the potential φ. The occurrence of a level (for $Z = Z_C$) corresponds to a singularity (resonance) of these states.

Let us assume the hypothesis that the summed charge density of the two bound electrons and the polarization of the vacuum are localized for $Z \to Z_C$. Since the bound electrons are delocalized, it follows that for $Z \to Z_C$ (but such that $Z < Z_C$) the polarization of the vacuum also becomes delocalized. It seems to us that this conclusion fits naturally into the general picture of the phenomenon.

By calculating the polarization we determine the properties of the stationary state of the electron-positron field without real particles under the influence of a perturbing potential. We assume that this state is destroyed spontaneously for $Z > Z_C$—real electrons and positrons appear. Hence, the desired state (without real particles) does not exist as a stationary state. But as the parameter Z, on which the solution depends, approaches the boundary of the region of existence of the solution ($Z = Z_c$) it is completely natural that the properties of the solution become unusual: in the present case, delocalization sets in. The presence of a singularity means that this result can in principle not be obtained by considerations based on a perturbation expansion in powers of the potential or powers of $Ze^2/\hbar c$.

When the energy of the bound electron reaches the value $E_1 = -mc^2$ (for $Z > Z_c$), i.e., the boundary of the lower continuum, it is physically meaningless to distinguish the corresponding level from the other close-lying levels of the continuum. Therefore, it is meaningful only to speak of the summed charge distribution, i.e., the charge distribution of the electrons attaining the energy $E_1 = -mc^2$ together with the charge distribution describing the polarization of the vacuum. It is this quantity which, by our hypothesis, remains localized even when $Z > Z_c$ and the energy at the lowest orbit is $E_1 = -mc^2$.

Let us now apply the principle of continuity[5] in order to find the charge distribution describing the polarization of the vacuum for $Z < Z_c$ and in particular, as Z approaches Z_c.

We recall that in the coordinate representation the charge distribution, which is proportional to $\Delta\varphi$, i.e., to the density of the original charge, leads to a physically unobservable change (renormalization) of all charges in the same ratio; this ratio is given by a divergent integral.

An observable effect of the polarization of the vacuum in the static problem is the appearance of additional charges near the real charge (proton or nucleus), whose sum is identically equal to zero. The positive charge collects near the spherically symmetric nucleus in the center, while the negative charge is found on the periphery. The potential is not changed at large distances, $\varphi_p = \varphi_0 = Ze^2/r$, but at close distances $\varphi_p > \varphi_0$ (φ_0 is the potential corresponding to the charge distribution in the nucleus after the renormalization, while φ_p is the same with account of the polarization of the vacuum). It is for this reason that the contribution of the vacuum polarization to the Lamb shift [equal to $(-e)(\varphi_p - \varphi_0)\psi^2 dV = -25$ MHz] is negative (the sign of e comes from the negative charge of the electron).

At first glance this charge distribution seems paradoxical and unnatural; it seems that the positive proton should "pull in" the negative charges and repel the positive ones. How can this be reconciled with the results of the calculations of quantum electrodynamics, which have also been verified by experiment? Here one must recall that essentially one calculates only a correction which remains after the main effect, which is accompanied by the removal of the charge to infinity, has been taken out by the charge renormalization, or better, has been included in the definition of the observable charge and declared unphysical.

There is no doubt that the summed effect has a reasonable sign, $\varphi_p < \varphi_{00}$, where φ_{00} refers to the bare (unrenormalized) charge, but φ_{00} is a quantity which can neither be observed nor calculated.

All considerations above refer to the usual situation of small Z, where it is sufficient to restrict oneself to the first term in the expansion with respect to $Ze^2/\hbar c$, in calculating the polarization of the vacuum. Let us now turn to the proper subject of the paper and try to determine the charge distribution for $Z \sim Z_c$. For $Z > Z_c$ we assume that a stationary situation is given by two electrons on a nucleus with $Z_{eff} = Z - 2$ (we recall that Z is the number of protons in the nucleus, and Z_{eff} is the charge measured by a distant observer), and the charge distribution is localized. For $Z \lesssim Z_c$ the "analytic continuation" of this state is a nucleus with two K electrons in the lowest level. As Z approaches Z_c the wave function of the K electrons spreads out, and for $Z \to Z_c$, the quantity $\kappa \to 0$ in the asymptotic wave function $e^{-\kappa r}$.

However, if the continuity principle holds, the total charge density remains localized even in this situation. Of course, the charge of the electrons in the K orbit (with $E > -mc^2$, nonpathological energies) is compensated by the positive charge from the polarization of the vacuum at large distances from the nucleus; for $Z \to Z_c$ the radius of the orbit and the radius of the positive charge both increase. But for $Z < Z_c$ the electrons can be torn off from the K orbit (spontaneous pair production is forbidden energetically). A bare nucleus remains, $Z_{eff} = Z < Z_c$, and we assert that the polarization of the vacuum gave rise to a positive charge cloud at large distances around the nucleus which is compensated by a negative charge inside.

This result is legitimate: if two protons recede to infinity for $Z > Z_c$, then these two protons prepare for their exit when $Z < Z_c$, i.e., they recede to large (but finite!) distances, which are the larger, the closer Z to Z_c. Hence, the polarization of the vacuum changes qualitatively, and takes on the opposite character in the transition from small Z to values of Z close to, but smaller than, Z_c.

The contribution of the polarization of the vacuum to the Lamb shift of the S level depends on the quantity $Q_0 = \int r^2 \delta\rho dV$, the monopole moment of the charge distribution. This quantity has opposite signs for $Ze^2/\hbar c \ll 1$ and $Z \lesssim Z_c$, and goes to infinity for $Z \to Z_c$. Hence, somewhere in between, at a value of Z which is smaller than Z_c by a finite amount, we have $Q_0 = 0$.

For large Z, the contribution of the vacuum polarization to the Lamb shift must change sign! On the basis of extremely rough estimates, there is no reason that the value Z_r for which $Q_0 = 0$ be especially very close to Z_c; apparently, $Z_c - Z_r \gg 1$. However, without difficult calculations it is impossible to say whether Z_r is close to 120 or 170, say. We recall that $Z_c = 200$ according to[2], and for $Z = 96$ the binding energy of the K electron[2)] is as small as 128 keV

[2)]We give the asymptotic form for Q_0 when Z is close to Z_c:

$$\psi \sim e^{-\varkappa r}, \quad \varkappa \sim \frac{\hbar}{mc}\sqrt{1-\left(\frac{E}{mc^2}\right)^2},$$

$$\frac{dE}{dZ} = -e^2\left(\overline{\frac{1}{r}}\right) = -e^2\varkappa = -\frac{e^2}{\hbar c} mc^2\sqrt{1-\frac{E}{mc^2}}.$$

Since, by the definition of Z_c, $E = -mc^2$ for $Z = Z_c$, we find $E = -mc^2 + mc^2\,[Z_c - Z)e^2/2\hbar c]^2$. This implies $Q_0 \sim k_1 e(\hbar/mc)^2 \cdot [137/(Z_c - Z)]^2$. For small Z we have $Q_0 \sim k_2 e(\hbar/mc)^2 Ze^2/\hbar c$ with a small dimensionless factor; $k_2 = 1/300$, k_1 is unknown.

$\approx 0.25\ mc^2$.

In closing, we return to our remark that the effects considered refer to thought experiments which serve for a clarification of some subtleties of the theory, but not to the real, experimentally verifiable physics. Nevertheless, it is remarkable that new predictions can be made not only for $Z > Z_c$, but also for $Z < Z_c$.

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