

CONTRIBUTION TO THE THEORY OF COLLECTIVE EXCITATIONS OF NUCLEAR MATTER

A. I. AKHIEZER, I. A. AKHIEZER, and B. I. BARTS

Submitted January 14, 1969

Zh. Eksp. Teor. Fiz. 56, 2081-2086 (June, 1969)

Volume collective excitations of nuclear matter are investigated by taking into account the electromagnetic interaction between nucleons. It is shown that electromagnetic interaction leads to a coupling between density oscillations and charge oscillations and significantly modifies the collective excitation spectrum in the long-wave region.

1. INTRODUCTION

COLLECTIVE excitations of nuclei can be connected, as is well known, both with changes of the form of the surface of the nucleus and with oscillations of all the nuclear particles, i.e., they can have a volume character. If the nucleus is sufficiently heavy, then it is possible to disregard surface effects in the analysis of its volume oscillations, and these oscillations can be treated as oscillations of the nuclear matter (abbreviated NM).

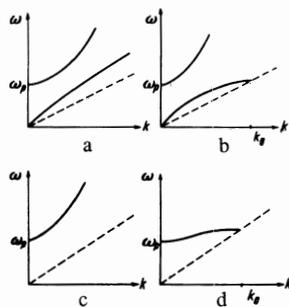
In principle, four types of volume collective excitations of NM are possible,^[1,2] in accordance with the fact that there are four different (spin and charge) states of the nucleon. Using the terminology adopted in Fermi-liquid theory,^[3] we can speak of four types of high-frequency (zero) sound in NM: density waves, spin waves, isospin waves, and coupled spin-isospin waves (we shall designate them by 0, s, i, and si, respectively). These excitations determine, in particular, the fine structure of the giant resonance, with the i-excitation corresponding to Goldhaber-Teller oscillations.^[4,5]

In the region of low-lying excitations, i.e., for sufficiently long waves, volume oscillations are characterized, as shown in^[1] and in^[2], by a linear dispersion law:

$$\omega_\alpha = s_\alpha k \quad (\alpha = 0, s, i, si), \tag{1}$$

where ω_α is the frequency of the α oscillation, \mathbf{k} is its wave vector, and s_α is the velocity of zero sound of type α (the minimum value of k is of the order of R^{-1} , where R is the dimensions of the nuclear system). In the foregoing formula, only purely nuclear forces were taken into account.

It is shown in the present paper that if electric forces are taken into account besides the nuclear forces, then the dispersion law (1) for density waves and for isospin waves may undergo appreciable modification, for the



Spectra of coupled oscillations of the density and of the charge density at various ratios between the amplitudes $F^{(0)}$: a- $F^{(0)} > 0, F^{(i)} > 0$; b- $F^{(0)} F^{(i)} < 0, F^{(s)} > 0$; c- $F^{(0)} F^{(i)} < 0, F^{(s)} < 0$; d- $F^{(0)} < 0, F^{(i)} < 0$. The dashed line corresponds to $\omega = kv_0$.

Coulomb interaction gives rise to a coupling between the density oscillations and the charge-density oscillations. The Coulomb interaction also makes it possible for oscillations to exist in those cases when oscillations of type 0 and i could not propagate without allowance for the electric forces.

The Coulomb interaction is essentially made up of 0 and i-type oscillations only in the region of sufficiently long waves. For the existence of such waves, the dimensions of the nuclear system should satisfy the condition $R \gtrsim 4 \times 10^{-13}$ cm.

The dependence of the frequencies of the coupled density oscillations and charge-density oscillations on the wave vector at different relations between the quantities $F^{(0)}$ and $F^{(i)}$ is schematically represented in the figure (the quantities $F^{(0)}$ and $F^{(i)}$ are determined by equation (10)).

2. KINETIC EQUATION

We shall describe the state of the NM by a distribution function of quasiparticles with respect to the momenta and coordinates $n(\mathbf{p}, \mathbf{r}, t)$, which is simultaneously the statistical matrix in the spin and charge variables. The unexcited state of the NM corresponds in this case to the equilibrium distribution function

$$n_0(\mathbf{p}) = \theta(\xi - \epsilon_p),$$

where ϵ_p is the energy of the quasiparticle with momenta \mathbf{p} , ξ is the endpoint energy, and $\theta(x) = \frac{1}{2}(1 + \text{sign } x)$.

At small deviations from equilibrium, the distribution function of the quasiparticles satisfies the equation

$$\left(\frac{\partial}{\partial t} + \frac{\partial \epsilon_p}{\partial \mathbf{p}} \frac{\partial}{\partial \mathbf{r}} - \frac{\partial U}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{p}} \right) n(\mathbf{p}, \mathbf{r}, t) = 0, \tag{2}$$

where $U \equiv U(\mathbf{p}, \mathbf{r}, t)$ is the potential energy of the quasiparticle.

If we confine ourselves to allowance for purely nuclear forces, then the change of the potential energy, occurring when the distribution function deviates from the equilibrium value, is given by

$$\delta U(\mathbf{p}, \mathbf{r}, t) = S_{\mathbf{p}\sigma\tau} \int \mathcal{F}(\mathbf{p}, \mathbf{p}') \delta n(\mathbf{p}', \mathbf{r}, t) \frac{d^3 p'}{(2\pi\hbar)^3}, \tag{3}$$

where $\delta n = n - n_0$ and $\mathcal{F}(\mathbf{p}, \mathbf{p}')$ is a function characterizing the interaction of two quasiparticles with momenta \mathbf{p} and \mathbf{p}' (it is a matrix with respect to the spin (σ) and isospin (τ) quantum numbers of these quasiparticles). Substitution of expression (3) for the potential energy into equation (2) leads to the well known Landau-Silin

equation^[3,6]

$$\left(\frac{\partial}{\partial t} + \frac{\partial \epsilon_p}{\partial p} \frac{\partial}{\partial r}\right) n(\mathbf{p}, \mathbf{r}, t) - \frac{\partial n_0(\mathbf{p})}{\partial p} S_{p\sigma\tau} \frac{\partial}{\partial \mathbf{r}} \int \mathcal{F}(\mathbf{p}, \mathbf{p}') n(\mathbf{p}', \mathbf{r}, t) \frac{d^3 p'}{(2\pi\hbar)^3} = 0. \quad (4)$$

We now take into account the electromagnetic interaction between the nucleons. This interaction leads to two effects: first, to electromagnetic splitting of the quasiparticle masses (more accurately, to a dependence of the quasiparticle energy on the isospin variable) and, second, to an additional potential energy of the charged quasiparticles:

$$U^{(c)}(\mathbf{r}, t) = e \frac{1 + \tau_3}{2} \varphi(\mathbf{r}, t), \quad (5)$$

where $\varphi(\mathbf{r}, t)$ is the potential of the electric field connected with the distribution function of the quasiparticles by the Poisson equation

$$\Delta \varphi(\mathbf{r}, t) = -4\pi e S_{p\sigma\tau} \int \frac{1 + \tau_3}{2} n(\mathbf{p}, \mathbf{r}, t) \frac{d^3 p}{(2\pi\hbar)^3}, \quad (6)$$

and τ_3 is a Pauli matrix (since the ratio of the velocity of the nucleon in the nucleus to the velocity of light is small, no account is taken of magnetic forces in formula (5)).

The first of these effects is small—the relative electromagnetic splitting of the quasiparticle masses is of the same order of magnitude as the fine structure constant, and we shall not take into account this effect. On the other hand, the second effect can be quite appreciable, for in the region of small momentum transfers the amplitude of the Coulomb scattering may become comparable with or even exceed the amplitude of the purely nuclear scattering. Therefore the additional potential energy due to the Coulomb interaction of the quasiparticles leads to a significant change of the dispersion laws of the NM oscillations in the long-wave region.

Returning to relation (3), we represent the matrix $\mathcal{F}(\mathbf{p}, \mathbf{p}')$ in the form

$$\mathcal{F} = \mathcal{F}^{(0)} + \mathcal{F}^{(s)}(\sigma, \sigma') + \mathcal{F}^{(i)}\tau\tau' + \mathcal{F}^{(si)}(\sigma, \sigma')\tau\tau', \quad (7)$$

where $\mathcal{F}^{(0)}$ and $\mathcal{F}^{(i)}$ are scalar functions of the momenta \mathbf{p} and \mathbf{p}' , while $\mathcal{F}^{(s)}$ and $\mathcal{F}^{(si)}$ are functions of the momenta, which are simultaneously matrices in spin space.

Recognizing that at low excitation energies, $\hbar\omega \ll \zeta$, the fluctuations of the distribution function are possible only near the Fermi surface, we can represent δn in the form

$$\delta n = \delta(\epsilon_p - \zeta) \{v^{(0)} + \sigma v^{(s)} + \tau v^{(i)} + \sigma_\mu \tau_\nu v_{\mu\nu}^{(si)}\}, \quad (8)$$

where $v^{(\alpha)} \equiv v^{(\alpha)}(\mathbf{n}, \mathbf{r}, t)$ are certain functions of the coordinates, of the time, and of the vector $\mathbf{n} = \mathbf{p}/p$.

Substituting (3) and (5) in (2), we can easily verify that the electric forces enter only in the equations for the quantities $v^{(0)}$ and $v_3^{(i)}$. On the other hand, the equations for the remaining quantities $v^{(\alpha)}$ (namely for the quantities $v^{(s)}$, $v^{(si)}$, $v_1^{(i)}$ and $v_2^{(i)}$) are not affected at all by the Coulomb interaction. We call attention to the fact that without allowance for the electromagnetic interaction the i -sound was triply degenerate, owing to the isotopic invariance of the nuclear forces; when the electric forces are taken into account, this degeneracy is partially lifted, since the electric forces separate the 3 axis in isotopic space.

3. OSCILLATION SPECTRA

Let us determine now the NM oscillation spectra. Putting

$$v^{(\alpha)}(\mathbf{n}, \mathbf{r}, t) = v^{(\alpha)}(\mathbf{n}) \exp\{i\mathbf{k}\mathbf{r} - i\omega t\}$$

and using relations (2), (3), (5), and (8), we obtain the following equations for the quantities $v^{(0)}$ and $v_3^{(i)}$:

$$(\omega - kv_0) v^{(0)}(\mathbf{n}) - kv_0 \int \frac{d\sigma'}{4\pi} \{ (F^{(0)} + F^{(c)}) v^{(0)}(\mathbf{n}') + F^{(c)} v_3^{(i)}(\mathbf{n}') \} = 0,$$

$$(\omega - kv_0) v_3^{(i)}(\mathbf{n}) - kv_0 \int \frac{d\sigma'}{4\pi} \{ (F^{(i)} + F^{(c)}) v_3^{(i)}(\mathbf{n}') + F^{(c)} v^{(0)}(\mathbf{n}') \} = 0, \quad (9)$$

where v_0 is the limiting velocity of the quasiparticles,

$$F^{(\alpha)} \equiv F^{(\alpha)}(\cos \chi) = \frac{2p_0 m^*}{\pi^2 \hbar^3} \mathcal{F}^{(\alpha)}(\mathbf{p}, \mathbf{p}') \Big|_{p=p'=p_0}$$

$$F^{(c)} \equiv F^{(c)}(k) = \frac{2p_0 m^*}{\pi \hbar^3} \frac{e^2}{k^2}, \quad (10)$$

p_0 is the limiting momentum, $m^* = p_0/v_0$ is the effective mass of the quasiparticle, χ is the angle between vectors \mathbf{p} and \mathbf{p}' , and $d\sigma'$ is the solid-angle element of the vector \mathbf{n}' . As to the equations for the remaining quantities $v^{(\alpha)}$, these, as noted above, have the same form as without allowance for the electric forces. Thus, allowance for the Coulomb interaction does not change the character of the spin sound, the spin-isospin sound, and two (out of three) branches of the isotopic sound; we shall therefore not consider these oscillations.

Going over to the study of the oscillations of the quantities $v^{(0)}$ and $v_3^{(i)}$, we assume for simplicity that $F^{(0)}$ and $F^{(i)}$ do not depend on the angle χ . Equations (9) can then be reduced to the form

$$v^{(0)} \{1 + w(F^{(0)} + F^{(c)})\} + v_3^{(i)} w F^{(c)} = 0,$$

$$v^{(0)} w F^{(c)} + v_3^{(i)} \{1 + w(F^{(i)} + F^{(c)})\} = 0, \quad (11)$$

where

$$v^{(\alpha)} = \int v^{(\alpha)}(\mathbf{n}) \frac{d\sigma}{4\pi}, \quad w \equiv w(\eta) = \frac{1}{2} \int_{-1}^1 \frac{x dx}{x - \eta + i0} \quad (12)$$

and $\eta = \omega/kv_0$. Equating the determinant of the system (11) to 0, we obtain the dispersion equation for the oscillations in question:

$$\{1 + w(F^{(0)} + F^{(c)})\} \{1 + w(F^{(i)} + F^{(c)})\} - w^2 F^{(c)2} = 0. \quad (13)$$

If $|\omega| < kv_0$ then, according to (12), the function w has an imaginary part; solving in this case the dispersion equation (13), we obtain complex expressions for the frequencies corresponding to rapidly damped oscillations. The rapid damping of the oscillations with phase velocities $\omega/k < v_0$ is obviously due to the resonant absorption of these oscillations by the quasiparticles, the velocity of which is equal to the phase velocity of the oscillations.

When $|\omega| > kv_0$, the function w is real and has the form

$$w(\eta) = 1 - \frac{\eta}{2} \ln \left| \frac{\eta + 1}{\eta - 1} \right|.$$

In this case equation (13) can have real solutions corresponding to undamped oscillations.

At large values of the wave vector \mathbf{k} , the quantity $F^{(c)}(\mathbf{k})$ is small, and equation (13) breaks up into two equations:

$$F^{(0)}w(\eta) + 1 = 0, \quad F^{(i)}w(\eta) + 1 = 0. \quad (14)$$

These equations have solutions with $\eta > 1$, corresponding to undamped α oscillations if $F^{(\alpha)} > 0$ ($\alpha = 0, i$).

At small values of k , $k \lesssim k_c$, where $k_c^2 = e^2 p_0 m^* F^{-1} \hbar^{-3}$ and $F = \min \{F^{(0)}, F^{(i)}\}$, it is necessary to take $F^{(c)}$ into account. The two branches of the oscillations then become entangled, and since $F^{(c)}$ depends on the wave vector k , the dispersion law for the oscillations is no longer linear. The dependence of the oscillation frequencies on the wave vector is determined, in accordance with (13), by the equations

$$F_1(k)w(\eta) + 1 = 0, \quad F_2(k)w(\eta) + 1 = 0, \quad (15)$$

where

$$F_{1,2}(k) = \frac{F^{(0)}F^{(i)} + 2F^{(+)}F^{(c)}(k)}{F^{(+)} + F^{(c)}(k) \pm \sqrt{F^{(+)-2} + F^{(c)2}(k)}} \quad (16)$$

and $F^{(\pm)} = \frac{1}{2}(F^{(0)} \pm F^{(i)})$. These equations have the same structure as equations (14), and the role of the amplitudes $F^{(\alpha)}$ is played by the effective amplitudes $F_{1,2}(k)$, which depend on the wave vector k .

At small values of k , the effective amplitudes are given by

$$F_1(k) = F^{(+)}, \quad F_2(k) = 2F^{(c)}(k). \quad (17)$$

The first equation of (15) corresponds to an oscillation with a linear dispersion law, while the second corresponds to an oscillation with frequency $\omega = \omega_p$, where $\omega_p^2 = 4\pi e^2 n_p / m^*$ and n_p is the proton density. When $k \rightarrow 0$, the second oscillation is a plasma (Langmuir) oscillation of the proton component of the NM.

The condition for the existence of undamped oscillations when account is taken of the Coulomb interaction is that the effective amplitudes F_1 and F_2 be positive. In the long-wave region this condition can obviously be satisfied also at negative values of one or both quantities $F^{(0)}$ and $F^{(i)}$. In particular, according to equations (15) and (17), at small values of k there always exists an undamped plasma branch; if at the same time $F^{(*)} > 0$, then the oscillation with the linear dispersion law is likewise undamped.

Depending on the signs of the quantities $F^{(0)}$, $F^{(i)}$, and $F^{(*)}$, four different variants of the spectra of the

coupled oscillations of the density and of the charge density are possible (see the figure). The point k_0 in the figure corresponds to the vanishing of the effective amplitude, and when $k > k_0$ the corresponding oscillation attenuates rapidly.

According to (10) and (16), the influence of the Coulomb interaction on the oscillation spectrum is determined by the parameter

$$\xi = \frac{F^{(c)}}{\min \{F^{(0)}, F^{(i)}\}} \sim \frac{4\pi e^2 n}{k^2 r_c},$$

where ξ is the end-point energy of the nucleons and n is their density. The minimum value of k is obviously of the order of magnitude of R^{-1} , where R is the dimensions of the nuclear system. We note that $\xi \sim \hbar^2 / r_0^2 M$, where M is the mass of the nucleon and r_0 is the average distance between nucleons, so that we obtain for the parameter ξ a maximum value

$$\xi_{\max} = \frac{e^2}{\hbar c} \frac{r_0}{r_c} \left(\frac{R}{r_0} \right)^2,$$

where $r_c = \hbar / Mc$ is the Compton wavelength of the nucleon. Thus, the Coulomb interaction can greatly alter the character of the 0 and i -oscillations of the NM, if the dimensions of the nuclear system satisfy the condition

$$R \geq \sqrt{\frac{r_0 r_c}{e^2 / \hbar c}} \sim 4 \cdot 10^{-13} \text{ cm.}$$

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Translated by J. G. Adashko