ABSORPTION OF SOUND IN LIQUID He^3

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The relaxation times in the expressions for the absorption coefficients of various sounds in liquid He^3 are determined. For this purpose, a kinetic equation with a collision integral is solved under the assumption that the scattering cross section depends only on the collision angle of the quasiparticles. The good agreement with the experimental data confirms this assumption. The calculations in a number of similar problems can thus be considerably simplified.

SOUNDS of different types can propagate in a Fermi liquid. Besides the usual low-frequency sound propagating in accordance with the laws of hydrodynamics, there exists in liquid He^3 various high-frequency (zero) sounds[1].

Sound absorption is determined from the solution of a kinetic equation containing the collision integral I (n)

$$\frac{\partial n}{\partial t}+\frac{\partial n}{\partial \mathbf{r}}\frac{\partial \varepsilon}{\partial \mathbf{p}}-\frac{\partial n}{\partial \mathbf{p}}\frac{\partial \varepsilon}{\partial \mathbf{r}}=I(n). \tag{1}$$

If the collision integral is represented in the form of $\sim \delta n/\tau$ (the τ-approximation), then it is possible to obtain from the kinetic equation (1) formulas for the absorption coefficients of the ordinary $(\omega\tau \gg 1)$ and zero $(\omega\tau \gg 1)$ sounds[2]. The collision integral is written in a form such that the kinetic equation (1) lead to conservation laws for the number of excitations, for the momentum, and for the energy. It turns out here that the experimental data on sound absorption[3] are satisfied only if the formulas for the absorption coefficients of the ordinary and zero sounds contain different values of τ.

The value of τ in the high-frequency region can be determined by solving the kinetic equation (1) with a linearized collision integral $I_l(n)$ and comparing the resultant absorption coefficient with the corresponding limiting value, calculated in the τ-approximation for the collision integral.

In the Landau theory, the interaction of the quasiparticles is described by a function $F(\chi)$, which we do not know in detail. From different experimental data we can determine only the first few coefficients of the expansion of this function in Legendre polynomials. For the case when only the first two such coefficients are known, using an expression for a linearized collision integral, in which integration of the energy is carried out, we can rewrite (1) in the form[1)]

$$(s-\cos\chi)\nu-\cos\chi\int(F_0+F_1\cos\chi')\nu'\frac{d\Omega'}{4\pi}$$
$$=\frac{1}{i}\frac{T^2m^{*3}}{8\pi^4\hbar^6}\frac{\pi^2+t^2}{2}\frac{s}{\omega}\int W(\theta,\varphi)(\nu_1+\nu_2-\nu_1'-\nu_2')\frac{d\Omega d\varphi_2}{4\pi^2\cos(\theta/2)}, \tag{2}$$

where $s = \omega/kv$, $t = (\epsilon - \mu)/T$, and $m^* = 3.1\, m_{He^3}$ at a pressure 0.28 atm. In this transformation, in accordance with the paper of Abrikosov and Khalatnikov,[2] it is assumed that the function ν, which characterizes the deviation of the quasiparticle distribution function from the equilibrium distribution function n_0, does not depend on the energy:

$$\delta n(\mathbf{p})=\frac{\partial n_0}{\partial \varepsilon}\nu(\theta,\varphi).$$

The angles θ and φ characterize the direction of the particle momentum.

The cross section of quasiparticle scattering near the Fermi surface depends on two angles: on the quasiparticle collision angle θ and on the angle of rotation of the plane in which the collision takes place φ. The quantity $W(\theta, \varphi)$ can be represented in the form

$$W(\theta,\varphi)=\frac{\pi}{\hbar}\{a_{\uparrow\uparrow}(\theta,\varphi)^2+a_{\uparrow\downarrow}(\theta,\varphi)^2\}, \tag{3}$$

where $a_{\uparrow\uparrow}(\theta, \varphi)$ and $a_{\uparrow\downarrow}(\theta, \varphi)$ are the scattering amplitudes for the case when the colliding quasiparticles have respectively parallel and antiparallel spins.

It is possible to simplify the calculation greatly by assuming that the quasiparticle scattering cross section depends only on the quasiparticle collision angle. In this case, the corresponding scattering amplitudes turn out to be connected by a simple relation with the expansion coefficients of the function $F(\theta)$[2]. We then have[4]

$$W(\theta)=\frac{\pi}{\hbar}[a_{\uparrow\uparrow}(\theta)^2+a_{\uparrow\downarrow}(\theta)^2],\quad a_{\uparrow\uparrow}(\theta)=\frac{\pi^2\hbar^3}{p_0m^*}\Big\{\Big(B_0+\frac{1}{4}C_0\Big)+\Big(B_1+\frac{1}{4}C_1\Big)\cos\theta\Big\},$$
$$a_{\uparrow\downarrow}(\theta)=\frac{\pi^2\hbar^3}{p_0m^*}\Big\{\Big(B_0-\frac{1}{4}C_0\Big)+\Big(B_1-\frac{1}{4}C_1\Big)\cos\theta\Big\},$$
$$B_l=\frac{F_l}{1+F_l/(2l+1)},\qquad C_l=\frac{Z_l}{1+Z_l/4(2l+1)}. \tag{4}$$

The quantities F_0, F_1, and Z_0 are determined from the experimental values of the speed of sound, the specific heat, and the magnetic susceptibility. The coefficient Z_1 is chosen such as to satisfy the Pauli principle $a_{\uparrow\uparrow}(0) = 0$. The latest experimental data[4] yield for these coefficients the following values (at a pressure 0.28 atm)

$$F_0=10.77,\quad F_1=6{,}25,\quad Z_0=-2.66,\quad Z_1=-2.89.$$

The value of τ for the case $\omega\tau \ll 1$ (which we denote by τ_0) can be determined in the following manner manner[2]. Since the sound propagates in this case in accordance with the laws of hydrodynamics, the ab-

[1)]We use here the notation of Abrikosov and Khalatnikov [2] sorption coefficient determined by solving the kinetic equation in the τ-approximation should coincide exactly with the value obtained from the hydrodynamics formulas. This makes it possible to determine τ_0 in terms of the viscosity coefficient η:

$$\tau_0 = \frac{5\eta}{\rho v^2(1+F_1/3)}. \quad (5)$$

where ρ is the particle density ($\rho = 8.268 \times 10^{-2}$ g/cm^3 at 0.28 atm), and v is the speed of sound.

In turn, the viscosity coefficient is determined by solving the kinetic equation with a linearized collision integral, and its value in our approximation for $W(\theta)$ is[2]

$$\eta = \frac{16}{45}\frac{m^*\hbar^3 v_F^5}{T^2}\left[\int_0^\pi W(\theta)\frac{\sin\theta\sin^4(\theta/2)}{\cos(\theta/2)}\frac{d\theta}{2\pi}\int_0^\pi \sin^2\varphi d\varphi\right]^{-1}. \quad (6)$$

Substituting the value of the Fermi velocity v_F = 53.8 m/sec and the limiting Fermi momentum $p_0/\hbar$ = 7.88×10^7 cm^{-1}, we obtain

$$\eta T^2 = 2.31\cdot 10^{-6} \text{ poise-deg}^2$$
$$\tau_0 T^2 = 1.56\cdot 10^{-12} \text{ sec-deg}^2$$

The experimental value for τ_0[3] agrees with the calculated value:

$$\tau_{0\,\mathrm{exp}}\, T^2 = 1.48\cdot 10^{-12} \text{ sec-deg}^2$$

The experimental value of the viscosity[5]

$$\eta_{\mathrm{exp}}\, T^2 = 2.8\cdot 10^{-6} \text{ poise-deg}^2$$

which was measured with an appreciable error, is apparently too high[3], and a more accurate value of the coefficient of viscosity is determined from the results of experiments on sound absorption[3]:

$$\eta_{\mathrm{exp}}\, T^2 = 2.3\cdot 10^{-6} \text{ poise-deg}^2$$

Thus, for ordinary sound ($\omega\tau \ll 1$), the assumption that the quasiparticle scattering cross section depends only on the quasiparticle collision angle is satisfied with sufficient accuracy.

To investigate the other limiting case ($\omega\tau \gg 1$), we shall solve the kinetic equation (2) by iteration. The collision integral is proportional to $1/\omega\tau$, and therefore we can take as the zero-th approximation for ν the solution in the absence of the collision integral

$$\nu^{(0)} = \frac{\cos\chi}{s-\cos\chi}F_0\nu_0 + \frac{1}{3}\frac{\cos^2\chi}{s-\cos\chi}F_1\nu_1, \quad (7)$$

where $\nu_0 = \overline{\nu}$, $\nu_1 = \overline{3\nu\cos\chi}$ (the superior bar denotes averaging over the solid angle). We substitute (7) in the collision integral in (2). For all ν_i in the collision integral, the corresponding angles χ_i are expressed by means of Euler's formulas in terms of the angles θ, φ, and φ_2.

In such a calculation scheme, the iterations are carried out in powers of $1/\omega\tau$, so that this scheme is suitable for the investigation of the absorption of zero sound ($\omega\tau \gg 1$). Integrating over the angle φ_2 in the collision integral (φ_2 characterizes the direction of the collision momentum $p_1 + p_2$ in our coordinate system), we obtain the following equation for ν:

$$\nu = \frac{\cos\chi}{s-\cos\chi}F_0\nu_0 + \frac{1}{3}\frac{\cos^2\chi}{s-\cos\chi}F_1\nu_1$$
$$+ \frac{T^2 m^*\pi}{i8p_0^2\hbar}\frac{\pi^2+t^2}{2}\frac{s^2}{\omega}\frac{F_0\nu_0 + {}^1\!/_3 sF_1\nu_1}{s-\cos\chi}2\int_0^\pi d\theta\sin\frac{\theta}{2}(9.595$$
$$+ 14.992\cos\theta + 10.047\cos^2\theta)\int_0^\pi \frac{d\varphi}{2\pi}\left(\frac{1}{s-\cos\chi} + \frac{1}{\sqrt{R}} - \frac{2}{\sqrt{R'}}\right), \quad (8)$$

where

$$R = (s-\cos\theta\cos\chi)^2 - \sin^2\theta\sin^2\chi,$$
$$R' = (s-\cos\theta'\cos\chi)^2 - \sin^2\theta'\sin^2\chi,$$
$$\cos\theta' = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\cos\varphi.$$

As a result of the first iteration, a term that depends quadratically on $t = (\epsilon - \mu)/T$ appeared in the expression for ν. Since the collision integral was simplified in the initial equation (2) under the assumption that ν is independent of t, it is impossible to use this equation directly for the subsequent iterations.

The appearance of a dependence on t in formula (8) denotes that while the rate of propagation of the non-equilibrium addition to the distribution function does not depend on t, the imaginary part of the velocity, in other words the absorption coefficient, does depend on t. This means that the form of this non-equilibrium addition changes as the sound wave propagates. But we are interested actually in the density oscillations. We shall therefore integrate equation (8) over the energies (which, of course, are multiplied by $\partial n_0/\partial\epsilon$).

Defining the quantities ρ_0 and ρ_1, with the dimension of density, by means of

$$\rho_0 = \left\langle \frac{\partial n_0}{\partial\varepsilon}\nu_0\right\rangle, \quad \rho_1 = \left\langle \frac{\partial n_0}{\partial\varepsilon}\nu_1\right\rangle, \quad (9)$$

where the angle brackets denote integration over the energy, we obtain for these quantities from (8) the following system of equations

$$\rho_0 = wF_0\rho_0 + \frac{1}{3}wsF_1\rho_1 + \frac{1}{i}\left(F_0\rho_0 + \frac{1}{3}sF_1\rho_1\right)\frac{T^2 m^*\pi^3}{12p_0^2\hbar\omega}\alpha_1 s^2,$$
$$\frac{1}{3}\rho_1 = swF_0\rho_0 + \frac{1}{3}\left(s^2 w - \frac{1}{3}\right)F_1\rho_1 + \frac{1}{i}\left(F_0\rho_0\right.$$
$$\left. + \frac{1}{3}sF_1\rho_1\right)\frac{T^2 m^*\pi^3}{12p_0^2\hbar\omega}(s\alpha_1 - \alpha_2)s^2, \quad (10)$$

where

$$w = \frac{s}{2}\ln\frac{s+1}{s-1} - 1,$$

$$\alpha_1 = \frac{1}{2\pi}\int_0^\pi d\chi\int_0^\pi d\theta\int_0^\pi d\varphi\left(\frac{1}{s-\cos\chi} + \frac{1}{\sqrt{R}} - \frac{2}{\sqrt{R'}}\right)\sin\frac{\theta}{2}\frac{\sin\chi}{s-\cos\chi}$$
$$\times(9.595 + 14.992\cos\theta + 10.047\cos^2\theta),$$

$$\alpha_2 = \frac{1}{2\pi}\int_0^\pi d\chi\int_0^\pi d\theta\int_0^\pi d\varphi\left(\frac{1}{s-\cos\chi} + \frac{1}{\sqrt{R}} - \frac{2}{\sqrt{R'}}\right)\sin\frac{\theta}{2}\sin\chi$$
$$\times(9.595 + 14.992\cos\theta + 10.047\cos^2\theta).$$

From the condition that the system (10) have a solution, we obtain

$$\left(1+\frac{F_1}{3}\right)(1-wF_0) - ws^2F_1 = \frac{1}{i}\frac{T^2 m^*\pi^3}{12p_0^2\hbar\omega}s^2\left[\alpha_1 F_0\left(1+\frac{F_1}{3}\right)\right.$$
$$\left. + \alpha_1 s^2 F_1 - \alpha_2 sF_1\right]. \quad (11)$$

When $\omega\tau \to \infty$ we obtain

$$\left(1+\frac{F_1}{3}\right)(1-wF_0) - ws^2F_1 = 0. \quad (12)$$

In order to find the absorption in the case $\omega\tau \gg 1$, we write

$$s = s_0 + i\xi, \quad |\xi| \ll s_0$$

and determine ξ from (11). In this case the quantity s_0 is determined from (12). A numerical solution of (12) yields[3] $s_0 = 3.597$ and $w(s_0) = 0.027033$. Since $s_0 = u/v_F$, this value of s_0 makes it possible to determine the rate of propagation of zero sound in the Fermi liquid. The value u = 193.5 m/sec is in good agreement with the experimental value[3] $u_{exp} = 194.4$ m/sec.

It is easy to verify that the coefficient $\alpha_2(s) \equiv 0$. The coefficient $\alpha_1(s)$ for the case $s > 1$ can be calculated with any degree of accuracy by expanding $1/(s - \cos\chi)$ in powers of $1/s$. Then ξ is expressed in terms of $\alpha_1(s)$ by means of the following formula:

$$\xi = -\frac{\pi^3 s_0^2 m^* T^2}{12 p_0^2 \hbar\omega}\frac{1}{w(s_0)}\left(1+\frac{F_1}{3}\right)\alpha_1(s_0)\left\{\frac{1}{w(s_0)}\left(1+\frac{F_1}{3}\right)\right.$$
$$\left.\times\left(\frac{s_0}{s_0^2-1}-\frac{w(s_0)+1}{s_0}\right)-2s_0 w(s_0)F_1\right\}^{-1}. \quad (13)$$

The absorption coefficient is determined as the imaginary part of the wave vector

$$\gamma = \operatorname{Im} k = \frac{\pi^3 m^* T^2}{12 p_0^2 v\hbar}\frac{1}{w(s_0)}\left(1+\frac{F_1}{3}\right)\alpha_1(s_0)\left\{\frac{1}{w(s_0)}\left(1\right.\right.$$
$$\left.\left.+\frac{F_1}{3}\right)\left(\frac{s_0}{s_0^2-1}-\frac{w(s_0)+1}{s_0}\right)-2s_0 w(s_0)F_1\right\}^{-1}. \quad (14)$$

The absorption coefficient of the zero sound, obtained from the solution of the kinetic equation in the τ-approximation, is of the form[2]

$$\gamma = \frac{1}{sv\tau_\infty}\left\{1-\left[\frac{1}{s_0}\left(1+\frac{F_1}{3}\right)(1+w(s_0))-s_0 w(s_0)(F_1-3)\right]\right.$$
$$\left.\times\left[\frac{1}{w(s_0)}\left(1+\frac{F_1}{3}\right)\left(\frac{s_0}{s_0^2-1}-\frac{w(s_0)+1}{s_0}\right)-2s_0 w(s_0)F_1\right]^{-1}\right\}. \quad (15)$$

We denote the value of τ in the case $\omega\tau \gg 1$ by τ_∞. Comparing (14) and (15), we obtain an expression for τ_∞:

$$\tau_\infty = \frac{12 p_0 v_F \hbar}{T^2}\,\frac{s_0/(s_0^2-1)-3s_0 w^2(s_0)-(w(s_0)+1)^2/s_0}{s_0\alpha_1(s_0)\pi^3}. \quad (16)$$

The coefficient $\alpha_1(3.597)$ is calculated with sufficient accuracy by using the first four terms of the expansion of $1/(s - \cos\chi)$

$$\alpha_1(3.597) = (3.75 \pm 0.07)\cdot 10^{-4};$$

this yields for the time τ_∞ the value

$$\tau_\infty T^2 = (1.17 \pm 0.02)\cdot 10^{-12}\ \text{sec-deg}^2$$

The experimental value for τ_∞ is[3]

$$\tau_{\infty\,\exp} T^2 = 1.1\cdot 10^{-12}\ \text{sec-deg}^2$$

Recognizing that the difference between the "low-frequency" τ_0 and the "high-frequency" τ_∞ amounts to 30%, the agreement can be regarded as very good. This justifies the assumption that the quasiparticle scattering cross section depends only on the collision angle. This conclusion is also of more general interest, since the model considered here makes it possible to simplify the calculations greatly, and on the other hand, owing to the connection between the scattering amplitude and the function $F(\theta)$, it makes it possible to decrease the number of unknown numerical parameters.

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