DIAMAGNETIC RESONANCE IN A MAGNETIC FIELD NORMAL TO THE SURFACE OF THE METAL

V. P. NABEREZHNYKH and N. K. DAN'SHIN

Donets Physico-technical Institute, Ukrainian Academy of Science

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Some properties of the surface impedance of a metal in a magnetic field **H** perpendicular to its surface are studied. Theoretical estimates of the contribution to the impedance of the wave vectors near single branching points of the conductivity Fourier components $\sigma(q)$ indicate that a peculiar type of resonance manifest as a single resonance line should exist. This type of resonance has been termed diamagnetic resonance. Resonance of this type was experimentally observed in cadmium at $\mathbf{H} \parallel \mathbf{n} \parallel [0001]$ and $\mathbf{H} \parallel \mathbf{n} \parallel [11\bar{2}0]$ (n is the normal to the sample surface). In this case the resonance is due to reference point electrons and to electrons of the cross sections in which extremum of $\partial S/\partial k_H$ is attained.

1. The surface impedance of a semi-infinite metal in a magnetic field normal to its surface was considered many times. It was shown[1] that in the first approximation the surface impedance is independent in this case of the magnetic field, and all the expressions of the theory of anomalous skin effects for a circularly-polarized external electric field $E_\pm = E_x \pm iE_y$ remain valid if ω is replaced by $\omega \mp \Omega$, where ω is the frequency of the external field and Ω is the cyclotron frequency of revolution of the electrons in the magnetic field. Considering the anomalous skin effect for both specular and diffuse reflection of the electrons from the surface of a metal, for a spherical Fermi surface, Dingle[2] obtained expressions for the surface impedance, in which account is taken of several approximations in δ/l (δ—depth of skin layer, l—mean free path). When the magnetic field is taken into account, these expressions assume the following form: for specular reflection

$$z_\pm \approx \frac{4\pi i\omega l}{c^2}\{0.7698(\pi i\alpha)^{-1/3} + 0.6534(\pi i\alpha)^{-2/3}a_\pm + (\pi i\alpha)^{-1}a_\pm^2[0.1318\ln(\pi i\alpha/a_\pm^3) + 0.852] + \ldots\}, \tag{1}$$

and for diffuse reflection

$$z_\pm \approx \frac{4\pi i\omega l}{c^2}\{0.866(\pi i\alpha)^{-1/3} + (\pi i\alpha)^{-2/3}a_\pm \times [0.1013\ln(\pi i\alpha/a_\pm^3) + 0.3991] + \ldots\}, \tag{2}$$

where

$$a_\pm = 1 + i(\omega \mp \Omega)\tau, \quad \alpha = {}^3/_2 l^2/\delta^2,$$

τ is the free path time of the electron.

As seen from these expressions, the first term does not depend on the magnetic field and is practically the same for specular and diffuse reflection. However, the third term in (1) and the second term in (2) have respective resonant singularities

$$\Delta z_\pm \approx \frac{4\pi i\omega l}{10c^2}(\pi i\alpha)^{-1}a_+^2\ln\left(\frac{\pi i\alpha}{a_+^3}\right) \tag{3}$$

and

$$\Delta z_\pm \approx \frac{4\pi i\omega l}{10c^2}(\pi i\alpha)^{-2/3}a_+\ln\left(\frac{\pi i\alpha}{a_+^3}\right). \tag{4}$$

The diffuse reflection gives a singularity larger by $(l/\delta)^{2/3}$ than specular.

2. The independence of the main part of the impedance on the magnetic field and the occurrence of small resonant additions can be understood in this case on the basis of the following reasoning. We assume that the electrons under the influence of the magnetic field move along such trajectories, that $v_x = v_\perp \cos\theta$, $v_y = v_\perp \sin\theta$, and v_z does not depend on θ ($\theta = \Omega t$ is the dimensionless time of motion of the electron on the orbit, $v_\perp^2 = v_x^2 + v_y^2$). This means that with the exception of the electrons of the central cross section all the remaining electrons will be ineffective ($v_z \neq 0$).

The sharp inhomogeneity of the electric field in the metal can be represented as a superposition of electromagnetic waves with different wave numbers q. For circularly polarized components of the field in specular reflection, we can write

$$E_\pm(z) = \frac{1}{\pi}\int_0^\infty E_\pm(q)\cos qz\, dq, \tag{5}$$

where

$$E_\pm(q) = E_\pm'(0)[q^2 - 4\pi i\omega c^{-2}\sigma_\pm(q)]^{-1}$$

is the Fourier component of the field, $\sigma_\pm(q)$ is the Fourier component of the conductivity and

$$E_\pm'(0) = \frac{\partial E_\pm}{\partial z}\bigg|_{z=0} \qquad E(z,t) = E(z)e^{-i\omega t}.$$

Inasmuch as in the anomalous skin effect the mean free path l is much larger than δ, the attenuation of the waves has a collisionless character and is due mainly to the electrons, for which the Doppler-shifted frequency of the external electromagnetic field $\omega \pm qv_z$ coincides with the cyclotron frequency Ω, i.e., electrons moving at velocity v_z will interact the most with that spatial harmonic of the field, whose wave number is

$$q = \left|\frac{\omega \mp \Omega}{v_z}\right| = 2\pi\left|\frac{\gamma \pm 1}{u}\right|$$

where $u = 2\pi v_z/\Omega$ is the shift of the electron along z over the cyclotron period, and $\gamma = \omega/\Omega$. The only role of the magnetic field is that when the field H is different, the given electron interacts maximally with a different Fourier component of the electric field. If the metal contains electrons with different velocities v_z and frequencies Ω, then for a continuous wave spectrum no group of electrons is preferred, (with the exception of the electrons with $v_z = 0$). Here, of course, the main contribution to the formation of the skin layer is made by electrons with small v_z, which interact with the Fourier component of the field, the wave number of which is $\sim\delta^{-1}$ and is determined by the roots of the dispersion equation $q^2 - 4\pi i\omega c^{-2}\sigma_{\pm}(q) = 0$. The change of the magnetic field can in this case lead only to a monotonic change of the surface impedance.

A somewhat different situation arises if there exists in the metal a sufficiently large group of electrons having the same cyclotron frequency and the same Doppler frequency shift qv_z at a given q. Since the number of such electrons is not small, they make an appreciable contribution to the impedance, and a change of the magnetic field may change this contribution. If the magnetic field is sufficiently large, such that $q = |(\omega - \Omega)/v_z|$ coincides with the real part of the root of the dispersion equation, the contribution made to the impedance by this group of electrons will be maximal and will depend in a resonant manner on the magnetic field. In this case a resonant increase of the impedance should be observed at $\Omega \approx \omega + \delta^{-1}_{v_z}$ [3].

If the frequencies are equal, $\omega = \Omega$, the electrons will interact maximally with the Fourier component having $q = 0$. And since the amplitude of this component is minimal, the contribution of these electrons to the impedance will also be minimal. Such electrons are, so to speak, eliminated from the game and should cause a resonant singularity of the impedance at $\omega = \Omega$. This resonance differs from the ordinary cyclotron resonance and is called diamagnetic.

The condition under which many electrons have the same value of

$$\left|\frac{\omega-\Omega}{v_z}\right| = 2\pi\left|\frac{\gamma-1}{u}\right|,$$

is that the quantities u and γ be simultaneously extremal as functions of the projection of the wave number of the electron on the direction of z, which is equivalent to the requirement that $S'(k_z) \equiv \partial S(k_z)/\partial k_z$ and $m^*(k_z)$ be extremal, where $S(k_z)$ is the area of the intersection of the Fermi surface by the plane $k_z = \text{const}$[1)]. The extremum of $u(k_z)$ and $\gamma(k_z)$ must be reached at the elliptical limiting point of the Fermi surface. It is precisely the electrons near the limiting point that cause the singularities of the impedance (3) and (4), rather than the electrons of the central section, as proposed in[4]. Although the central-section electrons with $v_z = 0$ can interact with all the Fourier components of the field, their number is so small that their contribution to the impedance is insignificant (we bear in mind, of course, that $v_z' \neq 0$).

[1)]It will be shown later that a resonant change of the impedance is possible if S′ has an extremum and if m* changes sufficiently little within the limits of the extremum of S′.

By virtue of the symmetry of the Fermi surface ($\epsilon(k_z) = \epsilon(-k_z)$), the metal contains electrons that move both towards and away from the metal surface. Some electrons will carry the field to the interior of the metal, and others will, to the contrary, carry it to the surface. However, the roles of the different electrons are not equal, and the net contribution to the impedance will differ from zero.

The dependence of the resonant additions of the impedance on the character of reflection of the electrons from the surface of the metal is connected with the fact that the electrons that carry the field to the surface will again carry the field to the interior after specular reflection, and their contribution will be much smaller than in diffuse reflection.

3. As shown in[5], at low frequencies, when ω can be neglected compared with Ω, the ineffective electrons, which have a displacement extremum within the cyclotron period $u = 2\pi v_z/\Omega$, cause the appearance of single branch points in the Fourier component of the conductivity $\sigma(q)$. This makes it possible for an electromagnetic wave with wave number $q = \Omega/v_z$ to penetrate into the metal to a large depth.

At high frequencies, the branch points $\sigma(q)$ are determined not by the frequency Ω but by the difference $\omega - \Omega$. It can be shown that because of this, both the distribution of the field in the interior of the metal and the surface impedance will change near the resonance $\omega - \Omega$.

Let us consider a Fermi surface that is axially symmetrical with respect to the z axis, on which there is at $k_z = k_{z0}$ a section with a displacement extremum within the cyclotron period. To simplify the calculations we assume that $m^*(k_z) = \text{const}$.

The Fourier components of the conductivity $\sigma_{\pm}(q)$, in a magnetic field normal to the surface of the metal ($H \parallel n \parallel z$), for circularly polarized waves, are given by[5]:

$$\sigma_{\pm}(q) = \frac{e^2}{(2\pi\hbar)^2}\int\limits_{-k_{zmax}}^{k_{zmax}} \frac{m^*}{\Omega} v_{\perp}^2 \left[\gamma_1 + i(\gamma \mp 1) + iq\frac{u(k_z)}{2\pi}\right]^{-1} dk_z, \quad (6)$$

where $\gamma_1 = \nu/\Omega$, ν is the frequency of the collisions between the electrons and the scatterers. Expanding $u(k_z)$ in powers of $k_z - k_{z0}$, we obtain approximately for $\sigma_{\pm}(q)$

$$\sigma_{\pm}(q) \approx \frac{e^2}{(2\pi\hbar)^2}\frac{m^*}{\Omega}v_{\perp}^2\left\{\int\limits_{-\infty}^{\infty} dk_z\left[\gamma_1 + i(\gamma \mp 1) + iq\frac{u_0}{2\pi} - iq\frac{u''}{4\pi}(k_z - k_{z0})^2\right]^{-1}\right.$$

$$\left. + \int\limits_{-\infty}^{\infty} dk_z\left[\gamma_1 + (\gamma \mp 1) - iq\frac{u_0}{2\pi} + iq\frac{u''}{4\pi}(k_z - k_{z0})^2\right]^{-1}\right\}$$

$$= \frac{e^2}{(2\pi\hbar)^2}\frac{m^*}{\Omega}v_{\perp}^2\frac{4\pi^2 i q^{1/2}}{q(2u_0u'')^{1/2}}\left[\frac{i}{(q_{\pm} - q)^{1/2}} - \frac{1}{(q_{\pm} + q)^{1/2}}\right]. \quad (7)$$

$$q_{\pm} = \frac{2\pi}{u_0}[i\gamma_1 - (\gamma \mp 1)].$$

Assuming that $u'' \sim u_0/k^2$ and $v_{\perp} \sim v_z \sim v$, we can rewrite the expression for $\sigma_{\pm}(q)$, accurate to a numerical coefficient, in the form

$$\sigma_{\pm}(q) \approx \frac{3\pi}{4}\frac{Ne^2 i}{\hbar k q}\left[i\left(\frac{q}{q_{\pm} - q}\right)^{1/2} - \left(\frac{q}{q_{\pm} + q}\right)^{1/2}\right], \quad (8)$$

where $N = k^3/3\pi^2$ is the electron density. As seen from (8), the branch points for $\sigma_{\pm}(q)$ are located at $q = \pm q_{\pm}$, and their position is determined by the relation between

ω and Ω. Substituting (8) in (5) and finding, as in[5], the asymptotic value of the integral at large z, which is determined mainly by the value near the point $q_\pm$ we obtain an expression for the electric field inside the metal:

$$E_\pm(z) \approx \frac{\delta_0^3}{2\sqrt{\pi}} E_\pm'(0)[1+(\omega\mp\Omega)^2\tau^2]^{1/4} \frac{e^{-z/l_1}}{l_1^{1/2} z^{3/2}} e^{i\psi(z)}, \quad (9)$$

where

$$\psi(z) = -\frac{2\pi z}{u_0}(\gamma\mp 1) + \frac{1}{2}\operatorname{arctg}(\omega\mp\Omega)\tau - \frac{\pi}{2}; \quad (9a)$$

$l_1 = u_0/2\pi\gamma_1$, δ_0 is the depth of the skin layer at $H = 0$, and $\delta_0^3 = c^2\hbar k/3Ne^2\omega\pi^2$.

The obtained field distribution is a helical wave whose phase velocity is $v_{ph\pm} = -u_0\omega/2\pi(\gamma \pm 1)$.

The phase velocity of the wave $E_-(z)$ is always negative, whereas the waves $E_+(z)$ reverse sign, depending on the sign of $(\omega - \Omega)$, becoming infinite at $\omega = \Omega$. This means that the length of the wave E_+ becomes infinitely large and the field oscillates only in time, varying in amplitude as a function of z like $e^{-z/l_1}/z^{3/2}$.

The amplitude of the electric field also depends on the difference $\omega - \Omega$, decreasing greatly when $\omega = \Omega$. In the case of a limiting point, branch points of the logarithmic type appear in $\sigma_\pm(q)$[5]. Analogous calculations yield the following expression for the field:

$$E_\pm(z) \approx \frac{\delta_0^3}{\pi} E_\pm'(0)\frac{e^{-z/l}}{z^2}\exp\left\{-i\left[\frac{\omega\mp\Omega}{v_{\lim}}z - \frac{\pi}{2}\right]\right\}, \quad (10)$$

where $l = v_{\lim}/\nu$, and $v_{\lim}$ is the velocity at the limiting point. Let us estimate the contribution made to the impedance by the Fourier components $\sigma_\pm(q)$ near the branch points q_+ for electrons moving along helical trajectories. In the case of diffuse reflection of the electrons from the surface, the impedance is given by[6]

$$z_+ = \frac{4\pi^2 i\omega}{c^2}\left\{\int_0^\infty \ln\left[\frac{q^2 - 4\pi i\omega c^{-2}\sigma_+(q)}{q^2}\right]dq\right\}^{-1}. \quad (11)$$

Substituting in (11) $\sigma_+(q)$ from (8) and transforming, we obtain for z_+^{-1}

$$z_+^{-1} = \left(\frac{4\pi^2 i\omega}{c^2}\right)^{-1}\int_0^\infty \ln\left\{\left[\frac{q^3 - i\delta_0^{-3}}{q^3}\right][1-f(q)]\right\}dq$$

$$= \left(\frac{4\pi^2 i\omega}{c^2}\right)^{-1}\left\{\int_0^\infty \ln\left[\frac{q^3 - i\delta_0^{-3}}{q^3}\right]dq + \int_0^\infty \ln[1-f(q)]dq\right\}, \quad (12)$$

where

$$f(q) = i\delta_0^{-3}\left\{iq^{1/2}\left[\frac{i}{(q_+-q)^{1/2}} - \frac{1}{(q_++q)^{1/2}}\right] - 1\right\}(q^3 - i\delta_0^{-3})^{-1}. \quad (12a)$$

The first integral in (12) gives the value of the high-frequency conductivity at $H = 0$

$$z_0^{-1} = \left(\frac{4\pi^2 i\omega}{c^2}\right)^{-1}\frac{2\pi}{\sqrt{3}}\delta_0^{-1}e^{i\pi/6}. \quad (13)$$

At large q, the f(q) dependence tends to zero, and therefore the main contribution to the second integral (12) is made by $q < \delta^{-1}$.

Then, expanding $(q^3 - i\delta_0^{-3})^{-1}$ in (12a) in a series, and confining ourselves to the first term of the expansion, we obtain for the addition to the high-frequency conductivity

$$z_1^{-1} \approx \left(\frac{4\pi^2 i\omega}{c^2}\right)^{-1}\int_0^M \ln\left\{iq^{1/2}\left[\frac{i}{(q_+-q)^{1/2}} - \frac{1}{(q_++q)^{1/2}}\right]\right\}dq, \quad (14)$$

where $\operatorname{Re} q_+ < M < \delta_0^{-1}$. Calculating and omitting terms that are independent of q_+ and the linear terms, we obtain

$$z_1^{-1} \approx \left(\frac{8\pi^2 i\omega}{c^2}\right)^{-1} q_+ \ln q_+. \quad (15)$$

Using the smallness of z_1^{-1} compared with z_0^{-1}, we can write

$$z = z_0 + \Delta z,$$

where

$$\Delta z \approx -z_1^{-1}z_0^2 = \frac{3}{2}\frac{\omega}{c^2}\delta_0^2 e^{-i\pi/3} q_+ \ln q_+. \quad (16)$$

Analogous calculations for specular reflection of the electrons yield an addition to the impedance of the order of

$$\Delta z \sim \delta_0^3 q_+^2 \ln q_+. \quad (17)$$

The obtained expressions (16) and (17) coincide, apart from coefficients with expressions (4) and (3) calculated for a spherical Fermi surface.

Figure 1 shows the derivative dR/dH, calculated from the formula

$$\frac{dR}{dH} = B\{\ln[1-(\Omega-\omega)^2\tau^2] + 2\sqrt{3}\operatorname{arctg}(\Omega-\omega)\tau\}, \quad (18)$$

where $B = (3/2)\omega\delta_0^2 e\tau/c^3 m l_1$ and R is the real part of the additional impedance Δz in (16).

So far we have considered the simplest case, when $m^*(k_z) = \text{const}$. The group of "resonance" electrons was distinguished by the extremal character of their displacement within the cyclotron period, i.e., by the extremal character of $S'(k_z)$. It is perfectly obvious that nothing changes if the effective mass depends on k_z but the extremum is reached at $k_z = k_{z0}$, i.e., on that section where $S'(k_z) = S'_{ext}$. The number of electrons taking part in the diamagnetic resonance will be determined in this case not only by the sharpness of the extremum of $S'(k_z)$ (i.e., by the value of S'''), but also by the sharpness of the extremum of the mass or by the value of $m^{*\prime\prime}$. If the effective mass is not extremal at $k_z = k_{z0}$, then in the calculation of $\sigma_\pm(q)$ in (7) it is necessary to substitute in the first approximation in place of γ the quantity $\gamma_0 + \gamma'(k_z - k_{z0})$. The

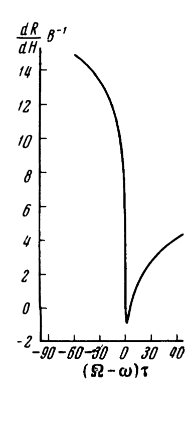

FIG. 1. Theoretical dependence of dR/dH on $(\Omega-\omega)\tau$ in the presence of a section with extremum $S(k_H)$, $m^* = \text{const}$, and diffuse reflection of the electrons from the surface of the metal. Here $B = (3/2)\,\omega\delta_0^2\, e\tau/c^3 l_1 m^*$.

expression for $\sigma_{\pm}(q)$ will in this case be of the form

$$\sigma_{\pm}(q) \approx \frac{3\pi}{4}\,\frac{Ne^2 i}{\hbar k q}\left\{\frac{qi}{[q(q_{\pm}-q)+\beta]^{1/2}} - \frac{q}{[q(q_{+}+q)-\beta]^{1/2}}\right\}, \qquad (19)$$

where $\beta = 2(y')^2/u''u_0$. Substituting (19) in (11) and calculating the integral, we obtain terms of the type

$$\Delta z \propto \left(\frac{q_+}{2} + \sqrt{\frac{q_+^2}{4}+\beta}\right)\ln\left(\frac{q_+}{2} + \sqrt{\frac{q_+^2}{4}+\beta}\right). \qquad (20)$$

This means that the non-constancy of m^* will lead to a suppression of the resonant singularity of the impedance. When

$$\frac{m^{*\prime}}{m^*} \ll \left(\frac{S'''}{2S'}\right)^{1/2}\frac{\pi}{\omega\tau}$$

the constancy of m^* can be neglected.

4. Diamagnetic resonance of this type can be observed experimentally in cadmium[4] and zinc[7,8] on the electronic "lens like" Fermi surface. Figure 2a shows plots of $\mu(k_H) \equiv m^*/m_0$, $S'(k_H)$, and $v_H(k_H)$ for cadmium in the model of almost free electrons at $H \perp [0001]$. Although in this model S' has a broad extremum at $k_H \approx 0.6\ \text{Å}^{-1}$, the effective mass at this value of k_H changes quite appreciably. This circumstance should be apparently smear out the resonance completely. In a real metal, the situation is somewhat different. An investigation of the radio frequency side effect at $H \perp [0001]$ and at different inclinations of the magnetic field to the surface of the sample has shown[9,10] that $S'(k_H)$ remains practically constant in the interval $k_H \approx (0.4-0.7)\,\text{Å}^{-1}$. In addition, from the results of the study of the cyclotron resonance and the radio frequency side effect in a magnetic field parallel to the surface of the metal[11] it can be concluded that the velocity of the electrons changes on the Fermi surface and amounts to 1.15×10^8 and 0.64×10^8 cm/sec respectively at the center of the "lens" and on the edge.

If we disregard the rounding off of the edges of the "lens" and assume that the velocity is the same throughout and is equal to the velocity at the center, then the velocity component $\bar{v}_H$ averaged over the period would depend on k_H like $\bar{v}_H = v_H = \hbar k_H/m^*_{\text{lim}}$, where m^*_{lim} is the effective mass at the limiting point at the center of the "lens" (shown by the thin line in Fig. 2b). On the other hand, if the edges are rounded off, the "lens" becomes similar to an ellipsoid of revolution with an axis ratio ~ 3. If the velocity on the edge is then assumed to be the velocity on the entire surface, then $\bar{v}_H$ should vary as a function of k_H in the manner shown by the dashed line in Fig. 2b.

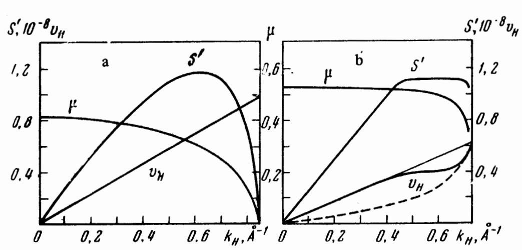

FIG. 2. Dependence of μ, S', and v_H on k_H for the "lens" of cadmium. a–In the model of almost free electrons; b–approximate form of real dependence which agree with the data on cyclotron resonance and the radio-frequency side effect.

Since at small values of k_H the electron is located during its course of motion in the magnetic field mostly on Fermi-surface sections with large velocity (the role of the edges is small), the real dependence will approximately correspond to the first case. With increasing k_H, the role of the edges will increase and the growth of $\bar{v}_H$ will slow down. Directly at the edge of the "lens" the real dependence of $\bar{v}_H$ will be close to that corresponding to an ellipsoid.

The heavy line in Fig. 2b shows the approximate form of the real dependence of $\bar{v}_H$ on k_H. Since $\mu(k_H)$, $S(k_H)$, and $\bar{v}_H(k_H)$ are connected by the relation $\bar{v}_H = \hbar S'/2\pi\mu m_0$, then when $S(k_H)$ is constant and $\bar{v}_H(k_H)$ decreases in a certain interval of k_H, the dependence of μ on k_H should also decrease. Figure 2b shows the mutually-compatible dependences $\mu(k_H)$, $S'(k_H)$ and $\bar{v}_H(k_H)$ (the value $\mu(0) = 0.53$ was taken from cyclotron-resonance data[11]).

Thus, the changes of the real Fermi surface compared with the model of almost free electrons are such that we can expect realization of the condition for the occurrence of diamagnetic resonance.

Indeed, in zinc[8] and cadmium, at approximately the same direction of the magnetic field at which the radio-frequency side effect of a quasiharmonic type takes place[9], a single peak of dR/dH is observed at high frequencies. The cadmium experiment was performed on a sample whose plane coincided with the crystallographic $(11\bar{2}0)$ plane. The ratio of the resistance at room and helium temperatures for the cadmium used in the sample was $\rho(4.2°\text{K})/\rho(300°\text{K}) \approx (3-5) \times 10^{-5}$. The frequency of the electromagnetic field was $f = 3.67 \times 10^{10}$ Hz, and $T = 1.7°$K. The sample served as the bottom of a cylindrical resonator with H_{011} mode.

Figure 3 shows plots of the dependence of dR/dH on H for cadmium at different angles between the magnetic field and the surface of the sample. We see that the greatest resonance-line intensity is observed at $H \parallel n \parallel [11\bar{2}0]$. When H deviates from the normal direction, the line changes its shape and decreases in intensity. The maximum interval of inclination angles in which resonance can be observed is $\sim \pm 20°$, which is somewhat smaller than the angle interval where the size effect is observed. The effective mass at $H \parallel n$, determined from the value of the field at the minimum of dR/dH, is $m^* = 0.49\, m_0$. The mass anisotropy, as in the case of zinc, is insignificant.

At the magnetic-field direction $H \parallel n \parallel [0001]$, where there are circular limiting points on the "lens" with large curvature radii, there is also observed a single resonance line, shown in Fig. 3e. The magnetic field in which it is observed corresponds to the effective mass of the limiting point ($m^*_{\text{lim}} = 1.32\, m_0$).

A detailed comparison of the experimental and theoretical dR/dH curves is difficult, since the conditions under which they were obtained are somewhat different. Thus, for example, the field employed in the experiment was not circularly polarized, the magnetic field was not strictly perpendicular to the surface of the sample, the Fermi surface was not axially symmetrical, $m^* \neq$ const, etc.

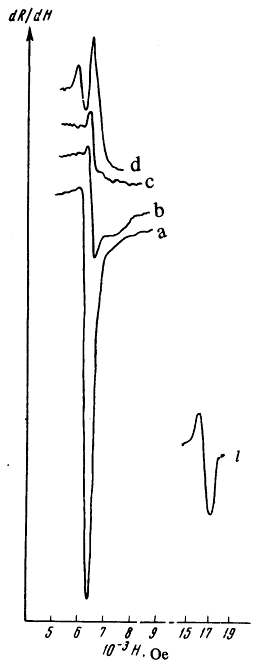

FIG. 3. Plot of dR/dH in cadmium. Curves a, b, c, and d were obtained for **n** ∥ [11$\bar{2}$0] at angles φ between **H** and **n** equal respectively to 0, 8, 11, and 13°. The gain used to plot lines b, c, and d, compared with line a, is approximately 10:1, 10:1, and 20:1. Curve e was obtained for **H** ∥ **n** [0001]. The gain was the same as for curve d. The accuracy of setting of **H** was ±1°.

Nonetheless, as seen from Figs. 1 and 3, the general character of the theoretical and experimental curves is quite close.

5. Thus, in metals, a unique resonance on the ineffective electrons is possible in a magnetic field normal to the surface of the metal. Unlike the cyclotron resonance on the effective electrons, which has harmonics at $\omega = n\Omega$, this resonance appears in the form of a single resonance line and is called diamagnetic. Taking part in the resonance are the electrons near the elliptic limiting points and the electrons of the helical trajectories, for which $S'(k_H)$ has an extremum with respect to k_H, and the effective mass changes little within the limits of the extremum of S'.

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