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# SECOND VISCOSITY AND ATTENUATION OF SECOND SOUND IN SOLIDS

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It is shown that processes which occur without conservation of the number of quasiparticles in ferrodielectrics or metals result in the additional absorption of second sound, similar to the absorption associated with second viscosity in hydrodynamics.

As is well known, temperature waves can propagate in solids at low temperatures. These are called second sound and represent oscillations of the density of elementary excitations, i.e., ordinary sound in a gas of quasiparticles. For the existence of second sound, it is necessary that the normal collisions between quasiparticles, which lead to the establishment of local thermodynamic equilibrium in the system of quasiparticles, be more probable than processes that establish equilibrium between the quasiparticles and the lattice (transport processes, scattering by lattice defects and so forth). The attenuation $\omega''$ of second sound is connected with collisions of both types and is equal, in order of magnitude, to

$$\frac{\omega''}{\omega} \sim -\left(\omega\tau_N + \frac{1}{\omega\tau_V}\right),$$

where $\tau_N$ and $\tau_V$ are the relaxation times of the normal collisions and of the processes which establish equilibrium of the quasiparticles with the lattice, and $\omega$ is the frequency of second sound, which satisfies the conditions $\tau_V^{-1} \ll \omega \ll \tau_N^{-1}$.

In this paper we take into account the influence of processes connected with the nonconservation of the number of quasiparticles on the absorption of second sound in ferrodielectrics and metals with multiply connected Fermi surfaces. The sound absorption mechanism considered is similar to the mechanism proposed by Mandel'shtam and Leontovich for liquids,[1] in which, as is well known, it is assumed that the presence of slow processes of establishment of equilibrium is macroscopically equivalent to the presence of a second viscosity with dispersion. In our case, the role of the slow processes is played by the processes which take place with nonconservation of the number of particles, and the parameter which characterizes the slow processes is the chemical potential of the system, $\mu$.

1. In ferrodielectrics, second sound is the oscillation of the magnon density.[2] For not too low temperatures, $T \gtrsim 10°K$, the role of normal collisions is played by processes with exchange interaction between the quasiparticles, as is well known. For this case, the number of magnons is conserved, as is their chemical potential $\mu$. Account of weak relativistic interactions, which do not conserve the quasiparticles, leads to relaxation of $\mu$ under these conditions and hence to an attenuation of the second sound.

Let us consider a uniaxial ferromagnet with a preferred axis along the direction of the external magnetic field $H_0$, i.e., $H_0 \| n | M_0$. The law of dispersion of magnons differs from a power law and has the form

$$\tilde{\varepsilon}(\mathbf{p}, \mathbf{r}, t) = \varepsilon(\mathbf{p}) + \mu_B(\mathbf{nh}), \tag{1}$$

where

$$\varepsilon(\mathbf{p}) = \Theta_c\left(\frac{ap}{\hbar}\right)^2 + \mu_B H^{ef}, \quad H^{ef} = H_0 + \beta M_0.$$

Here the following notation is used: $\Theta_c$ is a constant on the order of the Curie temperature, a the lattice constant, h Planck's constant, $\mu_B$ the Bohr magneton, $\beta$ the constant of magnetic anisotropy, and h the variable magnetic field produced by the spin wave. The kinetic equation for the magnon distribution function

$$\frac{dN}{dt} = \hat{I}_4(N) + \hat{I}_3(N), \quad \frac{dN}{dt} = \frac{\partial N}{\partial t} + \mathbf{v}\frac{\partial N}{\partial \mathbf{r}}, \quad \mathbf{v} = \frac{\partial \varepsilon}{\partial \mathbf{p}}, \tag{2}$$

can be solved by the method of successive approximations, as is well known ($I_4(N)$ is the integral of fourfold exchange interactions, $I_3(N)$ is the integral of threefold relativistic interactions). The zero approximation $\hat{I}_4(N^0) = 0$ leads to the drift solution $N^0 = N_0(\bar{\epsilon} - \mathbf{p}\cdot\mathbf{u} - \mu)/(1 + v)T$, where u is the drift velocity, $\vartheta$ the relative change in the temperature, $N_0(\epsilon) = [\exp(\epsilon/T) - 1]^{-1}$ is the equilibrium Bose distribution function.

We write down the conditions for solution of the kinetic equation in first approximation, which consist of the conservation of the number of particles in the exchange collisions with conservation of energy and quasimomentum for collisions which include both exchange and relativistic interactions:

$$\int \hat{I}_4 d\mathbf{p} = \int \varepsilon(\mathbf{p})(\hat{I}_4 + \hat{I}_3)\,d\mathbf{p} = \int \mathbf{p}(\hat{I}_4 + \hat{I}_3)d\mathbf{p}. \tag{3}$$

The conditions of the solvability of (3), the expression for the density of magnetic moment **M** and Maxwell's equations, in which the displacement current can be neglected because of the smallness of the ratio $V/c \ll 1$ (V is the speed of second sound, c the speed of light), form a complete set of equations for the drift parameters **u**, $\mu$, $\vartheta$, the magnetic moment **M**, and the field **h**:

$$\langle 1\rangle \dot{\mu} + \langle\varepsilon\rangle\dot{\vartheta} + \langle p_\varkappa v_\varkappa\rangle \operatorname{div}\mathbf{u} - \mu_B\langle 1\rangle(\mathbf{n}\dot{\mathbf{h}}) + \frac{\langle 1\rangle}{\tau_3}\mu = 0, \tag{4}$$

$$\langle\varepsilon\rangle\dot{\mu} + \langle\varepsilon^2\rangle\dot{\vartheta} + \langle p_\varkappa v_\varkappa \varepsilon\rangle \operatorname{div}\mathbf{u} - \mu_B\langle\varepsilon\rangle(\mathbf{n}\dot{\mathbf{h}}) = 0,$$

$$\langle p_i p_k\rangle \dot{u}_k + \langle p_\varkappa v_\varkappa\rangle\frac{\partial\mu}{\partial x_i} + \langle p_\varkappa v_\varkappa\varepsilon\rangle\frac{\partial\vartheta}{\partial x_i} - \mu_B\langle p_\varkappa v_\varkappa\rangle\frac{\partial(\mathbf{nh})}{\partial x_i} = 0,$$

$$\mathbf{M} = \frac{\mathbf{H}}{H}\left(M_0 - \frac{\mu_B}{h^3}\int N^0\,d\mathbf{p}\right), \quad \operatorname{rot}\mathbf{h} = 0, \quad \operatorname{div}(\mathbf{h} + 4\pi\mathbf{M}) = 0,$$

where

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \quad \langle \varphi(\mathbf{p}) \rangle = -\int \varphi(\mathbf{p}) \frac{\partial N_0}{\partial \varepsilon} d\mathbf{p}.$$

We have introduced $\tau_3$ above as the mean relaxation time in relation to processes of threefold relativistic interactions,

$$\frac{\langle 1 \rangle \mu}{\tau_3} = \int \hat{I}_3(N^0) d\mathbf{p}, \tag{5}$$

which is obtained as the result of substitution of the drift solution $N^0$ in the integral of triplet collisions, using the linearization

$$\hat{I}_3(N^0) = -\frac{N_0(\varepsilon)}{T} \frac{1}{\tau_3(\mathbf{p}_1)} \mu$$

and then averaging over the momenta; $\tau_3(\mathbf{p}_1)$ is the relaxation time of relativistic scattering of magnons with momentum $\mathbf{p}_1$.

$$\frac{1}{\tau_3(\mathbf{p}_1)} = \int d\mathbf{p}_2 d\mathbf{p}_3 \{ |\Psi(1, 2; 3)|^2 N_0(\varepsilon_2)(N_0(\varepsilon_3) + 1) \cdot \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3)\delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3) - |\Psi(2, 3; 1)|^2 (N_0(\varepsilon_2) + 1) \cdot (N_0(\varepsilon_3) + 1)\delta(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)\delta(\varepsilon_1 - \varepsilon_2 - \varepsilon_3) \}, \tag{6}$$

where $\Psi(1, 2; 3) \sim \Psi(2, 3; 1) \sim \mu_B M_0/\sqrt{N}$ is the amplitude of relativistic scattering of the magnons, N the number of magnons, $1 \equiv \mathbf{p}_1$, $2 \equiv \mathbf{p}_2$, $3 \equiv \mathbf{p}_3$. In obtaining (5), it was assumed that $\partial N_0/\partial \epsilon = -N_0/T$.

According to[3],

$$\frac{1}{\tau_3} \sim \frac{(\mu_B M_0)^2}{\hbar \Theta_c} \left( \frac{T}{\Theta_c} \right)^{1/2} \ln^2 \frac{2\mu_B H^{ef}}{T}, \quad \mu H^{ef} \ll T \tag{7}$$

In the case under consideration, the sound wave is, as is known,[2] principally a temperature wave ($\mu_B h/\vartheta T \ll 1$); therefore, in obtaining the dispersion equation of the system (4), the variable magnetic field h can be left out of consideration. Assuming $\mathbf{u}$, $\mu$, $\vartheta \sim \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, we obtain the following dispersion equation of the system (4):

$$\omega = k \left[ \frac{(V_0^2 - V_\infty^2 i\omega\tau_3)(\langle \varepsilon^2 \rangle \langle 1 \rangle - \langle \varepsilon \rangle^2) + V_0^2 \langle \varepsilon \rangle^2}{\langle \varepsilon^2 \rangle \langle 1 \rangle - i\omega\tau_3(\langle \varepsilon^2 \rangle \langle 1 \rangle - \langle \varepsilon \rangle^2)} \right]^{1/2}, \tag{8}$$

where $V_0^2 = (\hat{\alpha}^{-1})_{\kappa\kappa} \langle p_\kappa v_\kappa \epsilon \rangle^2 / \langle \epsilon^2 \rangle$ is the speed of second sound in the case in which the number of quasiparticles is not conserved and $\mu = 0$,

$$V_\infty^2 = \frac{(\hat{\alpha}^{-1})_{\kappa\kappa}(\langle p_\kappa v_\kappa \rangle^2 \langle \varepsilon^2 \rangle + \langle p_\kappa v_\kappa \varepsilon \rangle^2 \langle 1 \rangle - 2\langle p_\kappa v_\kappa \varepsilon \rangle \langle p_\kappa v_\kappa \rangle \langle \varepsilon \rangle)}{(\langle \varepsilon^2 \rangle \langle 1 \rangle - \langle \varepsilon \rangle^2)}$$

is the speed of sound when the number of particles is conserved, $\alpha_{ik} = \langle p_i p_k \rangle$, $(\hat{\alpha}^{-1}) = (\hat{\alpha}^{-1})_{ik} \kappa_i \kappa_k$, $\kappa = \mathbf{k}/k$ is the unit vector along the direction of propagation of the wave.

In the case of small frequencies $\omega\tau_3 \ll 1$ (here, naturally, we must satisfy the conditions $\omega\tau_N \ll 1$, $\omega\tau_V \gg 1$, $\tau_N < \tau_3$) from the dispersion equation (8) we get

$$\omega = kV_0 - \frac{i\tau_3\omega^2}{2V_0^2}(V_\infty^2 - V_0^2)\left(1 - \frac{\langle \varepsilon \rangle^2}{\langle \varepsilon^2 \rangle \langle 1 \rangle}\right). \tag{9}$$

It follows from Eq. (9) that the speed of propagation of sound in the case of low frequencies $\omega\tau_3 \ll 1$ is equal to $V_0$, since in this case the period $1/\omega$ of the sound wave is large in comparison with the relaxation time $\tau_3$ and equilibrium can be established in the system relative to processes of threefold relativistic interactions. The attenuation $\omega''$ in this case is proportional to $\omega^2$.

In the opposite case of high frequencies $\omega\tau_3 \gg 1$, the solution of the dispersion equation (8) has the form

$$\omega = kV_\infty - i\frac{(V_\infty^2 - V_0^2)}{2\tau_3 V_\infty^2}\left(1 - \frac{\langle \varepsilon \rangle^2}{\langle \varepsilon^2 \rangle \langle 1 \rangle}\right)^{-1}. \tag{10}$$

The sound wave in this case is propagated with a speed $V_\infty$, inasmuch as three-particle interactions cannot take place within the time of a single period and the number of particles in the system is conserved. The attenuation $\omega''$ in the case $\omega\tau_3 \gg 1$ does not depend on the frequency $\omega$.

In both limiting cases the relative attenuation is small, $\omega''/\omega \ll 1$; it has a maximum at some intermediate frequency, equal to

$$\omega_{ext} = \frac{V_0}{\tau_3 V_\infty}\left(1 - \frac{\langle \varepsilon \rangle^2}{\langle \varepsilon^2 \rangle \langle 1 \rangle}\right)^{-1}.$$

The maximum attenuation is given by (Fig. 1):

$$\left(\frac{\omega''}{\omega}\right)_{ext} = \frac{V_\infty - V_0}{V_\infty + V_0}.$$

It is not difficult to show that

$$V_\infty^2 - V_0^2 = \frac{(\hat{\alpha}^{-1})_{\kappa\kappa}(\langle p_\kappa v_\kappa \rangle \langle \varepsilon^2 \rangle - \langle p_\kappa v_\kappa \varepsilon \rangle \langle \varepsilon \rangle)^2}{\langle \varepsilon^2 \rangle (\langle \varepsilon^2 \rangle \langle 1 \rangle - \langle \varepsilon \rangle^2)}. \tag{11}$$

As follows from (11), the attenuation of sound $\omega''$ differs from zero only for a non-power-law dispersion for the magnons $\epsilon(\mathbf{p})$; in the opposite case,[2] $\langle \epsilon^2 \rangle \langle p_\kappa v_\kappa \rangle - \langle p_\kappa v_\kappa \rangle \langle \epsilon \rangle = 0$.

We estimate the value of the attenuation at the maximum. Substitution of the magnon dispersion law (1) in (11) yields the result

$$\left(\frac{\omega''}{\omega}\right)_{ext} \sim \left(\frac{\mu_B H^{ef}}{T}\right)^2, \tag{12}$$

$$\omega_{ext}\tau_3 \sim 1 - \left(\frac{\mu_B H^{ef}}{T}\right)^2.$$

We set $T \sim 10°$K, $\beta \sim 3$–$5$, $\mu_B M_0 \sim 1°$K; then $(\omega''/\omega)_{ext} \sim 0.1$–$0.2$. Such an absorption is entirely susceptible of experimental observation.

We shall now consider the case of low temperatures, $T < 10°$K, when the threefold relativistic interactions, more probable than the fourfold exchange interactions ($\tau_3 < \tau_4$), will play the role of normal processes. In this case, the fourfold interactions will not lead to an additional absorption of second sound, since they do not change the states of the magnon system (they change neither the energy, the momentum nor the number of magnons). Formally, this follows from the circumstance that the solution of the equation $\hat{I}_3(N_0) = 0$, which has the form

$$N^0 = N_0\left(\frac{\tilde{\varepsilon} - \mathbf{pu}}{(1 + \vartheta)T}\right), \quad \mu = 0,$$

also causes the integral of the four-particle exchange interactions to vanish; i.e., $\hat{I}_4(N^0) = 0$.

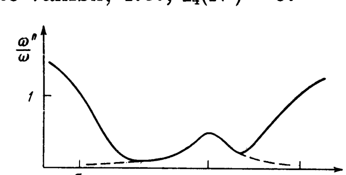


FIG. 1. Relative attenuation of second sound in ferrodielectrics as a function of the period of the sound wave, with account of processes leading to second viscosity; the value of the attenuation at maximum amounts to $(\mu H/T)^2$.

2. In metals at low temperatures, second sound is possible in the system of conduction electrons, the interaction among which takes place by way of exchange of thermal phonons.[4] The necessary condition for the existence of second sound is the equality of the number of electrons and holes in the metal: $n_+ = n_-$. This is necessary in order that the drift regime of the kinetic equation does not lead to the appearance of an electric current. We consider metals with multiply-connected Fermi surfaces, which can consist of separate surfaces (or groups of surfaces), separated from one another by a distance much greater than the temperature momentum of the phonon. The role of the normal collisions here is played by the processes of migration of electrons within the limits of a single Fermi surface due to the absorption of thermal phonons. The transitions of the electrons between the separate groups of surfaces lead to an additional attenuation of the second sound in metals, which in its nature is similar to the second viscosity in hydrodynamics. This attenuation can be found by means of the simultaneous solution of the kinetic equations for electrons $f(\mathbf{p}, \mathbf{r}, t)$ and phonon $N(\mathbf{q}, \mathbf{r}, t)$ distribution functions by the method of successive approximations. The kinetic equations for the electrons and phonons in zeroth approximation will have drift solutions with identical values of the parameters $\mathbf{u}$ and $\vartheta$. However, we should consider the change in the chemical potential of the electrons $\delta\mu^a$ to be different for the separate groups[4] (a is the index denoting the group). The solvability conditions of the kinetic equations in first approximation lead, as above, to a set of equations for the drift parameters $\mathbf{u}$, $\vartheta$ and $\delta\mu^a$:

$$\langle 1\rangle^a\delta\mu^a + \langle\bar\varepsilon\rangle^a\vartheta + u_i\Big(\langle p_i\rangle^a - \frac{1}{V}\langle p_i v_\varkappa\rangle^a\Big) = \frac{i\langle 1\rangle^a}{\omega}\sum_{a'}\frac{(\delta\mu^a - \delta\mu^{a'})}{\tau_{aa'}},$$

$$\sum_a \delta\mu^a\langle\bar\varepsilon\rangle^a + \vartheta(\langle\bar\varepsilon^2\rangle + \langle\Omega^2\rangle) + u_i\Big[\langle p_i\bar\varepsilon\rangle - \frac{1}{V}(\langle p_i v_\varkappa\bar\varepsilon\rangle + \langle q_i s_\varkappa\Omega\rangle)\Big] = 0,$$

$$\sum_a \delta\mu^a\Big(\langle p_i\rangle^a - \frac{1}{V}\langle p_i v_\varkappa\rangle^a\Big) + \vartheta\Big[\langle p_i\bar\varepsilon\rangle - \frac{1}{V}(\langle p_i v_\varkappa\bar\varepsilon\rangle + \langle q_i s_\varkappa\Omega\rangle)\Big] + u_k\Big[\langle p_i p_k\rangle + \langle q_i q_k\rangle - \frac{1}{V}\langle p_i p_k v_\varkappa\rangle\Big] = 0. \quad (13)$$

Here we have used the following notation: $\omega = kV$, $\mathbf{k} = \varkappa k$; $\epsilon(\mathbf{p})$ and $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ are the dispersion law and speed of electrons, respectively; $\Omega(\mathbf{q})$ and $s = \partial\Omega/\partial\mathbf{q}$ are the dispersion law and speed of phonons, respectively, $\bar\epsilon = \epsilon - \mu$;

$$\langle\varphi(\mathbf{p})\rangle^a = \frac{2}{h^3}\int d\mathbf{p}^a\varphi(\mathbf{p}^a)\Big(-\frac{\partial f_0}{\partial\varepsilon}\Big),$$

$$\langle\psi(\mathbf{q})\rangle = \frac{1}{h^3}\int d\mathbf{q}\psi(\mathbf{q})\Big(-\frac{\partial N_0}{\partial\Omega}\Big),$$

$f_0 = [\exp(\bar\epsilon/T) + 1]^{-1}$ is the electron distribution function, $N_0 = [\exp(\Omega/T) - 1]^{-1}$ is the phonon distribution function.

We introduced $\tau'_{aa}$ above as the mean relaxation time for transition of electrons between groups a and a′, defined by the formula

$$\frac{\langle 1\rangle^a}{\tau_{aa'}(T)} = \int d\mathbf{p}^{a'}\Big(-\frac{\partial f_0}{\partial\varepsilon}\Big)\int d\mathbf{q}\,|U_{\mathbf{p}+\mathbf{q},a';\mathbf{p},a}|^2 N_0(\Omega) \times (1 - f_0(\varepsilon_{\mathbf{p}}^a + \Omega))\,\delta(\varepsilon^{a'}_{\mathbf{p}+\mathbf{q}} - \varepsilon_{\mathbf{p}}^a - \Omega), \quad (14)$$

where $U_{p+q,a';p,a}$ is the amplitude of the electron transition and it has been assumed that $\partial f_0/\partial\epsilon = -f_0/T$.

At low temperatures, the number of phonons which generate a transition of electrons between separate groups is exponentially small; therefore, $\tau_{aa'} \sim \exp(\Omega_0/T)$, where $\Omega_0$ is the phonon threshold energy, associated with the transition. In what follows, we shall limit ourselves for simplicity to the case of two groups of Fermi surfaces, when processes connected with second viscosity can be characterized only by a single relaxation $\tau$, which describes electron transitions between two groups of surfaces.

Initially, we shall consider the limiting cases of high ($\omega\tau \gg 1$) and low ($\omega\tau \ll 1$) frequencies, when the additional damping brought about by the second viscosity is small. In the case of high frequencies $\omega\tau \gg 1$, the Fermi surface can be considered as composed of several separate groups, for the reason that within the time of a single period electron transitions between groups do not occur and the number of electrons is conserved in the limits of each group. The dispersion equation of the system (13) in this case has the form

$$\omega = k\Big[(\hat a^{*-1})_{\varkappa\varkappa}\sum_a\frac{(\langle p_\varkappa v_\varkappa\rangle^a)^2}{\langle 1\rangle^a}\Big]^{1/2}, \quad (15)$$

where the notation

$$a_{ik}^* = a_{ik} + \frac{2i}{\omega\tau}\alpha_{ik}, \quad a_{ik} = \alpha_{ik} - \sum_a\frac{\langle p_i\rangle^a\langle p_k\rangle^a}{\langle 1\rangle^a},$$
$$\alpha_{ik} = \langle p_k p_i\rangle + \langle q_i q_k\rangle.$$

is used. Solution of the dispersion equation (15) will be

$$\omega = kV_\infty - i/\tau, \quad (16)$$

where

$$V_\infty^2 = (\hat a^{-1})_{\varkappa\varkappa}\sum_a\frac{(\langle p_\varkappa v_\varkappa\rangle^a)^2}{\langle 1\rangle^a}$$

is the speed of second sound when $\omega\tau \gg 1$. In order of magnitude,

$$V_\infty \sim v_F\Big[1 + \Big(\frac{T}{T_1}\Big)^2\Big]^{-1}, \quad T_1 = \varepsilon_F\Big(\frac{s}{v_F}\Big)^{5/4}.$$

At temperatures $T \gtrsim T_1$, the phonon averages become important in the expression for the speed of sound, the speed decreases and tends to the speed of the phonon sound (the upper curve in Fig. 2).

The low-frequency case $\omega\tau \ll 1$ is characterized by the fact that in a single period of the wave $1/\omega$ one can establish equilibrium in the system over the electron transitions between the separate groups of surfaces and therefore the entire set of separate groups can be considered as a single group. The dispersion equation of the set (13) in this case has the form

$$\omega = k\frac{(\langle p_\varkappa v_\varkappa\bar\varepsilon\rangle + \langle q_\varkappa s_\varkappa\Omega\rangle)}{[\langle\bar\varepsilon^2\rangle + \langle\Omega^2\rangle]^{1/2}}\sqrt{(\hat a^{*-1})_{\varkappa\varkappa}}, \quad (17)$$

where

$$a_{ik}^* = a_{ik} + \frac{i\omega\tau}{2}\sum_a\frac{\langle p_i\rangle^a\langle p_k\rangle^a}{\langle 1\rangle^a}$$

and has the solution

$$\omega = kV_0 - i\omega^2\tau. \quad (18)$$

Here

$$V_0^2 = \frac{(\langle p_\varkappa v_\varkappa\bar\varepsilon\rangle + \langle q_\varkappa s_\varkappa\Omega\rangle)^2}{(\langle\bar\varepsilon^2\rangle + \langle\Omega^2\rangle)}(\hat a^{-1})_{\varkappa\varkappa} -$$

is the speed of second sound in the case of low frequen-

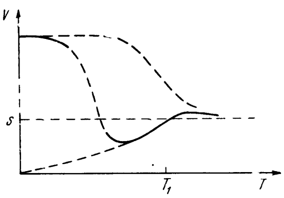


FIG. 2. Dependence of the speed of second sound in metals on the temperature in the case of a "temperature breakdown". The upper curve refers to the case of several groups of Fermi surfaces, the lower to a single group. The mean curve describes the "temperature breakdown"—the transition from the upper curve to the lower as a result of the union of several groups into one upon increase in temperature.

cies.[4] In order of magnitude, $V_0$ is

$$V_0 \sim v_F\left(\frac{T}{\varepsilon_F}\right)\left[1+\left(\frac{T}{T_1}\right)^2\right].$$

For $T \gtrsim T_1$, the sound becomes purely phonon (the lower curve in Fig. 2). In the intermediate case, when $\omega\tau \sim 1$, the absorption of second sound due to second viscosity becomes large and amounts to

$$\left(\frac{\omega''}{\omega}\right)_{ext} \sim \frac{V_\infty - V_0}{V_\infty + V_0} \sim 1,$$

since $V_\infty \gg V_0$. Consequently, the propagation of sound waves is impossible in the region of frequencies $\omega\tau \sim 1$. (One can reach a similar conclusion by considering the solution of the dispersion equation (13) for $\omega\tau \sim 1$.)

In metals, for which the distance between the separate groups of surfaces is much less than the Fermi momentum of the electron $p_F$, "temperature breakdown" can be observed. This phenomenon is such that with increase in temperature the probability of electron transitions between the separate groups of surfaces increases, such that they can be regarded as a single group. Here the phase velocity of the sound wave of frequency falls with increase in temperature from $V_\infty$, when $\omega\tau \gg 1$, to $V_0$ for $\omega\tau \ll 1$ (in the intermediate region $\omega\tau \sim 1$, the wave cannot be propagated in view of its large attenuation). For observation of the temperature breakdown, in addition to the smallness of the viscous attenuation $\omega\tau_N(v_F/V_0)^2 \ll 1$[4] it is necessary that the damping associated with transport processes be small and the transition to the lower branch of the speed of sound take place before the phonon averages in Eq. (15) become important. Fulfillment of the last two conditions guarantees the topology of the Fermi surfaces chosen above.

The region of existence of second sound in metals at constant temperature, in distinction from the ferrodielectrics, generally consists of two intervals of high ($\tau^{-1} \ll \omega \ll \tau_N^{-1}$) and low ($\tau_V^{-1} \ll \omega \ll \tau^{-1}$) frequencies, with different speeds of propagation of the sound inside each of the intervals. However, the presence of a large factor $(v_F/V_0)^2$ in the expression for viscous damping of the sound in the case of low frequencies[4] can lead to the result that the corresponding branch of the oscillations will not exist.

I take this opportunity to express my gratitude to R. N. Gurzhi and V. M. Kontorovich for posing the problem and for useful advice.

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