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INFLUENCE OF A TRANSVERSE ELECTRIC FIELD ON SONDHEIMER OSCILLATIONS OF ELECTRONS IN SEMICONDUCTORS

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The transverse magnetoresistance tensor is calculated for the surface space-charge layer in semiconductors. The scattering of electrons by the surface is assumed to be diffuse. It is shown that, when the band curvature is such as to cause degeneracy of the electron gas in a potential well at the surface, the components of the magnetoresistance tensor oscillate not only as a function of the magnetic field but also as a function of the surface potential. It is suggested that this effect can be used to investigate the nature of the scattering of electrons by the surface and the electron dispersion law well within the conduction band of a semiconductor.

STUDIES of the galvanomagnetic effects in metal films exhibiting diffuse reflection from the surface, carried out first by Sondheimer,[1] showed that when a magnetic field is applied normally to the surface of the film, the diagonal components of the electrical conductivity tensor oscillate with d/r, where d is the film thickness and r is the Larmor radius. It is known that, under certain conditions, a potential well for carriers may be established near the surface of a semiconductor. If the band curvature at the surface is stronger than kT, the motion of the carriers is localized in the well and the conductivity of such a surface channel has much in common with the conductivity of a film. We may expect that, in accordance with the results of Sondheimer, the transverse magnetoresistance of such surface channels should oscillate not only as a function of the magnetic field but also as a function of the surface potential which, in this case, governs the width of the potential well. Information on the Fermi energy of electrons in such a well, i.e., information on the constant-energy surfaces lying high in the conduction band of a semiconductor, can be obtained from the magnetoresistance oscillations by establishing such a band curvature at the surface that the electron gas in the potential well becomes degenerate.

We must mention that these oscillations are purely classical. They occur in magnetic fields which do not satisfy the quantization conditions and they are observed in the absence of the size quantization of electrons in the surface potential well. The physical cause of the appearance of the oscillations in the dependence of the magnetoresistance on the magnetic field under such classical conditions is as follows: in the case of diffuse scattering from the surface, the principal contribution to the channel conductivity is made by a narrow "beam" of electrons moving almost parallel to the surface and the drift velocity of such electrons oscillates with the magnetic field. These oscillations disappear in the case of specular reflection of electrons from the surface because of the averaging of the electron velocities over all possible directions.

The channel surface conductivity in the case of diffuse scattering by the surface was considered, together with the Hall effect, in[2,3]. However, these treatments are limited to the case of weak magnetic fields: $\Omega\tau \ll 1$. The present paper reports a calculation of the components of the electrical conductivity tensor of surface channels in semiconductors in arbitrary magnetic fields and at those surface potentials which ensure degeneracy of the electron gas. It is shown that these components oscillate as a function of the magnetic field as well as a function of the electric field of the space charge.

Let us consider a semiconductor (to be specific, we shall assume that it is p-type) which carries a positive charge on its surface. This positive charge is compensated by electrons in the surface layer and consequently the Fermi level shifts, relative to the band edges, in such a way that a potential well of depth $e\psi_s$ is formed at the surface. We shall consider the following geometry of the problem (cf. Fig. 1): the z axis is directed normally to the surface into the sample and the surface lies at z = 0. The electric field of the space charge is directed along the z axis: $E_z > 0$. An external electric field, E_x, is applied along the surface (along the x axis). A magnetic field is directed along the z axis.

The dispersion law of the semiconductor is assumed to be isotropic and quadratic: $\epsilon = p^2/2m$. The energy is measured from the bottom of the conduction band in the interior of the semiconductor; ϵ_v is the position of the top of the valence band.

In order to calculate the electrical conductivity tensor, it is necessary to solve a transport equation of the

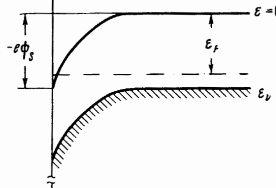

FIG. 1

type

$$v_z \frac{\partial f}{\partial z} - e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} - \frac{e}{c}[\mathbf{vH}]\frac{\partial f}{\partial \mathbf{p}} + If = 0. \qquad (1)^*$$

The electron distribution function is represented, as usual, in the form

$$f = f_0 + f_1, \qquad (2)$$

where f_0 is the equilibrium distribution function and f_1 is a small correction. We shall assume that the collision integral can be replaced by the relaxation time:

$$If = \frac{f - f_0}{\tau} = \frac{f_1}{\tau}.$$

The equilibrium distribution of the electrons in the space-charge layer is represented, in general, by the function

$$f_0 = \left[\exp\left\{\frac{\mu_0 - \varepsilon}{kT}\right\} + 1\right]^{-1}. \qquad (3)$$

Here, μ_0 is the position of the Fermi level in the interior of the semiconductor, measured from the bottom of the conduction band; $\epsilon = p^2/2m - e\psi(z)$ is the total electron energy.

The space-charge potential is defined as follows

when $z \to \infty \quad \psi(z) = 0,$
when $z = 0 \quad \psi(z) = \psi_s > 0.$

We shall assume that the scattering of electrons by the surface is diffuse, i.e., that $f_1 = 0$ when $z = 0$ for electrons with $v_z > 0$. The solution of Eq. (1) was obtained by Zemel[3] using all these assumptions and diffuse-scattering boundary conditions. Following the notation employed by Zemel,[3] we obtain an expression for f_1:

$$f_1 = \frac{ev\tau\partial f_0/\partial\varepsilon}{1 + \Omega^2\tau^2}\Big\{(1 - e^{\Delta K}\cos\Omega\tau\Delta K)(a\sin\theta + b\cos\theta) - e^{\Delta K}\sin\Omega\tau\Delta K(b\sin\theta - a\cos\theta)\Big\}. \qquad (4)$$

Here, $v = \sqrt{v_x^2 + v_y^2}$; θ is the polar angle in the (v_x, v_y) plane; $\tan\theta = v_x/v_y$;

$$K(v_z, \varepsilon_z) = \frac{m}{e}\int_0^{v_z}\frac{dv_z'}{\tau(\varepsilon_z, v_z')E_z(\varepsilon_z, v_z')};$$

$\epsilon_z = (1/2)mv_z^2 - e\psi(z)$ is the energy of the electron motion along the z axis, which is one of the integrals in Eq. (1);

$$\Delta K = K - K_0; \quad K_0 = \frac{m}{e}\int_0^{v_{zs}}\frac{dv_z'}{\tau(\varepsilon_z, v_z')E_z(\varepsilon_z, v_z')}$$

is the value of K on the surface;

$$v_{zs} = \sqrt{\frac{2}{m}(\varepsilon_z + e\psi_s)}, \qquad \Omega = \frac{eH}{mc},$$

$$a = E_x - \Omega\tau E_y, \quad b = E_y + \Omega\tau E_x.$$

We shall now calculate the current. By definition, the total electron current per unit surface area is

$$\mathbf{j} = -e\int_0^\infty dz\int dv_x\,dv_y\,dv_z\,\mathbf{v}f. \qquad (5)$$

Substituting Eq. (4) into Eq. (5) and isolating the terms proportional to E_x and E_y, we obtain

$$j_x = \sigma_{xx}E_x + \sigma_{xy}E_y, \qquad (6)$$

where

$$\sigma_{xx} = -e^2\int_0^\infty dz\int dv_x\,dv_y\,dv_z\frac{v^2\tau}{1 + \Omega^2\tau^2}\frac{\partial f_0}{\partial\varepsilon} \times\{(1 - e^{\Delta K}\cos\Omega\tau\Delta K)(\sin^2\theta + \Omega\tau\cos\theta\sin\theta) - e^{\Delta K}\sin\Omega\tau\Delta K(\Omega\tau\sin^2\theta - \cos\theta\sin\theta)\}; \qquad (7)$$

$$\sigma_{xy} = -e^2\int_0^\infty dz\int dv_x\,dv_y\,dv_z\frac{v^2\tau}{1 + \Omega^2\tau^2}\frac{\partial f_0}{\partial\varepsilon} \times\{(1 - e^{\Delta K}\cos\Omega\tau\Delta K)(\cos\theta\sin\theta - \Omega\tau\sin^2\theta) - e^{\Delta K}\sin\Omega\tau\Delta K(\sin^2\theta + \Omega\tau\cos\theta\sin\theta)\}. \qquad (8)$$

Similarly,

$$j_y = \sigma_{yx}E_x + \sigma_{yy}E_y. \qquad (9)$$

It follows from Onsager's reciprocity principle that

$$\sigma_{yx}(-\Omega) = \sigma_{xy}(\Omega), \qquad (10)$$

$$\sigma_{yy} = -e^2\int_0^\infty dz\int dv_x\,dv_y\,dv_z\frac{v^2\tau}{1 + \Omega^2\tau^2}\frac{\partial f_0}{\partial\varepsilon} \times\{(1 - e^{\Delta K}\cos\Omega\tau\Delta K)(\sin^2\theta - \Omega\tau\cos\theta\sin\theta) - e^{\Delta K}\sin\Omega\tau\Delta K(\Omega\tau\sin^2\theta + \sin\theta\cos\theta)\}. \qquad (11)$$

The components of the tensor σ_{ik} can be calculated conveniently using the following integration variables: v, θ, ϵ_z, and K. The Jacobian for this transformation is $\tau v/m$.

We shall assume that the potential well is very deep and that the total electron conductivity is due solely to those electrons which are located in the space-charge layer at the surface. After some transformations, we obtain

$$\sigma_{xx} = -\frac{\pi e^2}{m}\langle A_H(1 - \Omega^2\tau^2) - 2\Omega\tau B_H\rangle_{v,\varepsilon_z} \qquad (12)$$

$$\sigma_{xy} = +\frac{\pi e^2}{m}\langle 2\Omega\tau A_H + (1 - \Omega^2\tau^2)B_H\rangle_{v,\varepsilon_z}; \qquad (13)$$

$$\sigma_{yy} = \sigma_{xx}.$$

Here,

$$A_H = e^{-2K_0}\cos 2\Omega\tau K_0 + 2K_0 - 1,$$
$$B_H = e^{-2K_0}\sin 2\Omega\tau K_0 - 2\Omega\tau K_0,$$

and $\langle\ldots\rangle_{v,\epsilon_z}$ is defined as follows

$$\langle f(x)\rangle_{v,\varepsilon_z} = \int_{-e\psi_s}^\infty d\varepsilon_z\int_0^\infty dv\frac{v^3\tau^2\partial f_0/\partial\varepsilon}{(1 + \Omega^2\tau^2)^2}f(x).$$

We shall calculate the integrals in Eqs. (12) and (13) introducing the following simplifying assumptions: 1) we shall assume that the relaxation time is constant; 2) we shall use a linear model of the space-charge potential, i.e., E_z = const. Moreover, we shall specify the explicit form of the function $\partial f_0/\partial\epsilon$. We shall assume that the electron gas in the surface potential well is degenerate so that $\partial f_0/\partial\epsilon = -\delta(\epsilon - \epsilon_F)$, where ϵ is the total energy given by $\epsilon = \epsilon_z + mv^2/2$. After integration we finally obtain

$$\sigma_{xx} = \sigma_0 + \sigma_1\cos 2\Omega\tau\alpha + \sigma_2\sin 2\Omega\tau\alpha,$$
$$\sigma_{xy} = \sigma_3 + \sigma_2\cos 2\Omega\tau\alpha - \sigma_1\sin 2\Omega\tau\alpha. \qquad (14)$$

Here,

$$\alpha = \sqrt{2m(\varepsilon_F + e\psi_s)}\,/\,e\tau E_z,$$

$$\sigma_0 = \frac{4\pi e^2\tau^2(\varepsilon_F + e\psi_s)^2}{m^3(1 + \Omega^2\tau^2)^2}\Big\{\frac{4}{15}\alpha(1 + \Omega^2\tau^2) - \frac{1}{4}(1 - \Omega^2\tau^2) + \frac{1 - 6\Omega^2\tau^2 + \Omega^4\tau^4}{4\alpha^2(1 + \Omega^2\tau^2)^2} - \frac{6(1 - \Omega^2\tau^2)(1 - 14\Omega^2\tau^2 + \Omega^4\tau^4)}{16\alpha^4(1 + \Omega^2\tau^2)^4}\Big\}$$

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*$[\mathbf{vH}] \equiv \mathbf{v}\times\mathbf{H}$.

$$\sigma_1 = \frac{4\pi e^2\tau^2(\varepsilon_F + e\psi_s)^2}{m^3(1+\Omega^2\tau^2)^2} 2e^{-2\alpha}\left\{\frac{1-6\Omega^2\tau^2+\Omega^4\tau^4}{4\alpha^2(1+\Omega^2\tau^2)^2}\right.$$
$$\left. + 3\frac{1-10\Omega^2\tau^2+5\Omega^4\tau^4}{8\alpha^3(1+\Omega^2\tau^2)^3} + 3\frac{(1-\Omega^2\tau^2)(1-14\Omega^2\tau^2+\Omega^4\tau^4)}{16\alpha^4(1+\Omega^2\tau^2)^4}\right\}$$
$$\sigma_2 = -\frac{4\pi e^2\tau^2(\varepsilon_F + e\psi_s)^2}{m^3(1+\Omega^2\tau^2)^2} 2e^{-2\alpha}\,\Omega\tau\left\{\frac{1-\Omega^2\tau^2}{\alpha^2(1+\Omega^2\tau^2)^2}\right.$$
$$\left. + 3\frac{5-10\Omega^2\tau^2+\Omega^4\tau^4}{8\alpha^3(1+\Omega^2\tau^2)^3} + 3\frac{3-10\Omega^2\tau^2+3\Omega^4\tau^4}{8\alpha^4(1+\Omega^2\tau^2)^4}\right\}$$
$$\sigma_3 = -\frac{4\pi e^2\tau^2(\varepsilon_F + e\psi_s)^2}{m^3(1+\Omega^2\tau^2)^2} 2\Omega\tau\left\{\frac{2}{15}\alpha(1+\Omega^2\tau^2) - \frac{1}{4}\right.$$
$$\left. + \frac{1-\Omega^2\tau^2}{2\alpha^2(1+\Omega^2\tau^2)^2} - 3\frac{3-10\Omega^2\tau^2+3\Omega^4\tau^4}{8\alpha^4(1+\Omega^2\tau^2)^4}\right\}.$$

In an analysis of the expressions in Eq. (14), we shall exploit the following fact: in the triangular-well model $E_z = \psi_s/L$ (L is the total width of the well) and α can be represented in the form

$$\alpha = 2L_0 / l(\varepsilon_F + e\psi_s),$$

where $L_0 = (e\psi_s + \epsilon_F)L/e\psi_s$ is the effective width of the well for electrons whose energy is equal to the Fermi energy and $l(\epsilon_F + e\psi_s)$ is the mean free path of the same electrons.

We note that $\Omega\tau\alpha = 2L_0/r$, where r is the Larmor radius. It follows from Eq. (14) that, in order to observe oscillations, we must satisfy the condition

$$2L_0 \gtrsim r. \qquad (15)$$

On the other hand, these oscillations are rapidly damped when α increases because σ_1 and $\sigma_2 \propto \exp(-2\alpha)$. Therefore, we must have $\alpha \lesssim 1$, i.e.,

$$2L_0 \lesssim l(\varepsilon_F + e\psi_s). \qquad (16)$$

The results obtained are in agreement with those reported by Sondheimer.[1] It is known that the position of the first maximum of the Sondheimer oscillations determines the value of the momentum at the Fermi surface. In the case considered here, the position of the first maximum of the magnetoresistance curve corresponds to $\Omega\tau\alpha \sim 1$, i.e.,

$$\frac{H}{mcE_z}\sqrt{2m(\varepsilon_F + e\psi_s)} \sim 1,$$

and the above expression can be used to determine the effective electron mass when H, E_z, and the degeneracy energy $(\epsilon_F + e\psi_s)$ are known. This effect can be used to study the properties of the constant-energy surfaces in the conduction band. The advantage of this method is that the degree of degeneracy of the electron gas in the surface potential well is independent of the surface potential, which can be varied easily by an external field or by adsorption.

We shall now find numerically when conditions (15) and (16) are satisfied. Treating these conditions as one, we obtain $r \sim l$, i.e., $\Omega\tau \sim 1$. We shall assume that $\tau \sim 10^{-13}$ sec, $m = 0.1m_0$. The condition $\Omega\tau \sim 1$ is satisfied by a magnetic field $H \sim 6 \times 10^4$ Oe. The electric field E_z is related to the charge per unit surface area N_s in the following manner: $E_z = 4\pi e N_s/\epsilon$; the degeneracy energy is

$$e\psi_s + \varepsilon_F = eE_zL_0 = \left[\frac{15}{4}\frac{(2\pi\hbar)^3 e^2 N_s^2}{\varepsilon(2m)^{3/2}}\right]^{2/5}.$$

We thus find that α depends on the electron density per unit surface area and this dependence is given by:

$$\alpha = \frac{1}{4\pi}\left[\frac{15}{4}\frac{(2\pi\hbar)^3 2m\varepsilon^4}{e^8\tau N_s^3}\right]^{1/5}.$$

Substituting into the above expression the cited numerical values of τ and m and also assuming that $\epsilon = 16$, we find that $\alpha \leq 1$ when $N_s \sim 10^{11}$ cm^{-2}. When N_s increases, the value of α decreases. In order to reach appreciable degeneracy energies, for example, $\epsilon_F + e\psi_s \sim 0.1$ eV, we must have values $N_s \sim 10^{12}$ cm^{-2} and such values of N_s are known to satisfy condition (16).

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[2] J. S. Schrieffer, V sb. Problemy fiziki poluprovodnikov (in: Problems of Physics of Semiconductors), Russ. Transl., IIL, 1957, p. 287 [probably: Phys. Rev. 97, 641 (1955)].
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