ANISOTROPY OF THE HIGH FREQUENCY CONDUCTIVITY OF SIZE-QUANTIZED FILMS

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The transverse high frequency conductivity of size-quantized films is calculated. It is shown that if the thickness d of the film is sufficiently small the conductivity will be considerably smaller than that of a bulky sample. This results in pronounced anisotropy of the dielectric properties of the film. In particular, propagation of an electromagnetic wave polarized perpendicular to the film plane may become possible. An experiment is proposed which should make detection of the anisotropy possible.

As is well known, quantum size effects can occur in thin metallic and semiconducting films. They appear whenever the quantization of the motion of the conduction electron in a direction normal to the film becomes significant (we assume this direction to be the z axis). The influence of size quantization on the static conductivity has been considered in the literature with sufficient detail (see, for example, the review[1]). However, the discrete character of the transverse motion of the electrons can also affect the high-frequency properties of the film. In particular, if the following conditions are satisfied

$$T < \varepsilon_0, \tag{1}$$

$$\hbar\omega \ll \varepsilon_0, \tag{2}$$

$$\hbar\nu \ll \varepsilon_0, \tag{3}$$

(where $\epsilon_0 = \pi^2 h^2/2md^2$ is the energy of the size quantization, ω is the frequency of the external electromagnetic field, and ν is the collision frequency), one should expect an appreciable decrease of the component $\sigma_{zz}(\omega)$ of the high-frequency conductivity.

This effect should lead to an appreciable difference between the high-frequency properties of the film and those of a bulky sample. In particular, if the indicated inequalities are satisfied with a sufficiently large margin, σ_{zz} can reach values that are characteristic of dielectrics, and in this case the propagation of an electromagnetic wave polarized in the direction of the z axis becomes possible in such a film¹⁾. Estimates show that in a film of thickness $d \approx 5 \times 10^{-6}$ cm made of a material with $m \approx 10^{-2}\, m_0$, equations (1) and (2) are satisfied when $T < 30°$K, $\omega \ll 10^{13}$ sec^{-1}. Such an effective mass is possessed, in particular, by Bi (along the trigonal axis) and by InSb, in which the mobility is quite high, so that (3) is well satisfied.

We now proceed to a quantitative consideration of the problem. We start with the calculation of σ_{zz}. We note beforehand that the transverse conductivity of the size-quantized films was calculated in the resonant region ($h\omega \approx l^2\epsilon_0$, $l = 1, 2, 3, \ldots$)[2] and in the quasi-classical region ($h\omega \gg \epsilon_0$)[3]. We are interested in the case (2).

We shall neglect in the calculations the spatial dispersion. The validity of this procedure will be proved later. We represent the high-frequency electric field **E** by the vector potential

$$A = A_z = i\frac{c}{\omega}e^{i\omega t}e^{st}E_0 = i\frac{c}{\omega}E; \quad s \to +0. \tag{4}$$

In the space of the eigenfunctions of the unperturbed Hamiltonian

$$\psi_{k_x k_y l} = \frac{1}{\pi\sqrt{2d}}e^{i(k_x x + k_y y)}\sin\left(\frac{\pi l z}{d}\right), \quad l = 1, 2, 3, \ldots$$

the perturbation operator $\hat{V} = -(e/mc)A\hat{p}_z$ has matrix elements

$$\langle k_x k_y l|\hat{V}|k_x' k_y' l'\rangle = \frac{4e\hbar}{m\omega d}E\frac{ll'}{l^2 - l'^2}\delta(k_x - k_x')\delta(k_y - k_y') \tag{5}$$

at l and l' of different parity, and

$$\langle k_x k_y l|\hat{V}|k_x' k_y' l'\rangle = 0 \tag{5'}$$

when l and l' are of the same parity.

Using the density-matrix method[4], we obtain an expression for the current density in the form of the perturbation-theory series

$$j_z = i\frac{e}{m}\Big\{\sum_n f_0(\varepsilon_n)\psi_n^*\left(\hat{p}_z - \frac{e}{c}A\right)\psi_n + (i\hbar)^{-1}\sum_{n,n'}[f_0(\varepsilon_n) - f_0(\varepsilon_{n'})]$$
$$\times\int_{-\infty}^{t}\langle n|\hat{V}(t')|n'\rangle\exp[it'(\omega_n - \omega_{n'})]\,dt'\,\psi_{n'}^*\left(\hat{p}_z - \frac{e}{c}A\right)$$
$$\times\psi_n\exp[it(\omega_{n'} - \omega_n)] + \ldots\Big\}, \tag{6}$$

where n and n′ are the sets of all the quantum numbers, $\epsilon_n = \hbar\omega_n$ are the corresponding energies, and $f_0(\epsilon)$ is the Fermi function.

Let us average (6) over z and retain only the terms linear in E:

$$\sigma_{zz} = -\frac{ie^2}{2\pi^2 m\omega d}\Big\{\sum_l\iint f_0(\varepsilon_{k_x k_y l})\,dk_x\,dk_y + \frac{16\hbar}{md^2}\sum_{l,l'}{}'\left(\frac{ll'}{l^2 - l'^2}\right)^2 \cdot$$
$$\times\iint\frac{f_0(\varepsilon_{k_x k_y l}) - f_0(\varepsilon_{k_x k_y l'})}{\omega + \hbar^{-1}(\varepsilon_{k_x k_y l} - \varepsilon_{k_x k_y l'})}dk_x dk_y\Big\}$$

The prime at the summation sign denotes here that we are summing only terms in which l and l' have different parities. In the derivation of (7) we have taken the limit as $s \to +0$.

By virtue of (2), the second term in (7) can be expanded in a series in $\hbar\omega/\epsilon_0$. The zeroth term of the expansion cancels the first term completely. This is clear

¹⁾By polarization we mean here the direction of the intensity of the electric field of the wave.

from physical considerations, for when $\epsilon_0 \to \infty$ we should obviously have $j_z \to 0$. The linear term is an odd function of $l - l'$ and yields 0 upon summation. We confine ourselves to the quadratic term of the expansion:

$$\sigma_{zz} = -\frac{8ie^2\hbar^4\omega}{\pi^2m^2d^3}\sum_{l,\,l'}{}' \left(\frac{ll'}{l^2 - l'^2}\right)^2 \iint \frac{f_0(\varepsilon_{k_xk_yl}) - f_0(\varepsilon_{k_xk_yl'})}{(\varepsilon_{k_xk_yl} - \varepsilon_{k_xk_yl'})^3}\,dk_x dk_y. \quad (8)$$

To obtain the final results, let us consider individual particular cases.

Assume that the condition $Nd^3 \gg 1$ is satisfied, which is equivalent to the inequality $\zeta \gg \epsilon_0$ (N—carrier density, ζ—level of the chemical potential). By virtue of (1) it is obvious that in this case the electron gas should be regarded as completely degenerate. In the lowest order in ϵ_0/ζ, the calculations yield

$$\sigma_{zz} = i\frac{e^2md^2\omega}{12\pi\hbar^2}\left(\frac{3N}{\pi}\right)^{1/3}.$$

If we take into account the scattering of the electrons, by introducing the collision frequency ν, then it is necessary to replace ω in our expressions by $\omega - i\nu$. By virtue of (2), we can choose for ν the value of the collision frequency for the static conductivity. This quantity (more accurately, the relaxation time $\tau = \nu^{-1}$) was calculated by Tavger and Demikhovskiĭ for scattering by acoustic phonons[5,6], and by Sandomirskiĭ for scattering by a δ-like impurity potential[7].

The final expression for σ_{zz} has in this case the form

$$\sigma_{zz} = \frac{e^2md^2}{12\pi\hbar^2}\left(\frac{3N}{\pi}\right)^{1/3}(\nu + i\omega). \quad (9)$$

We now consider the case $Nd^3 < 3\pi/2$. By virtue of (1), only one level of the size quantization will then be filled. The answer will not depend on the degree of degeneracy of the electron gas, and therefore for simplicity in the calculations we shall assume the degeneracy to be complete. Then the Fermi energy is equal to[8]:

$$\zeta = \frac{\pi^2\hbar^2}{2md^2} + \frac{\pi^2\hbar^2 dN}{m}.$$

Taking this into account, we obtain

$$\sigma_{zz} = \frac{256e^2md^4N}{\pi^6\hbar^2}(\nu + i\omega)\sum_{l=1}^{\infty}\frac{4l^2}{(4l^2-1)^5}.$$

Since the series converges very rapidly, we can confine ourselves with good accuracy to the first term. We then obtain

$$\sigma_{zz} = \frac{1024}{243\pi^6}\frac{e^2md^4N}{\hbar^2}(\nu + i\omega). \quad (10)$$

It should be noted that if in a given substance there is a group of heavier carriers, which makes no noticeable contribution to the conductivity of the bulky sample, then it is necessary to take their presence into account in the case of a size-quantized film. If the conditions (1)–(3) are not satisfied for these carriers, then the contribution made to σ_{zz} by this group of carriers, calculated for example by means of the formulas of[3], may turn out to be larger than the values (9) and (10) obtained by us. If (1)–(3) are satisfied for them, then their presence can also be significant because $\sigma_{zz} \sim m$. We shall henceforth assume that the second group of carriers is missing.

Let us proceed to the propagation of electromagnetic waves in our film. From the point of view of the dielectric properties, this film can be regarded as a uniaxial crystal with an optical axis coinciding with the normal to the film. Two types of waves are possible in it: ordinary and extraordinary[9]. The first of them is polarized parallel to the surface of the film, i.e., perpendicular to the plane of incidence, which we denote by xz. This is the ordinary damped wave, the same as in the bulky conductor with conductivity $\sigma_{xx} = \sigma_{yy}$ (which generally speaking differs from the conductivity of a bulky sample of the same material). We shall not consider this wave.

The extraordinary wave is polarized in the plane of incidence. The dispersion law for this wave is of the form

$$\varkappa_0^2\frac{\omega^3}{c^2} + 4\pi i\frac{\omega^2}{c^2}\varkappa_0(\sigma_{zz} + \sigma_{xx}) - \left(\frac{4\pi}{c}\right)^2\omega\sigma_{zz}\sigma_{xx} - \varkappa_0\omega k^2 - 4\pi i(\sigma_{zz}k_z^2 + \sigma_{xx}k_x^2) = 0, \quad (11)$$

where κ_0 is the lattice part of the dielectric constant. In the case $k = k_x$ we obtain the usual expression:

$$k_x^2 = \varkappa_0\frac{\omega^2}{c^2} + 4\pi i\frac{\omega}{c^2}\sigma_{zz}. \quad (12)$$

Such a wave can have quite small damping. For example, for a film n-InSb ($\mu \approx 3 \times 10^5$ cm²/V-sec[10]) of thickness 3×10^{-6} cm at $N \approx 2 \times 10^{14}$ cm⁻³, the order of magnitude of Im k/Re k is 10^{-2} (in such a film, only one level is filled and σ_{zz} is calculated by means of formula (10))[2)].

We now have to prove that it is legitimate to neglect the spatial dispersion. This can be done when

$$\lambda k_x \approx \lambda\max\{\sqrt{\varkappa_0}\omega / c;\quad \sqrt{2\pi\omega\sigma_{zz}/c^2}\} \ll 1,$$
$$dk_z \approx d\sqrt{2\pi\omega\sigma_{xx}/c^2} \ll 1 \quad (13)$$

(λ is the mean free path). As a rule, these inequalities are satisfied quite well. For example, for the already mentioned InSb film or for a Bi film with $d \approx 10^{-5}$ cm, in which $\sigma_{xx} \approx 5 \times 10^2$ Ohm⁻¹ cm⁻¹ and $\lambda \approx 4 \times 10^{-5}$ cm[11], the inequalities of (13) are violated only at such frequencies, for which (2) no longer holds. It should be remembered that in (13) it is impossible, generally speaking, to substitute directly the experimental value of σ_{xx} characterizing the static conductivity, and it is necessary to introduce a factor $(1 + \omega^2\tau^2)^{-1}$, which takes into account the temporal dispersion.

The conductivity anisotropy described in the present paper can be observed experimentally in the following manner. Let us imagine a waveguide partitioned longitudinally with a dielectric substrate on which a size-quantized film is deposited. The presence of the film will cause attenuation of the radio waves propagating along the waveguide. The attenuation coefficient is[12]

$$\alpha = \frac{\omega}{2c}\left(\int_{S_n}\mathrm{Im}\,\varkappa\mathbf{E}\mathbf{E}^*dS\right)\Big/\left(\mathrm{Re}\int_S[\mathbf{E},\mathbf{H}^*]\mathbf{e}dS\right), \quad (14)^*$$

where **E** and **H** are the field intensities in the waveguide, **e** is a unit vector along the waveguide axis, S is the waveguide cross section, S_n is the transverse cross section of the film, and κ is its dielectric constant. If a wave with **E** parallel to the film is excited in such a

[2)] In view of the lack of experimental data on the mobility in thin films of InSb, we have chosen for estimating purposes the mobility in a bulky sample. In a film, obviously, μ is smaller. This changes correspondingly the estimates for N.

*$[\mathbf{E},\mathbf{H}^*] \equiv \mathbf{E} \times \mathbf{H}^*$.

waveguide, then for such a wave $\alpha \sim \sigma_{xx}d/cL$ (L—width of waveguide). For a bismuth film with $d \approx 10^{-5}$ cm, in the cm band, this yields $\alpha \sim 1$. On the other hand, if **E** is perpendicular to the film, then $\alpha \sim \sigma_{zz}d/cL$, i.e., smaller by several orders of magnitude. Such a difference in the attenuations can be readily observed.

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