ANALYSIS OF POSSIBLE DEVIATIONS FROM THE V–A THEORY OF WEAK INTERACTIONS

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Submitted November 6, 1968

Zh. Eksp. Teor. Fiz. 56, 1904–1913 (June, 1969)

A weak interaction model is considered in which the (e, ν_e) current has the unaltered V–A structure, whereas the most general expression for the (μ, ν_μ) current including derivatives is employed and allowance is made for a possible nonzero mass of the muonic neutrino. Expressions are obtained for the decay probability of polarized muons and of $\pi_{\mu 2}$ and $K_{\mu 2}$ decays. Estimates for the constants of the interactions introduced are discussed.

1. INTRODUCTION. FORMULATION OF THE PROBLEM

A. The universal V–A theory of weak interactions is apparently in good agreement with all known experimental data at least in those cases, where quantitative results can be obtained from the theory.

The muon decay

$$\mu \to e + \nu_e + \bar{\nu}_\mu \tag{1}$$

is of special importance in weak interaction physics, since it is so far the only experimentally accessible weak leptonic process. A quantitative theory for the process (1) has been constructed[1] after the discovery of parity and C invariance violation even before the advent of the V–A theory. In this theory it was sufficient to require that the emitted neutrinos are longitudinal and that a neutrino and an antineutrino be emitted in (1). At this stage the theory contained two (real) parameters. The V–A interaction fixes the ratio of these two parameters.

Together with the scheme of a current-current interaction which is local with respect to the lepton current, one of the most important assumptions of the present theory of the leptonic and the so-called semi-leptonic weak processes is the muon-electron universality. However, the data on the muon current (μ, ν_μ) are apparently less accurate than those on the structure of the electron current (e, ν_e). Moreover, it is not excluded that the muonic neutrino have a nonvanishing rest mass.

The appearance[2] of new, much more accurate experimental data on the spectrum and the asymmetry of the electrons in the decay (1) stimulated us to undertake a new analysis of the possible deviations from the usual theory of the process (1) in order to be able to test quantitatively the validity of the V–A theory and to draw conclusions on possible admixtures of interactions different from the V–A. To this end we consider a model of possible deviations from the V–A theory. The resulting estimates for the deviations depend, of course, on the model.

B. Below we assume a current-current interaction

$$\mathscr{L}_{int} \sim j_\alpha^{(e)} J_\alpha^{+(\mu)}, \tag{2}$$

where $j_\alpha^{(e)}$ is a V–A current constructed from the electron and the electronic neutrino:

$$j_\alpha^{(e)} = \bar{\psi}_e \gamma_\alpha (1 + \gamma_5) \psi_{\nu_e}. \tag{3}$$

For the muonic current $J_\alpha^{(\mu)}$ we employ the most general expression

$$J_\alpha^{+(\mu)} = \bar{\psi}_{\nu_\mu} \left[\left\{ \gamma_\alpha + \frac{b_+}{m_\mu}(p + q_2)_\alpha + \frac{c_+}{m_\mu}(p - q_2)_\alpha \right\} \Pi_+ + \left\{ a\gamma_\alpha + \frac{b_-}{m_\mu}(p + q_2)_\alpha + \frac{c_-}{m_\mu}(p - q_2)_\alpha \right\} \Pi_- \right] \psi_\mu = \bar{\psi}_{\nu_\mu} [\Gamma_+^\alpha \Pi_+ + \Gamma_-^\alpha \Pi_-] \psi_\mu. \tag{4}$$

With (2) and (4) we obtain for the effective Lagrangian for the process (1)

$$\mathscr{L}_{int} = \frac{G}{\sqrt{2}} \bar{\psi}_{\nu_\mu} [\Gamma_+^\alpha \Pi_+ + \Gamma_-^\alpha \Pi_-] \psi_\mu \bar{\psi}_e \gamma_\alpha (1 + \gamma_5) \psi_{\nu_e}, \tag{5}$$

where $\Pi_\pm = 1 \pm \gamma_5$, p(**p**, E_μ), and $q_2(\mathbf{q}_2, \omega_2)$ are the four-momenta of the muon and the muonic neutrino, and k(**k**, ω) and $q_1(\mathbf{q}_1, \omega_1)$ are the four-momenta of the electron and the electronic neutrino, and m_μ and m_e are the masses of the muon and the electron.

It is clear that only the first two terms remain in (4) if the V–A theory is true. For $b_\pm \neq 0$, $c_\pm \neq 0$, derivatives of the fields ψ_{ν_μ} and ψ_μ are included in the interaction.[1)] By a Fierz transformation, (5) can be brought into the form

$$\mathscr{L}_{int} = \frac{G}{\sqrt{2}} \Big\{ -\bar{\psi}_{\nu_\mu} \gamma_\alpha (1 + \gamma_5) \psi_{\nu_e} \bar{\psi}_e \gamma_\alpha (1 + \gamma_5) \psi_\mu + 2a \bar{\psi}_{\nu_\mu} (1 + \gamma_5) \psi_{\nu_e} \bar{\psi}_e (1 - \gamma_5) \psi_\mu + \frac{1}{2} \left[\frac{b_+}{m_\mu}(p + q_2)_\alpha + \frac{c_+}{m_\mu}(p - q_2)_\alpha \right]$$
$$\times [\bar{\psi}_{\nu_\mu} (1 + \gamma_5) \psi_{\nu_e} \bar{\psi}_e \gamma_\alpha (1 + \gamma_5) \psi_\mu - \bar{\psi}_{\nu_\mu} \sigma_{\alpha\beta} (1 + \gamma_5) \psi_{\nu_e} \bar{\psi}_e \gamma_\beta (1 + \gamma_5) \psi_\mu] + \frac{1}{2} \left[\frac{b_-}{m_\mu}(p + q_2)_\alpha + \frac{c_-}{m_\mu}(p - q_2)_\alpha \right]$$
$$\times [\bar{\psi}_{\nu_\mu} \gamma_\alpha (1 + \gamma_5) \psi_{\nu_e} \bar{\psi}_e (1 - \gamma_5) \psi_\mu + \bar{\psi}_{\nu_\mu} \gamma_\beta (1 + \gamma_5) \psi_{\nu_e} \bar{\psi}_e \sigma_{\alpha\beta} (1 - \gamma_5) \psi_\mu] \Big\}. \tag{6}$$

From (6) the difference between our model and the models considered earlier becomes clear. Bahcall and Curtis[6] allowed for a nonzero mass of the muonic neutrino $m_{\nu_\mu} = m_2$ in the calculation of the rate of the

[1)] We note at once that the introduction of derivatives differs from the method of Lee and Yang, [3] who admitted the existence of derivatives both in $j_\alpha^{(e)}$ and $J_\alpha^{(\mu)}$, and from the approaches of Bludman and Klein [4] and Bergia and Russo. [5]

process (1), assuming an $\mathscr{L}_{int}$ in accordance with the V—A theory. Friedberg[7] obtained formulas for the decay spectrum of the muon, admitting terms without derivatives in (5). Marshak et al.[8] left the structure of the (μ, ν_μ) current unchanged and admitted a term $\gamma_\alpha \Pi_-$ in the (e, ν_e) current.

The effect of an A + V admixture has recently been considered in another model by Lipmanov and Mikheev.[9]

After some discussion of the general expression for the interaction, we obtain below formulas for the spectrum of the electrons, their asymmetry and polarization, the helicity of the neutrino, and the total rate of the process (1). The calculations are carried out in two steps. First we allow for a nonzero mass m_2 in an interaction without derivatives (i.e., with $b_{+-} = c_{+-} = 0$, but $a \neq 0$). Then we consider the interaction (5) with derivatives with $m_2 \neq 0$. In the absence of derivatives we allow for complex values of a in order to estimate the transverse polarization of the electrons arising from CP violation. For interactions with derivatives (and $m_2 = 0$) this effect has been considered in[5].

The change in the structure of the muon current affects also other processes. Of these, we consider here the $\Pi_{\mu 2}$ and $K_{\mu 2}$ decays.

C. The general expression (4) for the current $J_\alpha^{(\mu)}$ agrees formally with that for the baryon current in semi-leptonic processes (without account of G invariance). The quantities a, b_{+-}, and c_{+-}, whose analogues in the baryon current are the form factors, are constant numbers in the case of the process (1). The truth of this assertion can be verified in the following way.

Since the current $j_\alpha^{(e)}$ is local, the quantities a, b_{+-}, and c_{+-} can only be functions of the momentum transfer $k - q_1$ to the pair $\mu - \nu_\mu$, but owing to the locality of the lepton current $J_\alpha^{(\mu)}$ these same quantities must be functions of the momentum transfer $p - q_2 = k + q_1$ to the pair $e - \nu_e$. These two conditions can be satisfied only when a, b_{+-}, and c_{+-} are constants. Neglecting small CP violations, these constants are real.

Not all of the terms introduced in (4) are equally effective in the process (1), even though their existence has been allowed for. It is easy to see from (3) and (5) that, because $p - q_2 = q_1 + k$ in (1),

$$\begin{aligned} c_+(\bar{\nu}_\mu (q_1+k)_\alpha \Pi_+ \mu)(\bar{e}\gamma_\alpha(1+\gamma_5)\nu_e) &= c_+ m_e (\bar{\nu}_\mu \Pi_+ \mu)(\bar{e}\Pi_+ \nu_e), \\ c_-(\bar{\nu}_\mu \Pi_- \mu)(\bar{e}(\hat{q}_1+\hat{k})\Pi_+ \nu_e) &= c_- m_e (\bar{\nu}_\mu \Pi_- \mu)(\bar{e}\Pi_+ \nu_e). \end{aligned} \tag{7}$$

The contribution of the quantities c_{+-} is proportional to the mass of the electron and is much less effective than the contribution of b_{+-}. Thus it is quite difficult to observe even the presence of the quantities c_{+-} in the investigation of (1).

For the other processes with emission of the pair $\nu_\mu - \mu$ the effectiveness of the different "new" terms can be different. Thus, for the $\pi_{\mu 2}$ decay, whose matrix element

$$M(\pi \to \mu) = f_\pi (p_\pi)_\alpha J_\alpha^{(\mu)} \tag{8}$$

(where $p_\pi = p + q_2$, and f_π is the pion form factor) can be rewritten as

$$\begin{aligned} M(\pi \to \mu) = f_\pi m_\mu \bar{u}_{\nu_\mu} \{ [1 + a(m_2/m_\mu) \\ + b_-(m_\pi/m_\mu)^2 + c_-(1 - m_2^2/m_\mu^2)] \Pi_- \\ + [a + m_2/m_\mu + b_+(m_\pi/m_\mu)^2 + c_+(1 - m_2^2/m_\mu^2)] \Pi_+ \} u_\mu \end{aligned} \tag{9}$$

the contributions from b_{+-} and c_{+-} are comparable.

For the amplitude for the $K_{\mu 2}$ decay an analogous expression is obtained with the obvious replacement $\pi \to K$, $m_\pi \to m_K$.

2. MUON DECAY

A. Let us first consider process (1), setting $b_{+-} = c_{+-} = 0$ in (4) but allowing for $m_2 \neq 0$ and complex values of a (CP violation). We introduce the usual four-vectors for the polarization of the electron:

$$s' \equiv \begin{cases} s_4' = (\mathbf{k}\boldsymbol{\zeta})/m_e \\ \mathbf{s}' = \boldsymbol{\zeta} + (\mathbf{k}\boldsymbol{\zeta})\mathbf{k}/m_e(E+m_e) \end{cases}$$

and the muon:

$$s \equiv \begin{cases} s_4 = (\mathbf{p}\boldsymbol{\eta})/m_\mu \\ \mathbf{s} = \boldsymbol{\eta} + (\mathbf{p}\boldsymbol{\eta})\mathbf{p}/m_\mu(E_\mu + m_\mu) \end{cases}.$$

For the decay rate in the rest system of the muon we have

$$\begin{aligned} dW(E,\theta) = \frac{G^2 m_\mu}{16\pi^4}(1-\alpha)^2 (E^2 - m_e^2)^{1/2}\, dE\, d\Omega \\ \times \Big\{ \Big[(1 + \alpha + 2|a|^2) E(E_0 - E) - m_2\Big(E - \frac{m_e^2}{m_\mu}\Big)\mathrm{Re}\, a \\ + \frac{1+2\alpha}{3}(E^2 - m_e^2)\Big] + (\mathbf{k}\boldsymbol{\eta})\Big[\frac{1-\alpha-6|a|^2}{3}(E_0 - E) + m_2\, \mathrm{Re}\, a \\ - \frac{1+2\alpha}{3}\Big(E - \frac{m_e^2}{m_\mu}\Big)\Big] - (\mathbf{k}\boldsymbol{\zeta})\Big[(1+\alpha+2|a|^2)(E_0 - E) \\ - \frac{1+2\alpha}{3}\Big(\frac{m_e^2}{m_\mu} - E\Big) - m_2\, \mathrm{Re}\, a - (\mathbf{k}\boldsymbol{\eta})\Big\{\frac{1}{3}\Big(1 + 2\alpha - 3\frac{m_2}{m_\mu}\mathrm{Re}\, a\Big) \\ - \frac{1-\alpha-6|a|^2}{3}\frac{E_0 - E}{E + m_e} - \frac{m_2}{E+m_e}\Big(1 - \frac{E}{m_\mu}\Big)\mathrm{Re}\, a\Big\}\Big] \\ - m_e(\boldsymbol{\zeta}\boldsymbol{\eta})\Big[\frac{1-\alpha-6|a|^2}{3}(E_0 - E) + m_2\Big(1 - \frac{E}{m_\mu}\Big)\mathrm{Re}\, a\Big] \\ - \frac{m_2 m_e}{m_\mu}(\boldsymbol{\zeta}[\mathbf{k}\boldsymbol{\eta}])\, \mathrm{Im}\, a \Big\}; \end{aligned} \tag{10*}$$

here

$$\alpha = \frac{m_2^2}{q^2} = \frac{m_2^2}{(p-k)^2} = \frac{m_2^2}{2m_\mu(T_e^{max} - T_e) + m_2^2}, \quad E_0 = \frac{m_\mu^2 + m_e^2}{2m_\mu},$$

and the energy of the electron varies from $E_{min} = m_e$ to

$$E_{max} = E_0 - \frac{m_2^2}{2m_\mu} = T_e^{max} + m_e.$$

We note that α is small for all values of the electron energy except for the small region near T_e^{max}, where $\alpha \cong 1$. The "width" of this energy region is

$$\Delta T^e \cong m_2^2/2m_\mu \lesssim 20 \text{ keV}.$$

For $\zeta = 0$, (10) goes over into the result of Friedberg, and for a = 0, into the result of Bahcall and Curtis. Since there is no term proportional to the electron mass in the part of the spectrum (10) which is independent of the polarization, the Michel parameter $\eta = 0$ in our model.

The spectrum of the electrons is sensitive to m_2 and to the values $a = \pm 0.1$ only for very small ($\epsilon = E/2m_\mu$

*$[\mathbf{k}\eta] \equiv \mathbf{k} \times \eta$.

< 0.05) and very large ($\epsilon > 0.9$) energies of the electrons. The accuracy of the measurement of dW(E) must be some ten percent. It is seen from the estimates that the study of the μ decay spectrum does not allow one to lower appreciably the limit on m_2 (compared to $m_2 \cong 2$ Mev).

The energy dependence of the asymmetry of the decay of a polarized muon is somewhat more sensitive to m_2 and a than the spectrum of the decay of an unpolarized muon (also for low and high energies).

We note that evidently it is not always possible to regard the quantities a and m_2 as nonzero independently of each other. For $m_2 \neq 0$, we necessarily have $a \neq 0$. Finally, there may be reasons for which $m_2 = 0$ when $a \neq 0$. In any case, the kinematic effects remaining in (10) for a = 0 are quadratic in m_2. The assumption $a \neq 0$ leads to additional terms of order am_2.

For the entire consideration of this paper (for small a and m_2) it is very important to know the effect of radiative corrections. We assume that the theory can be compared with experiment after the radiative corrections have been duly taken into account. The last to consider the radiative corrections (for $a \neq 0$) has been Allcock.[10] He arrived at the conclusion that their effect does not exceed 0.2%.

Expression (10) can be written in the form

$$dW = dW_0 + (\mathbf{k}\boldsymbol{\eta})dW_1 + (-\mathbf{k}\boldsymbol{\zeta})dW_2 + (\boldsymbol{\eta}\boldsymbol{\zeta})dW_3 + (\boldsymbol{\zeta}[\mathbf{k}\boldsymbol{\eta}])dW_4. \tag{11}$$

The expression proportional to $(\boldsymbol{\xi} \cdot \mathbf{k} \times \boldsymbol{\eta})$ determines the polarization of the electrons from the decay of polarized muons perpendicular to the decay plane $\mathbf{k}, \boldsymbol{\eta}$.

It is seen from (10) that

$$dW_4 \sim 6\frac{m_2 m_e}{m_\mu^2}\,\mathrm{Im}\,a$$

and the component of the electron polarization connected with CP violation contains a small factor 10^{-3} besides Im a. It is interesting to note that dW_4 vanishes for $m_2 = 0$.

Thus, in our model which allows for deviations from the V—A theory, the effects of CP violation are connected with the nonzero mass of the muonic neutrino.

As is seen from (10) and (11), the expression for $dW_0 + (\mathbf{k}\boldsymbol{\eta})dW_1$ does not correspond to the expression usually considered (with $\eta = 0$)

$$\frac{dW}{d\Omega}(\varepsilon, \rho, \delta) = \frac{1}{\pi\tau}\left\{\left[3(1-\varepsilon) + 2\rho\left(\frac{4}{3}\varepsilon - 1\right)\right] \mp \xi\cos\theta\left[(1-\varepsilon) + 2\delta\left(\frac{4}{3}\varepsilon - 1\right)\right]\right\}\varepsilon^2 d\varepsilon. \tag{12}$$

If we determine the parameters ρ and δ by quadratic approximation to the two expressions in the square brackets in (12), then

$$\rho = \frac{3}{4}\frac{1 - 6\beta\,\mathrm{Re}\,a}{1 + |a|^2 - 4\beta\,\mathrm{Re}\,a}, \quad \beta = \frac{m_2}{m_\mu}, \tag{13}$$

which for real a agrees with the result of Allcock,[10], and

$$\delta = \frac{3}{4}\frac{1 - 6\beta\,\mathrm{Re}\,a}{1 + 3|a|^2 - 12\beta\,\mathrm{Re}\,a}, \tag{14}$$

and

$$\xi = \frac{1 + 3|a|^2 - 12\beta\,\mathrm{Re}\,a}{1 + |a|^2 - 4\beta\,\mathrm{Re}\,a}. \tag{15}$$

It is seen from (13) to (15) that the following relation holds between the parameters ρ, δ, and ξ in our model:

$$\rho/\xi = \delta.$$

The total decay rate of the muon is equal to

$$W = \frac{1}{\tau} = \frac{G^2 m_\mu^5}{192\pi^3}[1 + |a|^2 - 4\beta\,\mathrm{Re}\,a]. \tag{16}$$

For the polarization of the neutrino H_{ν_μ} we have (for real a) with $\omega_2 \gg m_2$

$$H_{\nu_\mu} = -(1 - a^2)/(1 + a^2).$$

Thus the most difficult measurements of H_{ν_μ} must be more accurate than by a few percent in order to yield new information on the quantity $a \lesssim 0.1$.

For the polarization of the electrons

$$\mathbf{P}_e = P_e \mathbf{v}/c \tag{17}$$

we have

$$P_e = (\varepsilon^2 - 4\lambda^2)^{1/2}[3u + 3\beta^2 + 6a^2 u - 6a\beta + (1 + 2\beta^2/u)(\varepsilon - 2\lambda^2)] \times [(1 + 2\beta^2/u)(\varepsilon^2 - 4\lambda^2) - 6a\beta(\varepsilon - 2\lambda^2) + (3u + 6a^2 + 3\beta^2)\varepsilon]^{-1},$$

where

$$u = 1 - \varepsilon + \lambda^2, \quad \lambda = m_e/m_\mu, \quad \beta = m_2/m_\mu,$$

It is seen from (17) that $P_e \sim 1$ up to terms of order $\lambda^2 \sim 10^{-5}$.

B. Let us now consider process (1) by assuming (4) and (5) and the reality of a, $b_\pm$, and $c_\pm$, neglecting terms of order m_e^2/m_μ^2 and m_2^2/m_μ^2.

For the decay rate we obtain

$$dW = dW_0(1 - \mathbf{n}\boldsymbol{\zeta}) + (\mathbf{k}\boldsymbol{\eta})dW_1 + (\mathbf{k}\boldsymbol{\zeta})(\mathbf{k}\boldsymbol{\eta})dW_2 + (\boldsymbol{\eta}\boldsymbol{\zeta})dW_3, \tag{18}$$

where

$$dW_0 = \frac{G^2 m_\mu^5}{64\pi^4}d\Omega\,\varepsilon^2 d\varepsilon\left\{\left[\frac{3 - 2\varepsilon}{6} + a^2(1 - \varepsilon) - a\beta\right] + [\beta(b_+ + ab_-) + 2\beta b_+ b_- + (ab_+ + b_-)](1 - \varepsilon) + \frac{b_-^2 + b_+^2}{6}(3 - 4\varepsilon + \varepsilon^2)\right\}. \tag{19}$$

We note that (19) contains terms which are proportional to the first power of $\beta = m_2/m_\mu$, but as before, $\eta = 0$. Even if we neglect terms of order β in (19), the main effect of the terms with $b_\pm$ reduces to the replacement of the decay spectrum of unpolarized muons, which in the V—A theory has the form

$$\varepsilon^2 d\varepsilon(3 - 2\varepsilon),$$

by the expression

$$\varepsilon^2 d\varepsilon[3A - 2B\varepsilon + C\varepsilon^2] = \varepsilon^2 d\varepsilon[\{3 + 6a^2 + 6(b_- + ab_+) + b_-^2 + b_+^2\} - \{2 + 6a^2 + 6(b_- + ab_+) - 4(b_-^2 + b_+^2)\}\varepsilon + (b_-^2 + b_+^2)\varepsilon^2]. \tag{19'}$$

Comparison of (19) with the existing experimental data yields the result that a and b_{+-} do not exceed 0.1. The form of the spectrum depends appreciably on the value of b_{+-}. A more careful analysis may lead to a more accurate determination of a and b_{+-}.

Integration of (19) leads to the following expression for the total decay rate of the muon:

$$W=\frac{G^2m_\mu^5}{192\pi^3}L,\quad L=1+a^2-4a\beta+\beta(b_++ab_-)+2\beta b_+b_-+ab_++b_-+\frac{2}{5}(b_-^2+b_+^2). \tag{20}$$

Determining the parameter ρ, as before, by quadratic approximation of (19) to the Michel function (12), we obtain

$$\rho={}^3/_4[1-6a\beta-{}^1/_3(b_-^2+b_+^2)]L^{-1} \tag{21}$$

[L is defined by (20)].

The expression for dW_1

$$dW_1=\frac{G^2m_\mu^4}{192\pi^4}d\Omega\,\varepsilon\,d\varepsilon(\mathbf{k}\boldsymbol{\eta})\{1-2\varepsilon-6a^2(1-\varepsilon)+6a\beta+6\beta(b_+-ab_-)(1-\varepsilon)+(b_+^2-b_-^2)(\varepsilon^2-1)+(1-\varepsilon)(2b_+-6ab_-)\}=\frac{G^2m_\mu^4}{192\pi^4}d\Omega\,\varepsilon\,d\varepsilon(\mathbf{k}\boldsymbol{\eta})\{D-2F\varepsilon+R\varepsilon^2\} \tag{22}$$

in the model allowing for deviations from the V—A theory replaces the usual expression for the energy dependence of the asymmetry of the decay of polarized muons

$$(\mathbf{k}\boldsymbol{\eta})\,\varepsilon\,d\varepsilon\,(1-2\varepsilon).$$

For the integral asymmetry coefficient ξ defined by

$$dN(\theta)=W(1\mp{}^1/_3\xi\cos\theta)\,d\Omega/4\pi, \tag{23}$$

we obtain from (18), (19) and (21)

$$\xi=\mathcal{M}L^{-1}=L^{-1}[1+3a^2-12a\beta-2\beta(b_+-ab_-)+{}^4/_5(b_+^2-b_-^2)-(b_+-3ab_-)]. \tag{24}$$

For the parameter δ defined similarly as before, we have

$$\delta={}^3/_4[1-6a\beta+{}^1/_{30}(b_+^2-b_-^2)]\mathcal{M}^{-1}, \tag{25}$$

where $\mathcal{M}$ is determined by (24).

It is seen from the expressions for dW_2 and dW_3,

$$\begin{aligned}(\mathbf{k}\boldsymbol{\eta})(\mathbf{k}\boldsymbol{\zeta})dW_2=&\frac{G^2m_\mu^5}{192\pi^3}\frac{d\Omega}{4\pi}\varepsilon^3d\varepsilon(\mathbf{n}\boldsymbol{\zeta})(\mathbf{n}\boldsymbol{\eta})\\ &\times\Big\{\Big[1-3a\beta-\frac{3\beta}{2}(ab_--b_+-c_++ac_-)-\frac{1}{2}(1-\varepsilon)(b_+c_+-b_-c_-\\ &-3b_+^2+3b_-^2)-\frac{b_-}{2}(1-2\varepsilon)-\frac{3ac_+}{2}(1-\varepsilon)+\frac{c_-}{2}+\frac{3ab_+}{2}(1-\varepsilon)\Big]\\ &-\frac{1}{2}\frac{m_\mu}{(E+m_e)}\Big[(1-6a^2)(1-\varepsilon)+3a\beta(2-\varepsilon)-3\beta\Big\{2(ab_--b_+)\\ &-\frac{\varepsilon}{2}(c_+-ac_-+3ab_--3b_+)\Big\}-\frac{1}{2}\{(1-\varepsilon)(b_+c_+-b_-c_-)\varepsilon\\ &+(1-\varepsilon)(2-\varepsilon)(b_+^2-b_-^2)\}+\frac{\varepsilon}{2}\{c_--3ac_+(1-\varepsilon)\}\\ &+b_-\Big(2+\varepsilon^2-\frac{5}{2}\varepsilon\Big)-ab_+\Big(6-\frac{15}{2}\varepsilon+\frac{3}{2}\varepsilon^2\Big)\Big]\Big\}\end{aligned} \tag{26}$$

and

$$\begin{aligned}dW_3(\boldsymbol{\eta}\boldsymbol{\zeta})=&\frac{G^2m_\mu^5}{192\pi^3}\frac{d\Omega}{2\pi}\varepsilon\,d\varepsilon\Big(-\frac{m_e}{m_\mu}\Big)(\boldsymbol{\eta}\boldsymbol{\zeta})\\ &\times\Big\{(1-6a^2)(1-\varepsilon)+3a\beta(2-\varepsilon)-6\beta(ab_--b_+)\\ &+\frac{3\beta\varepsilon}{2}(3ab_--3b_+-ac_++c_-)-\frac{1}{2}[(1-\varepsilon)(b_+c_+\\ &-b_-c_-)\varepsilon+(1-\varepsilon)(2-\varepsilon)(b_+^2-b_-^2)]+\frac{1}{2}[\varepsilon c_--3ac_+\varepsilon(1-\varepsilon)]\\ &+b_-\Big(2+\varepsilon^2-\frac{5}{2}\varepsilon\Big)-ab_+\Big(6-\frac{15}{2}\varepsilon+\frac{3}{2}\varepsilon^2\Big)\Big\},\end{aligned} \tag{27}$$

that the polarization of the electrons from the decay of polarized muons is sensitive to the values of c_- and c_+, which do not enter in dW_0 and dW_1. Measurement of dW_2 is somewhat easier since dW_3 is proportional to m_e/m_μ.

3. MUONIC DECAYS OF THE PION AND THE K MESON

In this section we discuss the changes in the properties of the decays of pions and K mesons arising from the generalized structure (4) of $J^{(\mu)}_\alpha$.

For the rate of the $\pi_{\mu 2}$ decay (in standard notation) we have

$$\begin{aligned}W(\pi\to\mu)=&\frac{G^2f_\pi^2m_\mu^2}{8\pi}m_\pi\Big(1-\frac{m_\mu^2}{m_\pi^2}\Big)^2r;\\ r\cong&1+a^2+4a\frac{m_2}{m_\mu}\Big(1-\frac{m_\mu^2}{m_\pi^2}\Big)^{-1}+2\Big[b_+\Big(\frac{m_\pi}{m_\mu}\Big)^2\\ &+c_+\Big]\Big[a+\frac{m_2}{m_\mu}\Big(1+\frac{m_\mu^2}{m_\pi^2}\Big)\Big(1-\frac{m_\mu^2}{m_\pi^2}\Big)^{-1}\Big]+2\Big[b_-\Big(\frac{m_\pi}{m_\mu}\Big)^2\\ &+c_-\Big]+(b_-^2+b_+^2)\Big(\frac{m_\pi}{m_\mu}\Big)^4+2(b_-c_-+b_+c_+)\Big(\frac{m_\pi}{m_\mu}\Big)^2+c_+^2-c_-^2.\end{aligned} \tag{28}$$

The expression for the rate of the $K_{\mu 2}$ decay is obtained from (28) by making the replacements

$$f_\pi^2\to f_K^2,\qquad m_\pi\to m_K.$$

For the ratio

$$R_\pi=W(\pi\to e)\,/\,W(\pi\to\mu)$$

we obtain from (28)

$$R_\pi=\Big(\frac{m_e}{m_\mu}\Big)^2\Big(\frac{1-m_e^2/m_\pi^2}{1-m_\mu^2/m_\pi^2}\Big)^2r^{-1}=R_{\pi 0}r^{-1}, \tag{29}$$

where r is defined in (28) and $R_{\pi 0}=1.28\times10^{-4}$.

If the derivative terms in (4) are omitted, r differs from unity by no more than 1 to 2%. If (for $m_2\lesssim 2$ MeV) a, $b_\pm$, and $c_\pm$ all are smaller than or of the order of 0.1, r differs from unity by up to 30%, which is excluded by experiment. The main contribution to r from the derivative terms in (4) reduces for the $K_{\mu 2}$ decay [cf. (28)] to

$$1+2\Big[\Big(\frac{m_K}{m_\mu}\Big)^2b_-+c_-\Big]+(b_+^2+b_-^2)\Big(\frac{m_K}{m_\mu}\Big)^4,$$

comparison of the experimental data on $R_K=W(K\to e)/W(K\to\mu)$ with the predictions of the V—A theory yields $b_\pm\lesssim 10^{-2}$.

As already noted by Lipmanov,[9] and also recently by Arbuzov,[11] the deviations from the V—A theory lead to an incomplete polarization of the muon in $\pi_{\mu 2}$ and $K_{\mu 2}$ decays.

In our model, with $a\approx 0.1$ MeV and no derivative terms, the polarization of the muons differs from unity by 1 to 2%. For interactions with derivatives the polarization of the muons in the $K_{\mu 2}$ decay with $b_\pm$ and $c_\pm$ equal to 0.01 differs from unity by 5 to 7%.

4. CONCLUSION

It is seen from the above analysis that the present experimental data do not exclude admixtures of interactions different from V—A which reach several percent in the model considered. A more accurate analysis of the muon decay would allow one to lower these limits (or to observe their existence). From this point of view we recommend a more careful study of the decay spectrum of polarized muons, a comparison of the measured spectra with expressions (19′) and (22′), and the determination of the experimental values of A, ..., G.

The possible effect of the a finite mass of the muonic neutrino is quadratic in m_2 if there are no derivative terms in the muon current. In the opposite case new terms may appear which are proportional to the first power of m_2. In this connection it would be desirable to have experimental results at the very limit of the electron spectrum.

It is interesting to note that in the absence of derivative terms in (4) the possible effect of CP violation is very small since according to (10) the contribution to the probability is proportional to $6m_e m_2/m_\mu^2$, besides Im a.

It is seen from (27) that the polarization of the electrons from the decay (1) is sensitive to certain structures in (4) whose contribution to other quantities is small. If the interaction (4) is used for $\pi_{\mu 2}$ and $K_{\mu 2}$ decays, then already the existing data on K_{e2} and $K_{\mu 2}$ decays allow one to lower the limit on $b_\pm$ in (4) by an order of magnitude.

It would be highly desirable to have more accurate data on π_{e2}, $\pi_{\mu 2}$, and K_{e2}, $K_{\mu 2}$ decays as well on the polarization of the muons in these processes, and especially in the $K_{\mu 2}$ decay.

The authors are grateful to A. I. Mukhin and R. M. Sulyaev for useful discussions.

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Translated by R. Lipperheide
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