# EFFECTS OF P- AND T-PARITY VIOLATION IN RESONANCE SCATTERING OF γ RADIATION

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Resonance scattering of $\gamma$-radiation by nuclei in a magnetic field is considered. Expressions for the scattering amplitudes and cross sections of a mixed multiplicity nuclear transition are derived for the case when the line width of the incident $\gamma$-radiation is smaller than the Zeeman splitting of the nuclear levels. The scattering cross section is expressed in terms of resonance absorption cross sections in the Zeeman transitions. Consequences of violation of P- and T-parity in resonance scattering of $\gamma$-quanta are analyzed. Scattering in nuclear transitions which are a mixture of dipole and quadrupole multipoles is considered in detail.

## 1. INTRODUCTION

At present, T-invariance in various interactions is being intensively investigated.[1] In particular, T-invariance of nuclear and electromagnetic interactions is being investigated in nuclear $\gamma$-ray transitions (cf., e.g.,[2,3]). In relation to the demonstrated non-conservation of P-parity for nuclear states[4], there has appeared, over a period of time, a series of experimental studies of P-noninvariant effects.[5-8]

Experimental investigations, looking for evidence of violations of P- and T-parity in electromagnetic transitions in nuclei,[2,3,7] have demonstrated the relevance of using the Mössbauer effect in such studies. In the works noted above, the method used was that of resonance absorption of $\gamma$-radiation. It is of interest to study the possibility of using Mössbauer scattering, with the same goal, which might prove to be a more effective method of looking for violations of P- and T-parity (cf., e.g.,[1,9]). In this article, we analyze the consequences of violation of P- and T-parity for the case of resonance scattering of $\gamma$ quanta following a nuclear $\gamma$ transition of mixed multipolarity. We examine the possibilities of establishing violations of P- and T-parity via use of resonance scattering.

The violation of P-parity for nuclear states permits the existence of $\gamma$ transitions in which a contrary multiparity forbidden by parity is admixed to a parity-allowed multipolarity. As is well-known,[10] for radiative transitions of mixed multipolarity the condition of T-invariance leads to the value 0 or $\pi$ for the relative phase $\eta$ of the matrix elements of mixed multipolarities. In this fashion, the confirmation of the occurrence of transitions with an admixture forbidden by parity would serve as evidence of violation of P-parity, and the presence of transitions with $\eta \neq 0$ or $\eta \neq \pi$ would confirm violation of T-invariance. The analysis of effects due to interference between mixed multipolarities in resonance emission, absorption, or scattering of $\gamma$ radiation permits a decision as to the character of the mixture and relative phase $\eta$ and, thereby, also the investigation of P- and T-parity violation. We present below an analysis of P- and T-nonparity effects for resonance scattering.

## 2. THE AMPLITUDE OF RESONANCE SCATTERING

We shall study resonance scattering of $\gamma$ radiation by a nucleus in a magnetic field that induces Zeeman splitting of the nuclear levels. We assume that the scattering proceeds via a nuclear electromagnetic transition of mixed multipolarity and that the $\gamma$-radiation line width is less than the Zeeman splitting of the nuclear levels. The process of resonance scattering may be divided into two stages: 1) resonance absorption of the emitted $\gamma$ quantum, during which the nucleus is changed from the initial state i to the intermediate state $\nu$; 2) emission by the excited nucleus of a secondary $\gamma$ quantum, during which the nucleus is changed to the final state f. The assumed small line-width of the emitted radiation means that, for a fixed energy of the initial $\gamma$ quantum, the scattering proceeds via well-defined Zeeman levels of the initial and intermediate states of the nucleus.

Under the given assumptions, the expression for the scattering amplitude has the form

$$f_{fi}(\mathbf{k}, \mathbf{n}, \mathbf{k}', \mathbf{n}') = cH_{i\nu}(\mathbf{k}, \mathbf{n})H_{\nu f}(\mathbf{k}', \mathbf{n}'). \quad (1)$$

Here $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the initial and scattered $\gamma$ quantum, respectively, $\mathbf{n}$ and $\mathbf{n}'$ the corresponding polarization vectors (the latter being the one of particular interest to us), and $H_{i\nu}(\mathbf{k}, \mathbf{n})$ and $H_{\nu f}(\mathbf{k}', \mathbf{n}')$ the matrix elements of the nuclear-electromagnetic interaction Hamiltonian; the multiplier c, whose specific form[11,12] is irrelevant for our purposes, will be omitted in the following. The absorption matrix element $H_{i\nu}$ in Eq. (1) can be expressed in terms of $\mathbf{n}_{\nu i}$, the polarization vector, and $I_{\nu i}$, the intensity of the radiation emitted in the direction $\mathbf{k}$ during the Zeeman transition, $i \rightarrow \nu$, in the form:[13]

$$H_{i\nu}(\mathbf{k}, \mathbf{n}) = (\mathbf{n}^*\mathbf{n}_{\nu i})\sqrt{I_{\nu i}(\mathbf{k})}. \quad (2)$$

The matrix element for the emitted $\gamma$-quantum with wave-vector $\mathbf{k}'$ and polarization $\mathbf{n}'$ in the transition $\nu \rightarrow f$ takes the form

$$H_{\nu f}(\mathbf{k}', \mathbf{n}') = (\mathbf{n}'\mathbf{n}_{\nu f}'^{*})\sqrt{I_{\nu f}(\mathbf{k}')}. \quad (3)$$

Here, the quantities $\mathbf{n}'_{\nu f}$ and $I_{\nu f}(\mathbf{k}')$ refer to the transition $\nu \rightarrow f$, with the meaning given above. Substituting

Eq. (2) and Eq. (3) into Eq. (1), we obtain the scattering amplitude

$$f_{if}(\mathbf{k},\mathbf{n},\mathbf{k}',\mathbf{n}') = (\mathbf{n}^*\mathbf{n}_{\nu i})(\mathbf{n}'\mathbf{n}^*_{\nu f})\sqrt{I_{\nu i}(\mathbf{k})I_{\nu f}(\mathbf{k}')}. \quad (4)$$

For a transition in which there is a mixing of multipolarities $L_1$ and $L_2$, $\mathbf{n}_{\nu i}$ and $I_{\nu i}(\mathbf{k})$ can be expressed as:

$$\mathbf{n}_{\nu i} = (|E_1|^2+|E_2|^2)^{-1/2}\left(\frac{[\hat{\mathbf{k}}[\mathbf{h}\hat{\mathbf{k}}]]}{|[\mathbf{h}\hat{\mathbf{k}}]|}E_1 + i\frac{[\hat{\mathbf{k}}\mathbf{h}]}{|[\mathbf{h}\hat{\mathbf{k}}]|}E_2\right), \quad (5)^*$$

$$I_{\nu i}(\mathbf{k}) = |E_1|^2+|E_2|^2, \quad (6)$$

$$E_q = E_q(L_1)+E_q(L_2)e^{i\eta}, \quad q=1,2; \quad (7)$$

$$E_q(L) = \begin{pmatrix} j_i & L & j_\nu \\ m_i & M & -m_\nu \end{pmatrix} |\chi(L)|\, e_q(L,M); \quad (8)$$

$$\begin{aligned} e_1 = \Bigg\{&\Bigg[\begin{pmatrix} l & 1 & L \\ M+1 & -1 & -M \end{pmatrix} Y_l^{-M-1}(\hat{\mathbf{k}}) \\ &- \begin{pmatrix} l & 1 & L \\ M-1 & 1 & -M \end{pmatrix} Y_l^{1-M}(\hat{\mathbf{k}})\Bigg]\cos\theta \\ &+ \sqrt{2}\sin\theta \begin{pmatrix} l & 1 & L \\ M & 0 & -M \end{pmatrix} Y_l^{-M}(\hat{\mathbf{k}})\Bigg\}(2L+1)^{1/2}, \end{aligned} \quad (9)$$

$$\begin{aligned} e_2 = \Bigg[&\begin{pmatrix} l & 1 & L \\ M+1 & -1 & -M \end{pmatrix} Y_l^{-M-1}(\hat{\mathbf{k}}) \\ &+ \begin{pmatrix} l & 1 & L \\ M-1 & 1 & -M \end{pmatrix} Y_l^{1-M}(\hat{\mathbf{k}})\Bigg](2L+1)^{1/2}. \end{aligned}$$

In Eqs. (5–9) $\mathbf{h}$ is a unit vector in the magnetic field direction; $\theta$ is the angle between $\mathbf{h}$ and $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$; the spherical harmonics in Eq. (9) are functions of only the one angle $\theta$ because they are expressed in a coordinate system in which the polar axis coincides with $\mathbf{h}$ and the azimuthal angle for $\mathbf{k}$ is zero; $j_\nu$, $j_i$, $m_\nu$, and $m_i$ are the nuclear spins and projections in the magnetic-field direction for the states $\nu$ and i; $M = m_\nu - m_i$; $\chi(L)$ is the reduced nuclear matrix element. The expressions for $\mathbf{n}'_{\nu f}(\mathbf{k}')$ are given by Eqs. (5–9) with replacement of i by f and $\theta$ by $\theta'$. The scattering amplitude for a nuclear transition corresponding to the case of "pure" multipolarity can be obtained from Eqs. (5–9) by setting $E_q(L_2) = 0$ in these relations. For dipole- and quadrupole transitions the $e_q(L, M)$ are tabulated in[13].

## 3. RESONANCE-SCATTERING CROSS SECTION

With the use of Eq. (4) for the scattering amplitude, the differential cross section for scattering of $\gamma$ quanta in the direction $\mathbf{k}'$ with polarization vector $\mathbf{n}'$ takes the form

$$\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \sum_f |f_{if}(\mathbf{k},\mathbf{n},\mathbf{k}',\mathbf{n}')|^2 = \sum_f \Sigma_{i\nu}(\mathbf{k},\mathbf{n})\Sigma_{f\nu}(\mathbf{k}',\mathbf{n}'), \quad (10)$$

where $\Sigma_{cd}(\mathbf{k}, \mathbf{n})$ is the absorption cross-section for $\gamma$ quanta with wave-vector $\mathbf{k}$ and polarization $\mathbf{n}$ in the transition $c \to d$ (this quantity coincides with the emission cross section with the same $\mathbf{k}$ and $\mathbf{n}$ in the transition $d \to c$). For the case of unpolarized or partially-polarized incident radiation, the cross section formula (10) takes the form

$$\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \sum_f \overline{\Sigma_{i\nu}(\mathbf{k},\mathbf{n})}\,\Sigma_{f\nu}(\mathbf{k}',\mathbf{n}'), \quad (11)$$

where the bar over $\Sigma_{i\nu}(\mathbf{k}, \mathbf{n})$ indicates averaging over the initial polarization. Since under our assumed conditions the radiation is absorbed during a definite Zeeman transition, the polarization of the scattered radiation is independent of the polarization of the emitted radiation. In the case of a single final state f the scattered radiation is completely polarized; its polarization coincides with the polarization of the radiation emitted in the transition $\nu \to f$ in the direction $\mathbf{k}'$, and is described by the vector $\mathbf{n}_{\nu f}$. In the case of several final states f, the scattered radiation is partially polarized. The corresponding polarized density matrix is given by the expression

$$\rho(\mathbf{k}') = \left(\sum_f \rho(\mathbf{n}_{\nu f}')\Sigma_{f\nu}(\mathbf{k}',\mathbf{n}')\right)\Big/\sum_f \Sigma_{f\nu}(\mathbf{k}',\mathbf{n}_{\nu f}'), \quad (12)$$

where $\rho(\mathbf{n}'_{\nu f})$ is the polarized density matrix for $\gamma$ quanta with polarization vector $\mathbf{n}_{\nu f}$[14]. If the polarization of the scattered radiation is not measured, expression (10) for the cross section simplifies to:

$$\frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \sum_f \Sigma_{i\nu}(\mathbf{k},\mathbf{n})\Sigma_{f\nu}(\mathbf{k}',\mathbf{n}_{\nu f}') = \sum_f \Sigma_{i\nu}(\mathbf{k},\mathbf{n})I_{\nu f}(\mathbf{k}'). \quad (13)$$

In this way, the scattering cross section can be expressed as the product of the absorption cross section for the transition $i \to \nu$ by a certain sum of absorption and emission cross-sections for transitions $\nu \to f$. We shall therefore introduce below expressions for the cross section of resonance absorption of $\gamma$ quanta, from which the resonance scattering cross section can be obtained simply by multiplying and summing the appropriate quantities. We note that in expression (10) for the scattering cross section, the sum over f degenerates into one term either if the selection rules for radiation via the intermediate state allow only one final state of the nucleus or if the detection of the scattered radiation is realized by a resonance detector set for the energy of the precisely defined transition $\nu \to f$.

## 4. THE RESONANCE ABSORPTION CROSS SECTION

The nuclear resonance absorption cross section for a polarized $\gamma$ quantum in the transition $i \to \nu$ can, with the help of Eq. (2), be written as:

$$\Sigma_{i\nu}(\mathbf{k},\mathbf{n}) = |(\mathbf{n}\mathbf{n}_{\nu i}^*)|^2 I_{\nu i}(\mathbf{k}). \quad (14)$$

The polarization vector of the absorbed quantum can be expressed in the form:

$$\mathbf{n} = (\chi_2\cos\alpha + i\chi_1\sin\alpha)e^{i\beta}, \quad (15)$$

where $\chi_1$ and $\chi_2$ are unit vectors, mutually orthogonal and also orthogonal to $\mathbf{k}$, and $\alpha$ and $\beta$ are real parameters. The ratio of the axes of the corresponding polarization ellipse equals $\tan\alpha$, $\chi_1$ and $\chi_2$ specify their orientation, and $\exp(i\beta)$, is a phase-factor multiplier which is inessential here.

The value of the parameter $\alpha = \pm\pi/4$ in Eq. (15) corresponds to circular polarization, just as $\alpha = 0$ or $\pi/2$ corresponds to linear polarization. Using the expressions for $I_{\nu i}(\mathbf{k})$, $\mathbf{n}_{\nu i}$, and $\mathbf{n}$ (Eqs. (5, 6, 7, 15)), we obtain the absorption cross section of polarized $\gamma$ radiation in a $\gamma$ transition of mixed multiplicity:

$$\Sigma_{i\nu}(\mathbf{k},\mathbf{n}) = \frac{1}{2}\sum_{p,q} E_q^2(L_p)[1-(-1)^q\cos 2\alpha\cos 2\varphi]$$

$$+ \sin 2\alpha \sum_p E_1(L_p)E_2(L_p) + \cos\eta\left\{\sum_q E_q(L_1)E_q(L_2)[1\right.$$

*$[\mathbf{h}\hat{\mathbf{k}}] \equiv \mathbf{h}\times\hat{\mathbf{k}}$

$$-(-1)^q \cos 2\alpha \cos 2\varphi] + \sin 2\alpha [E_1(L_1)E_2(L_2) + E_2(L_1)E_1(L_2)]\Big\} + \sin\eta \cos 2\alpha \sin 2\varphi [E_1(L_1)E_2(L_2) - E_2(L_1)E_1(L_2)],$$

where p, q = 1, 2; $\varphi$ is the azimuthal angle of **h** relative to the axis $\hat{\mathbf{k}}$, measured from the direction of the vector $\chi_1$ (cf. Fig. 1).

If the absorbed $\gamma$ radiation is unpolarized, the absorption cross section has the form:

$$\Sigma_{iv}(\mathbf{k}) = \frac{1}{2} \sum_{p,q} E_q^2(L_p) + \cos\eta \sum_q E_q(L_1)E_q(L_2) = \frac{1}{2} I_{vi}(\mathbf{k}). \quad (17)$$

If the radiation is partially polarized and the degree of its polarization is P[14], the absorption cross section is equal to

$$\Sigma_{iv}(\mathbf{k}, \mathbf{n}) = \frac{1}{2}(1-P)I_{vi}(\mathbf{k}) + P\Sigma_{iv}(\mathbf{k}, \mathbf{n}), \quad (18)$$

where **n** is the vector describing the polarization partially represented in the absorbed radiation.

## 5. SCATTERING IN $\gamma$ TRANSITION WITH ADMIXTURE OF PARITY-FORBIDDEN MULTIPLICITY

We shall examine the manifestations of P-parity violation in resonance scattering. In this case, the scattering cross section is given by the formulas of the preceding sections, in which the resonance $\gamma$ transition contains an admixture of multipolarity forbidden by parity. As is evident from Eq. (13), the consequences of violation of P-parity in scattering can be found from the related effects in the absorption cross section. In the following, for definiteness, we shall assume that in the formulas of Sec. 3 there is only a single final state f.

To find evidence for violation of P-parity we can use the fact that in Eqs. (10), (11), (13), and (16) only the interference terms change sign (under certain transformation of the arguments $(\theta, \varphi)$); these terms are non-zero only because of admixtures forbidden by parity, and are proportional to the admixture parameter $\delta$.

For example, for linearly-polarized initial radiation, such transformations are: 1) the reversal of direction of the magnetic field ($\mathbf{h} \to -\mathbf{h}$, i.e., $\theta \to \pi - \theta$, $\varphi \to \pi + \varphi$), 2) the subsequent change of direction of the magnetic field: $\theta \to \pi - \theta$, $\varphi \to \pi - \varphi$ (cf. Fig. 2).

The latter transformation changes the scattering cross section if there is violation of P-parity and has no effect if there is conservation only in the case that the wave vector $\mathbf{k}'$ of the scattered radiation is orthogonal to the plane of polarization of the incident radiation. We can see this from Eqs. (10) and (16) if we set $\alpha = 0, \pi/2$ (linear polarization) in Eq. (16), and carry through the resulting transformation. These same transformations result in a change of cross section upon P-parity violation only when the incident radiation is completely unpolarized. If the initial radiation is polarized, but not linearly, $(\alpha \neq 0, \pi/2)$, then a change of the magnetic-field direction results in a change of the scattering cross section even in the absence of parity-forbidden admixtures. (In the expressions given above, the relative cross section change is proportional to $\sin 2\alpha$). Therefore, a transformation that changes the scattering cross section only upon violation of P-parity must, in this case, include not only a

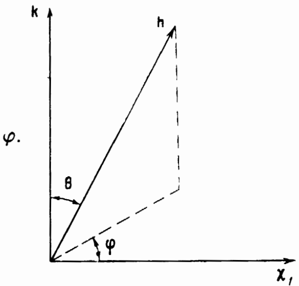


FIG. 1. Definition of the angles $\theta$ and $\varphi$.

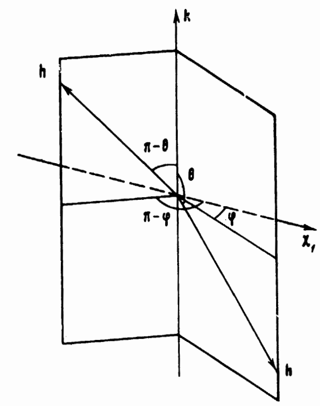


FIG. 2. Transformation of the magnetic-field direction from **h** to $\tilde{\mathbf{h}}$, corresponding to $\theta \to \pi - \theta$, $\varphi \to \pi - \varphi$.

change of the magnetic field but also a change of the polarization of the initial radiation[13] (e.g., simultaneous reversal of magnetic-field direction and change of the polarization vector of the incident radiation to the complex-conjugate).

In resonance scattering, the same nuclei usually serve as both source and scatterer and, therefore, there is simultaneous presence of parity-forbidden admixture in both radiative and scattering transitions. Hence, radiation from unpolarized nuclei appears to be circularly-polarized with degree of polarization $P \sim \delta$ (cf. e.g.,[15]), i.e., polarization is an inescapable consequence of P-parity violation. In this event, in the absence of line splitting of the source, the change in cross section under the transformations examined above appears entirely as a consequence of P-parity violation, and the relative difference between the scattering cross sections, under reversal of magnetic-field direction, in the approximation linear in the admixture parameter, is given by the expression

$$\left(\frac{d\sigma(\mathbf{h})}{d\Omega_{\mathbf{k}'}} - \frac{d\sigma(-\mathbf{h})}{d\Omega_{\mathbf{k}'}}\right) \Big/ \left(\frac{d\sigma(\mathbf{h})}{d\Omega_{\mathbf{k}'}} + \frac{d\sigma(-\mathbf{h})}{d\Omega_{\mathbf{k}'}}\right) = \frac{I(\mathbf{k},\mathbf{h})I(\mathbf{k}',\mathbf{h}) - I(\mathbf{k},-\mathbf{h})I(\mathbf{k}',-\mathbf{h})}{I(\mathbf{k},\mathbf{h})I(\mathbf{k}',\mathbf{h}) + I(\mathbf{k},-\mathbf{h})I(\mathbf{k}',-\mathbf{h})} + 4P\tau \frac{E_1E_2}{E_1^2 + E_2^2}, \quad (19)$$

where $\tau$ takes on the value 1 or −1 for right- or left-circular polarization, respectively.

The case of the mixtures E(1) – E(2) and E(2) – M(2) was examined in[16]. We give the absorption cross section for the mixture E(1) – M(1):

$$\Sigma(\mathbf{k}, \mathbf{n})_{M=0} = \sin^2\theta \left[\frac{a^2 + b^2}{2} + \frac{a^2 - b^2}{2} \cos 2\alpha \cos 2\varphi - ab \cos\eta \sin 2\alpha - ab \sin\eta \cos 2\alpha \sin\varphi\right],$$

$$\Sigma(\mathbf{k},\mathbf{n})_{M=\pm1}=(a^2+b^2)(1+\cos^2\theta)-(a^2-b^2)\cos 2\alpha\cos 2\varphi\sin^2\theta$$
$$\mp(a^2+b^2)\sin 2\alpha\cos\theta+ab\cos\eta[\pm\cos\theta-\sin 2\alpha(1+\cos^2\theta)]$$
$$-ab\sin\eta\cos 2\alpha\sin 2\varphi\sin^2\theta,$$

where the quantities a and b are defined by the relations:

$$a_{M=0}=\sqrt{\frac{3}{2}}\begin{pmatrix} j_i & 1 & j_\nu \\ m_i & 0 & -m_\nu \end{pmatrix}|\chi|,$$
$$a_{M=\pm1}=\frac{\sqrt{3}}{2}\begin{pmatrix} j_i & 1 & j_\nu \\ m_i & 1 & -m_\nu \end{pmatrix}|\chi|. \quad (20)$$

The symbol b, analogon to a, is similarly defined for the admixed multipolarity. The quantities $\chi$ in Eq. (20) differ from the previously introduced matrix elements (cf. Eq. (8)) by numerical factors and are connected with the total intensity of radiation from a polarized nucleus by the relationship:

$$\int I_0 d\Omega_{\mathbf{k}}=\frac{1}{2j_\nu+1}[|\chi(L_1)|^2+|\chi(L_2)|^2];\qquad I_0=\sum_{i\nu} I_{\nu i}(\mathbf{k}).$$

## 6. RESONANCE SCATTERING AND VIOLATION OF T-INVARIANCE

We shall examine $\gamma$-ray scattering for a nuclear $\gamma$ transition which is a mixture of two parity-allowed multipolarities, but with either $\eta \neq 0$ or $\eta \neq \pi$. As in the preceding section, we limit ourselves to the case of a single final state f. To determine that $\eta$ differs from 0 or $\pi$, we can use the fact that in Eqs. (10), (11), (12), and (16), only the term proportional to $\sin\eta$ is changed by certain transformations of their arguments. For arbitrary initial polarization, these are such transformations that change the sign of $\sin 2\varphi$ (i.e., $\varphi \to -\varphi$ or $\varphi \to \pi-\varphi$). These transformations can be realized by rotation of the plane of polarization for linear polarization or, in the general case, by rotation of the axes of the polarization-ellipse of the incident radiation or rotation of the magnetic field with respect to the direction $\hat{\mathbf{k}}$. In the case of linear polarization of the initial radiation, only the sign of the term containing $\sin\eta$ changes, just as with reversal of the magnetic-field direction. Let us recall that the replacement $\mathbf{h} \to -\mathbf{h}$ changes the scattering cross section in the presence of parity-forbidden mixed multipolarities in a $\gamma$ transition. Therefore, with this transformation, the effects of P-parity violation, if we do not specially take them into account, may imitate violation of T-invariance (for details, cf.[17]).

We write down the expression for the relative change of the scattering cross section for polarized radiation occurring when we substitute $\varphi \to -\varphi$:

$$\left(\frac{d\sigma(\varphi)}{d\Omega_{\mathbf{k}'}}-\frac{d\sigma(-\varphi)}{d\Omega_{\mathbf{k}'}}\right)\Big/\left(\frac{d\sigma(\varphi)}{d\Omega_{\mathbf{k}'}}+\frac{d\sigma(-\varphi)}{d\Omega_{\mathbf{k}'}}\right)=\frac{|(\mathbf{n}_\varphi{}^*\mathbf{n}_{\nu i})|^2-|(\mathbf{n}_{-\varphi}{}^*\mathbf{n}_{\nu i})|^2}{|(\mathbf{n}_\varphi{}^*\mathbf{n}_{\nu i})|^2+|(\mathbf{n}_{-\varphi}{}^*\mathbf{n}_{\nu i})|^2}. \quad (21)$$

We write down in explicit form the absorption cross section of polarized $\gamma$ radiation in the mixed-multipolarity transition E(2) – M(1):

$$\Sigma(\mathbf{k},\mathbf{n})_{M=0}=\sin^2\theta[{}^1\!/_2(a^2\cos^2\theta+b^2\sin^2\theta)$$
$$+{}^1\!/_2(a^2\cos^2\theta-b^2\sin^2\theta)\cos 2\alpha\cos 2\varphi$$
$$+ab\cos\eta\sin 2\alpha\cos\theta+ab\sin\eta\cos 2\alpha\sin 2\varphi\cos\theta],$$
$$\Sigma(\mathbf{k},\mathbf{n})_{M=\pm1}=a^2(\cos^2 2\theta+\cos^2\theta)+b^2(1+\cos^2\theta)$$
$$-(a^2\sin^2\theta-b^2\cos^2\theta)\cos 2\alpha\cos 2\varphi$$
$$\mp(a^2\cos 2\theta+b^2)\sin 2\alpha\cos\theta-ab\cos\eta[\cos 2\theta+\cos^2\theta$$
$$-\cos 2\alpha\cos 2\varphi\sin^2\theta\mp 2\sin 2\alpha\cos^3\theta]\mp ab\sin\eta\cos 2\alpha\sin 2\varphi\sin 2\theta\sin\theta,$$
$$\Sigma(\mathbf{k},\mathbf{n})_{M=\pm2}=a^2\sin^2\theta[1+{}^1\!/_2\cos 2\theta(1+\cos 2\alpha\cos 2\varphi)\mp\sin 2\alpha\cos\theta],$$

where, for the multipolarity E(2),

$$a_{M=0}=\sqrt{\frac{15}{2}}\begin{pmatrix} j_i & 2 & j_\nu \\ m_i & 0 & -m_\nu \end{pmatrix}|\chi|,$$
$$a_{M=\pm1}=\frac{\sqrt{5}}{2}\begin{pmatrix} j_i & 2 & j_\nu \\ m_i & 1 & -m_\nu \end{pmatrix}|\chi|, \quad (22)$$
$$a_{M=\pm2}=\frac{\sqrt{5}}{2}\begin{pmatrix} j_i & 2 & j_\nu \\ m_i & 2 & -m_\nu \end{pmatrix}|\chi|,$$

and b (the multipolarity M(1)) is defined by Eq. (20). From the expression for the resonance absorption cross section for a $\gamma$ transition of the type E(2) – M(1) it follows that the T-noninvariant term has its greatest value in the linearly polarized case, i.e., $\alpha = 0, \pi/2$, for $\theta \approx 55^\circ$ and $\varphi = \pm\pi/4$.

We note that the difference of $\eta$ from 0 to $\pi$ can arise not only from violation of T-invariance but also from influences of the atomic electronic shells on the radiative transition[18]. In the last-mentioned work, consideration is also given to the possibility of separating out purely T-noninvariant effects, and calculations are made for Mössbauer transitions of mixed multipolarity (E(2) – M(1)) in Ru and Ir.

## 7. CONCLUSIONS

Before we pass on to consideration of the results, we remark on one of the reasons, already noted in the Introduction, for the relevance of using the Mössbauer effect (both in absorption and in scattering) in investigations of violations of P- and T-parity.

Such investigations may be based on the analysis of the polarization of the radiation (cf. e.g.,[13]). In searches for violations of T-invariance, such an approach is the only one possible, inasmuch as the difference of $\eta$ from 0 or $\pi$ introduces into the various Zeeman components effects that are linear with respect to this property but introduces only quadratic effects in the angular distribution of the radiation.

With the help of the Mössbauer effect it is relatively easy to obtain the needed information as to polarization by studying the absorption (scattering) of radiation in $\gamma$ transitions between different Zeeman levels of the nucleus. To obtain the same information, for example with the help of angular correlations, requires the use of coincidence techniques together with direct measurements of polarization.

In Sec. 2 we introduced an expression for the resonance scattering amplitude which, for arbitrary initial and final polarizations, permits both a compact representation of it and is especially convenient for analysis of interference effects. In the case of standard polarizations (linear and circular) of the emitted and scattered radiation, the corresponding scattering amplitude has been investigated in a number of works (cf., e.g.,[12,20]).

The expressions (16), (17), and (18) for the absorption cross section, can be used not only for obtaining the resonance scattering cross section but also for describing resonance absorption of polarized and unpolarized $\gamma$ radiation in mixed-multipolarity transitions.

From a comparison of Eqs. (19) and (21) it follows that the use of scattering may turn out to be especially effective in investigations of P-parity violations, because it increases the observable effect (cf. Eq. (19)) in comparison to the effect for absorption. In investigations of violations of T-invariance, to obtain an analogous increase of the observable effect in comparison to the effect for absorption, it is necessary to detect the polarization of the scattered radiation.

The possibility of establishing violation of T-invariance, with the help of resonance scattering was discussed by Burgov and Lobov[9] who actually considered scattering by polarized nuclei for the case of large width of the emitted radiation line or unresolved Zeeman splitting in the scatterer. In the case examined by us, with resolved Zeeman-splitting, the scattering goes via a single initial state $m_i$ and intermediate state $m_\nu$.

Hence, in effect, the scattering is produced also by polarized nuclei; however, the selection of orientations of the scattering nuclei can be realized from a system of unoriented nuclei with the help of energy separation of a definite transition $m_i \rightarrow m_\nu$. This last-mentioned circumstance makes it essential, in this case, that low temperatures be used to obtain polarization of the nuclei and to eliminate disturbances of the angular correlations by the magnetic field.

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