# *SOME FEATURES OF THE HYDRODYNAMIC MECHANISM OF ELECTRIC CONDUCTIVITY OF METALS IN A MAGNETIC FIELD*

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The electric conductivity of a metallic plate in a magnetic field arbitrarily oriented relative to its surface is considered. It is found that the distribution of the current over the cross section of the plate depends on the magnetic field component $H_\perp$ perpendicular to the magnetic field. The resistance tensor $\rho_{ik}$ is calculated and its dependence on $H_\perp$ and $H_\parallel$ in investigated ($H_\parallel$ is the magnetic field component parallel to the surface of the plate).

GURZHI[1] has shown that a hydrodynamic mechanism of electric conductivity can exist in certain metals at low temperatures. This mechanism is realized under conditions when the normal electron-phonon collisions are much more frequent than the collisions in which the total quasimomentum is not conserved, and the normal collisions themselves do not produce resistance.

As the result of the frequent normal collisions, a local equilibrium is established in the system and its behavior can be described with the aid of hydrodynamic variables. In particular, to calculate the electric conductivity it is sufficient to know the drift velocity of the electron-phonon gas $\mathbf{u}(\mathbf{r})$. The hydrodynamic equations satisfied by $\mathbf{u}(\mathbf{r})$ were derived and investigated in[1] for the case of a metal in an electric field, and in[2] for a metal in electric and magnetic fields. In the latter case, a plate was considered and the magnetic field was assumed parallel to its surface.

In this paper we consider the hydrodynamic mechanism of electric conductivity of a metallic plate with the magnetic field arbitrarily oriented relative to its surface. It turns out here that if the "radius" $r_\perp = cp_F/eH_\perp$ ($p_F$ is the electron momentum on the Fermi surface, $H_\perp$ is the component of the magnetic field perpendicular to the surface of the plate) is larger than the electron mean free path $l_{ep}$ relative to the normal collisions with the phonons, then, as in the absence of a magnetic field, a simultaneous drift of the electron-phonon gas is possible in the system. The motion of this gas is described by the hydrodynamic equations in which the external force is the usual Lorentz force, which is connected with the electric field and with the component of the magnetic field $H_\perp$; on the other hand, the component $H_\parallel$ parallel to the surface leads to a change in the viscosity of the electron-phonon gas.

The character of the distribution of the current over the cross section of the sample depends on the relation between the lengths that enter in the problem. If $r_\perp \gg L \gg l_{ep}$ ($L \sim d^2/l_{ep}\eta$ is the length of the Brownian path covered by the electron from the interior of the sample to its surface, d is the thickness of the plate, and $\eta$ is a factor that takes into account the presence of the field $H_\parallel$), then the current is sufficiently uniformly distributed over the sample cross section. If $L \gg r_\perp \gg l_{ep}$, then an appreciable change of the current density occurs at a distance on the order of $(r_\perp l_{ep}\eta)^{1/2}$ from the surface. When $r_\perp \ll l_{ep}$, there are two characteristic scales of current variation, $r_\perp$ and $l_{ep}$.

The complete system of equations for the solution of the problem consists of the kinetic equations

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + e(\mathbf{E}\mathbf{v})\frac{\partial f_0}{\partial \varepsilon} + \frac{e}{c}[\mathbf{v}\mathbf{H}]\frac{\partial f}{\partial \mathbf{p}} = \hat{J}_{ep}{}^N\{f, N\} + \hat{J}_e{}^U\{f, N\}, \quad (1)^*$$

$$\mathbf{s}\frac{\partial N}{\partial \mathbf{r}} = \hat{J}_{pe}{}^N\{f, N\} + \hat{J}_p{}^U\{f, N\} \quad (1')$$

and the equation

$$\operatorname{rot}\mathbf{E} = 0. \quad (2)$$

Here f and N are the distribution functions of the electrons and the phonons; $\mathbf{v}$, $\mathbf{p}$, $\epsilon$ are the velocity, momentum, and energy of the electron; $\mathbf{s}$ is the velocity of the phonon; $\mathbf{E}$ is the electric field, which is the sum of the external electrostatic field and the Hall field; $f_0 = [\exp\{(\epsilon - \epsilon_F)/T\} + 1]^{-1}$; $\hat{J}_{ep}^N$ and $\hat{J}_{pe}^N$ are the electron-phonon and phonon-electron collision operators; $\hat{J}_e^U$ and $\hat{J}_p^U$ are operators describing collisions in which the quasimomenta of the electrons and phonons respectively are not conserved.

The hydrodynamic equations can be obtained from (1) and (1′) with the aid of the Chapman-Enskog method (see, for example,[3,4]). It is not particularly difficult to obtain in this case equations for an arbitrary cross section of the sample. However, when these equations are solved in the general case, after elimination of the Hall field, we arrive at rather complicated integro-differential equations for $\mathbf{u}(\mathbf{r})$. We therefore confine ourselves below to the simplest case of a plate, whose thickness 2d is much smaller than the remaining dimensions.

It is easy to show that if the electron mean free path relative to normal collisions with the phonons, $l_{ep}$, is connected with the "radius" $r_\perp$ by the inequality $r_\perp \ll l_{ep}$, then hydrodynamic flow of electron-phonon gas is impossible, for this gives rise to a Hall field that does not satisfy the condition curl $\mathbf{E} = 0$. We shall therefore assume that $l_{ep} \ll r_\perp$.

We choose a Cartesian system of coordinates with the z axis perpendicular to the surface of the plate,

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*$[\mathbf{v}\mathbf{H}] \equiv \mathbf{v} \times \mathbf{H}$.

and with the y axis perpendicular to the magnetic field. We denote by $H_\perp$ and $H_\parallel$ the projections of the vector **H** on the z and x axes.

Writing the distribution functions in the form

$$f = f^{(0)} + f^{(1)} + \ldots, \qquad N = N^{(0)} + N^{(1)} + \ldots \tag{3}$$

in accordance with the Chapman-Enskog procedure, we arrive at the following equations:

$$eE_{z1}v_z\frac{\partial f_0}{\partial \varepsilon} + \frac{eH_\parallel}{mc}\left[\frac{\partial f^{(0)}}{\partial \mathbf{p}}\mathbf{v}\right]_x = \hat{J}_{ep}{}^N\{f^{(0)}, N^{(0)}\},$$
$$J_{pe}{}^N\{f^{(0)}, N^{(0)}\} = 0; \tag{4}$$

$$v_z\frac{\partial f^{(0)}}{\partial z} + \frac{eH_\parallel}{mc}\left[\frac{\partial f^{(1)}}{\partial \mathbf{p}}\mathbf{v}\right]_x = \hat{J}_{ep}{}^N\{f^{(1)}, N^{(1)}\}, \tag{5}$$
$$s_z\frac{\partial N^{(0)}}{\partial z} = \hat{J}_{ep}{}^N\{f^{(1)}, N^{(1)}\}.$$

We have represented the field $E_z$ in the form $E_z = E_{z1} + E_{z2}$; $|E_{z1}| \gg |E_{z2}|$. It will be shown later that the need for such a breakdown is connected with the solvability of the equations of hydrodynamics (see (9)).

The solution of the pair of equations (4) is

$$f^{(0)} = \left[\exp\left\{\frac{\varepsilon - \varepsilon_F - \mathbf{u}(z)\mathbf{p}}{T}\right\} + 1\right]^{-1},$$
$$N^{(0)} = \left[\exp\left\{\frac{h\nu - \mathbf{u}(z)\mathbf{p}}{T}\right\} - 1\right]^{-1}. \tag{6}$$

Here **u** is the drift velocity of the electron-phonon gas parallel to the plate surface. It can be shown that for typical metals with large electron density, the chemical potential and the temperature, and consequently also the electron density, can be assumed to be the equilibrium values (provided that we are interested only in the electric conductivity tensor). It also follows from (4) that

$$E_{z1} = \frac{H_\parallel}{c}u_y. \tag{7}$$

The solution of the system of integral equations (5) for an arbitrary dispersion law entails appreciable difficulties. It was obtained in[2] for the case of isotropic electron and phonon dispersion laws. We shall not write out the corresponding expressions for $f^{(1)}$ and $N^{(1)}$, since they are too complicated.

We now multiply (1) and (1′) by the momenta **p** and **q** of the electron and phonon respectively, integrate, and add the resultant expressions. We obtain then [1])

$$\operatorname{div}\langle p_i \mathbf{v} f\rangle_e + \operatorname{div}\langle q_i \mathbf{s} N\rangle_p + ec^{-1}\langle[\mathbf{H}_\perp\mathbf{v}]_i f\rangle_e + enE_i = \langle p_i J_e{}^U\rangle_e + \langle q_i J_p{}^U\rangle_p, \tag{8}$$

where

$$\langle\ \rangle_e \equiv \frac{2}{h^3}\int \ldots d\mathbf{p}, \quad \langle\ \rangle_p \equiv \frac{3}{h^3}\int \ldots d\mathbf{q},$$

n is the electron density. Substituting in (8) $f = f^{(0)} + f^{(1)}$ and $N = N^{(0)} + N^{(1)}$, we obtain, accurate to terms of higher order in the small parameters $l_{ep}/r_\perp$, $l_{ep}/d$, $l_{ep}/(l^{U-1} \equiv l_e^{U-1} + l_p^{U-1}$, where $l_e^U$ and $l_p^U$ are the electron and phonon mean free paths relative to collisions with momentum loss)

$$v_x\frac{\partial^2 u_x}{\partial z^2} + \Omega_\perp u_y + \frac{e}{m}E_x = \frac{u_x}{\tau^U},$$
$$v_y\frac{\partial^2 u_y}{\partial z^2} - \Omega_\perp u_x + \frac{e}{m}E_y = \frac{u_y}{\tau^U}, \tag{9}$$
$$v_z\frac{\partial^2 u_y}{\partial z^2} + \frac{e}{m}E_{z2} = 0.$$

Here $\Omega_\perp = eH_\perp/mc$, $\tau^U = l^U/v_F$, $\nu_i = \nu_0\eta_i$, $\nu_0 \approx v_F l_{ep}$ is the kinematic viscosity in the absence of a magnetic field, and $\eta_i$ is a dimensionless factor describing the influence of the longitudinal magnetic field $H_\parallel$. Accurate to coefficients of the order of unity we have

$$\eta_x = [1 + (l_{ep}/r_\parallel)^2]^{-1} + (l_{pe}/l_{ep})^2,$$
$$\eta_y = [1 + 4(l_{ep}/r_\parallel)^2]^{-1} + (l_{pe}/l_{ep})^2, \tag{10}$$
$$\eta_z = -\frac{2l_{ep}/r_\parallel}{1 + 4(l_{ep}/r_\parallel)^2} + \left(\frac{l_{pe}}{l_{ep}}\right)^2.$$

In (10), $l_{pe}$ denotes the mean free path of the phonons relative to normal collisions with electrons. It can be shown (compared with[1]) that $l_{pe}/l_{ep} \sim (T/\Theta)^4$, so that the second term in (10) becomes appreciable only in exceedingly strong fields.

From (9) and (10) we see the already mentioned essentially different character of the influence of the components of the field **H** on the motion of the electron-phonon gas. The component $H_\parallel$ changes the viscosity of this gas, whereas $H_\perp$ contributes to the external force acting on the gas.

It is most important in the following that the fields $E_x$ and $E_y$ do not depend on z. Indeed, in our case of a thin plate, the dependence on the coordinates x and y can be neglected. It follows here from $(\operatorname{curl}\mathbf{E})_y = 0$ and $(\operatorname{curl}\mathbf{E})_z = 0$ that $\partial E_x/\partial z = 0$ and $\partial E_y/\partial z = 0$. This makes it easy to solve the system (9). The method for finding the constants $E_x$ and $E_y$ from the known external field will be described later.

The solution of (9) satisfying the boundary conditions $u_x(\pm d) = u_y(\pm d) = 0$, corresponding to diffuse scattering of the electrons from the surface of the sample, is

$$u_x = \frac{e}{m}\frac{\tau^U}{k_2^2 - k_1^2}\left\{\left(\frac{\operatorname{ch}k_1 z}{\operatorname{ch}k_1 d} - \frac{\operatorname{ch}k_2 z}{\operatorname{ch}k_2 d}\right)\frac{E_x}{v_x\tau^U} - \left[k_2^2\left(\frac{\operatorname{ch}k_1 z}{\operatorname{ch}k_1 d} - 1\right) - k_1^2\left(\frac{\operatorname{ch}k_2 z}{\operatorname{ch}k_2 d} - 1\right)\right]\frac{E_x + \gamma E_y}{1+\gamma^2}\right\}$$
$$u_y = \frac{e}{m}\frac{\tau^U}{k_2^2 - k_1^2}\left\{\left(\frac{\operatorname{ch}k_1 z}{\operatorname{ch}k_1 d} - \frac{\operatorname{ch}k_2 z}{\operatorname{ch}k_2 d}\right)\frac{E_y}{v_y\tau^U} - \left[k_2^2\left(\frac{\operatorname{ch}k_1 z}{\operatorname{ch}k_1 d} - 1\right) - k_1^2\left(\frac{\operatorname{ch}k_2 z}{\operatorname{ch}k_2 d} - 1\right)\right]\frac{E_y - \gamma E_x}{1+\gamma^2}\right\}, \tag{11}$$

where $\gamma = \Omega_\perp\tau^U$, $k_{1,2}$ are the positive roots of the equation

$$v_x v_y k^4 - \frac{1}{\tau^U}(v_x + v_y)k^2 + \left(\Omega_\perp^2 - \frac{1}{\tau^{U^2}}\right) = 0. \tag{12}$$

It can be verified, by directly calculating the expressions for $u_x$ and $u_y$ from formulas (11), that the inequality $d^2/l^U l_{ep}\eta \gg 1$ (we recall that $\eta_x \sim \eta_y$) corresponds to a bulky sample, and in this case the normal collisions do not exert a noticeable influence on the transport processes. When $d^2/l^U l_{ep}\eta \ll 1$, the results depend on the relation between the "radius" $r_\perp = cp_F/eH_\perp$ and the diffusion length $d^2/l_{ep}\eta$.

[1])Multiplying (1) and (1′) respectively by the electron energy $\epsilon(\mathbf{p})$ and the photon energy $h\nu(\mathbf{q})$, integrating, and adding we arrive at the equation $\operatorname{div}\mathbf{u} = 0$. The same equation results from the requirement that the number of electrons be conserved.

When the magnetic-field component perpendicular to the surface of the sample is sufficiently small, so that $r_\perp \gg d^2/l_{ep}\eta$, the components of the current density $j_i = neu_i$ depend on the coordinate $z$ in accordance with $j_i \sim 1 - (z/d)^2$. On the other hand, the components of the average current

$$\bar{j}_i = \frac{1}{2d}\int_{-d}^{+d} j_i(z)\,dz$$

are given by

$$\bar{j}_x = \frac{ne^2}{mv_F}\frac{d^2}{3l_{ep}\eta_x}\left\{E_x + \frac{2}{5}\frac{d^2}{r_\perp l_{ep}\eta_y}E_y\right\},$$
$$\bar{j}_y = \frac{ne^2}{mv_F}\frac{d^2}{3l_{ep}\eta_y}\left\{-\frac{2}{5}\frac{d^2}{r_\perp l_{ep}\eta_x}E_x + E_y\right\}. \qquad (13)$$

The dependence of the tensor $\sigma_{ik}$ on the field $H_\perp$ has in this case the natural form for weak fields: the diagonal elements $\sigma_{ik}$ coincide with their values in the absence of the field, $H_\perp$, and in the nondiagonal elements the effective mean free path is of the order of $(l^{\text{eff}})^2/r_\perp$, where $l^{\text{eff}}$ is the mean free path which enters in the expression for the diagonal components (see, for example[5]).

On the other hand, if $r_\perp \ll d^2/l_{ep}\eta$, then the current density varies with the coordinate $z$ in accordance with the law

$$j_x \sim E_x \exp\left\{(1+i)\frac{|z|-d}{(2r_\perp l_{ep})^{1/2}(\eta_x\eta_y)^{1/4}}\right\} + E_y. \qquad (14)$$

The expression for $j_y$ is obtained from (14) by making the substitutions $E_x \to E_y$ and $E_y \to -E_x$.

Figure 1 shows the dependence of the components $j_\nu = j_x \cos\theta + j_y \sin\theta$ and $j_\mu = -j_x \sin\theta + j_y \cos\theta$ on the coordinate $z$. The $\nu$ axis is chosen along the axis of the plate (the external electric field $E_\nu$), the $\mu$ axis is perpendicular to $\nu$ and $z$; $\theta$ is the angle between $H_\parallel$ and $E_\nu$ (in preparing the plots we took into account the fact that $\bar{j}_\mu = 0$).

The fact that the change of the current density occurs at distances of the order of $(r_\perp l_{ep}\eta)^{1/2}$ can be explained in the following manner. After colliding with the surface, the electron rotates around the magnetic field and penetrates into the metal. Since the electrons have a distribution with respect to the velocity $v_z$, they will "straggle" after passing a certain distance, and this will lead to an attenuation of the current. It is easy to see that the current attenuates at a depth at which the electrons rotate around $H_\perp$ through an angle on the order of unity. Since the angular velocity of rotation of the electron around $H_\perp$ is of the order of $v_F/r_\perp$, the electron will move, during the time that it rotates to an angle on the order of unity, in the absence of normal collisions, a distance on the order of $r_\perp$ from the surface. But if normal collisions are present, the electron will move like a Brownian particle with a pace $l_{ep}\eta$, and during the same time it will move from the surface a distance on the order of $(r_\perp l_{ep}\eta)^{1/2}$. Therefore the current connected with the collisions between the electrons and the surface will attenuate at just this distance from the surface. Together with the drift $c\mathbf{E}\times\mathbf{H}/H^2$, which does not depend on $z$, this produces the picture shown in Fig. 1. The oscillations of the current with depth are due to the rotation of the electron around the magnetic field.

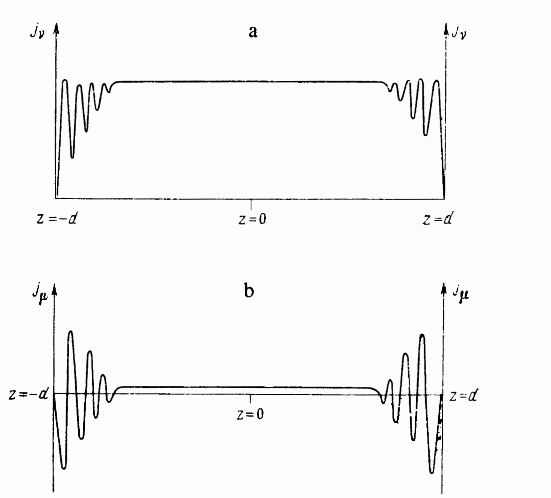


FIG. 1. Distribution of current density over the cross section of the sample when $d \gg (r_\perp l_{ep}\eta)^{1/2}$ and $r_\perp \gg l_{ep}$: a—current density on the order of $necE_\nu d/H_\perp\sqrt{r_\perp l_{ep}\eta}$; b—current density near the surface on the order of $necE_\nu d/H_\perp\sqrt{r_\perp l_{ep}\eta}$, and in the bulk of the volume $necE_\nu/H_\perp$. The current oscillations on both diagrams have a period $(r_\perp l_{ep}\eta)^{1/2}$.

The expressions for the average current at $(r_\perp l_{ep}\eta)^{1/2} \ll d$ are of the form

$$\bar{j}_x = \frac{ne^2}{mv_F}r_\perp\left\{\frac{(r_\perp l_{ep})^{1/2}\eta_y}{\sqrt{2}d(\eta_x\eta_y)^{1/4}}E_x + E_y\right\},$$
$$\bar{j}_y = \frac{ne^2}{mv_F}r_\perp\left\{-E_x + \frac{(r_\perp l_{ep})^{1/2}\eta_x}{\sqrt{2}d(\eta_x\eta_y)^{1/4}}E_y\right\}. \qquad (15)$$

It is now appropriate to note the following circumstance. Although all the calculations of the present paper, as already mentioned, pertain to the case $r_\perp \gg l_{ep}$, the considerations advanced above with respect to the character of the motion of the electron near the surface make it also possible to determine qualitatively correct expressions for the current density even when $r_\perp \ll l_{ep}$. In this case the current connected with the collisions between the electrons and the surface has two characteristic attenuation scales (compare with[6]): $r_\perp$ and $l_{ep}$. Indeed, the electrons for which $v_x, v_y \sim v_z$ "straggle" as the result of the distribution with respect to $v_z$, at a distance on the order of $r_\perp$ from the surface, i.e., the surface current due to such electrons will attenuate at distances on the order of $r_\perp$ from the surface, and since $l^{\text{eff}} \sim r_\perp$ for such electrons, their contribution to the average current is of the order of $r_\perp ne^2 r_\perp/dmv_F$. On the other hand, the electrons for which $v_x, v_y \lesssim r_\perp v_z/l_{ep}$, will move a distance of the order of $l_{ep}$ from the surface prior to turning around $H_\perp$ through an angle on the order of unity. After normal collisions, they are knocked out from this group, i.e., the surface current due to such electrons will attenuate at a distance on the order of $l_{ep}$ from the surface. Since $l^{\text{eff}} \sim r_\perp$ for such electrons, and their relative number is of the order of $r_\perp/l_{ep}$, the contribution of such electrons to the average current is of the order of

$$\frac{l_{ep}}{d}\frac{ne^2}{mv_F}r_\perp\frac{r_\perp}{l_{ep}}.$$

Thus, when $r_\perp \ll l_{ep}$ we have, in order of magnitude,

$$\bar{j}_x = \frac{ne^2}{mv_F} r_\perp \left\{ \frac{r_\perp}{d} E_x + E_y \right\},$$
$$\bar{j}_y = \frac{ne^2}{mv_F} r_\perp \left\{ -E_x + \frac{r_\perp}{d} E_y \right\}. \quad (16)$$

For convenience in comparing the results with experiment, we write out the components of the resistance tensor $\rho_{ik}$ in the coordinates system $(\nu, \mu, z)$ (we recall that the $\nu$ axis is chosen in the direction of the external field, and $\mu \perp \nu, z$).

Recognizing that $\bar{j}_\mu = \bar{j}_z = 0$, we have

$$E_\nu = \rho_{\nu\nu}\bar{j}_\nu, \quad E_\mu = \rho_{\mu\nu}\bar{j}_\nu, \quad \bar{E}_z = \rho_{z\nu}\bar{j}_\nu. \quad (17)$$

We emphasize that $E_\nu$ is the external field, and that $E_\mu$ and $E_z$ are the Hall fields. Simple calculations yield

$$\rho_{\nu\nu} = \frac{\sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta}{\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2}, \quad (18)$$

$$\rho_{\mu\nu} = \frac{\sigma_{xy} + \frac{1}{2}(\sigma_{xx} - \sigma_{yy})\sin 2\theta}{\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2}, \quad (18')$$

$$\rho_{z\nu} = \frac{H_\parallel}{nec}\sin\theta, \quad (18'')$$

where $\sigma_{ik}$ is determined by formulas (13)—(16) and $\theta$ is the angle between the axes $\nu$ and x; in determining $\rho_{z\nu}$, we used (7) and the fact that $|E_{z1}| \gg |E_{z2}|$.

Figures 2 and 3 show plots of $\rho_{\nu\nu}$ and $\rho_{\mu\nu}$ as functions of the components of the field **H**: Fig. 2 as functions of $H_\perp$ at fixed $H_\parallel$ and Fig. 3 as functions of $H_\parallel$ at fixed $H_\perp$.

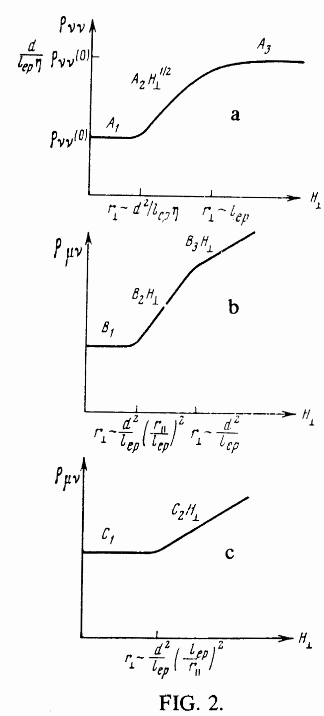

FIG. 2.

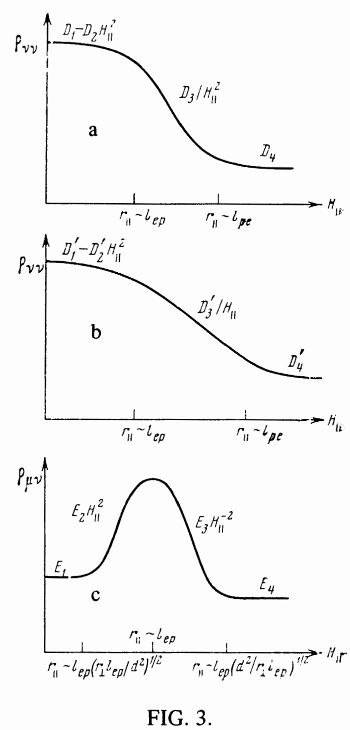

FIG. 3.

FIG. 2. Dependence of the tensor $\rho_{ik}$ on the field $H_\perp$ at fixed $H_\parallel$. a) $A_1 \sim T^{6}d^{-2}$, $A_2 \sim T^{-5/2}d^{-1}\eta^{1/2}$, $A_3 \sim d^{-1}$; b) $B_1 \sim d^{-2}T^{-15}H_\parallel^2$, $B_2 \sim B_3$ – constants, with $B_2/B_3 = 6/5$; c) $C_1 \sim d^{-2}T^5H_\parallel^{-2}$, $C_2 = B_3$. The dependence of $\rho_{\mu\nu}$ on $H_\perp$: b – at $r_\parallel \gg l_{ep}$, c – at $l_{ep} \gg r_\parallel \gg l_{pe}$.

FIG. 3. Dependence of the tensor $\rho_{ik}$ on the field $H_\parallel$ at fixed $H_\perp$: a) $D_1 \sim T^{-5}$, $D_2 \sim T^{-15}$, $D_3 \sim T^5$, $D_4 \sim T^3$, all $D_i \sim d^{-2}$; b) $D'_1 \sim T^{-2.5}$, $D'_2 \sim T^{-12.5}$, $D'_3 \sim T^{7.5}$, $D'_4 \sim T^{-0.5}$, all $D'_i \sim d^{-1}H_\perp^{1/2}$; c) $E_2 \sim T^{-15}d^{-2}$, $E_3 \sim T^{-5}d^{-2}$; $E_1 \sim E_4 \sim H_\perp$, with $E_1/E_4 = 6/5$. The dependence of $\rho_{\nu\nu}$ on $H_\parallel$ takes place: a) when $r_\perp \gg d^2/l_{ep}\eta$, b) when $r_\perp \ll d^2/l_{ep}\eta$.

It is seen from Fig. 2a that the diagonal component $\rho_{\nu\nu}$ tends to saturate with increasing $H_\perp$, as is the case in the usual electric conductivity mechanism for metals with closed Fermi surfaces. However, in the hydrodynamic mechanism of the electric conductivity, there is an intermediate region in which the resistance has an unusual growth, $\rho_{\nu\nu} \sim H_\perp^{1/2}$. The nondiagonal component of $\rho_{\mu\nu}$ (Figs. 2b, c), as in the absence of normal collisions, is proportional to $H_\perp$ at sufficiently strong fields $H_\perp$. But the presence of normal collisions in the case of weak ($r_\parallel \gg l_{ep}$) and very strong ($r_\parallel < l_{pe}$) fields causes the region $\rho_{\mu\nu} = \text{const}\cdot H_\perp$ to break up into two regions: $\rho_{\mu\nu} = B_2H_\perp$ and $\rho_{\mu\nu} = B_3H_\perp$. It is seen from Figs. 2b and c that when $r_\parallel > l_{ep}$ the increase of $H_\parallel$ leads to an increase of the region $\rho_{\mu\nu} = \text{const}\cdot H_\perp$, and to a decrease when $r_\parallel < l_{ep}$. It must be emphasized that the diagrams of Fig. 2b and c were plotted for angles $\theta$ larger than $\pi/2$. When $\theta < \pi/2$, the regions $\rho_{\mu\nu} = \text{const}$ lie below the abscissa axis.

We note that Fig. 2 does not show the dependence of $\rho_{\mu\nu}$ on $H_\perp$ when $r_\parallel < l_{pe}$. In this case (which is very difficult to achieve experimentally), the plot of $\rho_{\mu\nu}(H_\perp)$ has the same form as in Fig. 2b, with $B_1 \sim d^{-2}H_\parallel^{-2}T^5$ and $B_2 \sim B_3 = \text{const}$; the first two regions coming in contact when $r_\perp \sim (d^2/l_{ep})(l_{ep}/r_\parallel)^2$, while the second and third regions come in contact when $r_\perp \sim (d^2/l_{ep})(\Theta/T)^8$.

The dependence of $\rho_{\nu\nu}$ on the magnetic field parallel to the surface (Fig. 3a, b) is determined by the value of the field $H_\perp$. When $r_\perp \gg d^2/l_{ep}\eta$, the $\rho_{\nu\nu}(H_\parallel)$ dependence has the same form as when $H_\perp = 0$. On the other hand, if $r_\perp \ll d^2/l_{ep}\eta$, then the decrease of $\rho_{\nu\nu}$ in strong fields $H_\parallel$ is slower than when $H_\perp = 0$. It must also be borne in mind that the inequality $r_\perp \gg d^2/l_{ep}\eta$ can go over with increasing $H_\parallel$ into the inequality $r_\perp \ll d^2/l_{ep}\eta$ (see formulas (10)). Therefore it is possible to realize experimentally not only cases of the $\rho_{\nu\nu}(H_\parallel)$ dependence shown in Figs. 3a and b, but also the intermediate case (i.e., in the region $l_{pe} < r_\parallel < l_{ep}$ the decrease of $\rho_{\nu\nu}(H_\parallel)$ begins like $1/H^2$ and ends like $1/H$).

Finally, a characteristic feature of the $\rho_{\mu\nu}(H_\parallel)$ plot is the presence at $r_\perp < d^2/l_{ep}\eta$ of a minimum (at $\theta < \pi/2$) or a maximum (at $\theta > \pi/2$) at the point $r_\parallel \sim l_{ep}$. Figure 3c shows a plot of $\rho_{\mu\nu}(H_\parallel)$ for the case $\theta > \pi/2$.

It was already indicated above that the exceeding complexity of the integro-differential equations (5) does not make it possible to obtain their solution for arbitrary electron and phonon dispersion laws. However, from the qualitative considerations advanced above it is clear that the results obtained for the isotropic case are valid in order of magnitude for arbitrary closed Fermi surfaces and for an arbitrary phonon dispersion law. In formulas (13) and (15), and (16), $r_\parallel$, $r_\perp$, and $l_{ep}$ should be taken to mean their average values over the Fermi surface.

If the Fermi surface has open sections, the foregoing picture of the electron motion will apparently not occur (with the exception of the case when the open section is only in the z direction).

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[1] R. N. Gurzhi, Zh. Eksp. Teor. Fiz. **47**, 1415 (1964) [Sov. Phys.-JETP **20**, 953 (1965)].

[2] R. N. Gurzhi and S. I. Shevchenko, Zh. Eksp. Teor. Fiz. **54**, 1899 (1968) [Sov. Phys.-JETP **27**, 1014 (1968)].

[3] K. Huang, Statistical Mechanics, Wiley, 1963.

[4] G. E. Uhlenbeck and J. Ford, Lectures on Statistical Mechanics, Am. Math. Soc. 1966.

[5] J. M. Ziman, Theory of Solids, Cambridge U.P.

[6] M. Ya. Azbel', Zh. Eksp. Teor. Fiz. **44**, 980, 983 (1963) [Sov. Phys.-JETP **17**, 665, 667 (1963)].

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