LASERS WITH RESONATOR MIRROR TRANSMISSION VARYING OVER THE CROSS SECTION

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The spatial and energy characteristics of lasers with resonator mirrors whose transmission varies over the cross section are investigated. The interaction of transverse modes and the angular divergence of emission from lasers with such resonators are analyzed theoretically. It is shown that such lasers are weakly sensitive to misalignment of the resonator mirrors.

1. INTRODUCTION

IT was shown in[1] that lasers with flat resonators can have extremely small angular divergence ($\sim\lambda/D$) without a loss of power. However the methods of transverse (angular) mode selection in flat mirror lasers have two disadvantages: 1) mode selection is difficult when the resonator cross section is large, and 2) the alignment of flat mirrors is fairly critical.

Resonators with spherical mirrors are much less critical to align, although in this case predominantly higher transverse modes are excited, causing a large angular divergence. Thus it is of interest to consider a laser resonator with mirror transmission that is variable over the cross section. Some features of such resonators were considered in[2,3] where it was shown that if mirror transmission is distributed over the resonator cross section according to certain rules, the field in the resonator is concentrated along the resonator axis and the diffraction losses of transverse modes increase. Consequently, transverse mode selection is possible in lasers with such resonators.

In this paper, the special features of generation in lasers with cross section variable transmission of flat resonator mirrors is investigated theoretically and experimentally. The theoretical analysis is based on the assumption that modes with a single axial index are excited, i.e., the spectrum of transverse modes is under consideration.

2. THEORETICAL ANALYSIS

The angular divergence of laser emission as a whole is determined both by the field structure of the individual transverse modes and by their number and intensity ratio. We consider the features of formation of such a field in lasers with cross section variable transmission of flat resonator mirrors. The resonator mirrors are assumed to be two infinite planes at a distance $2l$ from each other. The origin of Cartesian coordinates lies on the surface of one of the mirrors; the z axis lies along the resonator axis.

Let the mirror reflectivity vary along one of the directions (the x axis, for example) on the mirror surface according to the rule

$$R = R_{max} \exp\left[-2(x/a)^2\right], \qquad (1)$$

where R_{max} is the reflection coefficient at the center of the mirror (x = 0), and a is a parameter characterizing the distribution of the reflection coefficient over the mirror surface. Then according to Vakhitov[2] the field distribution of modes with transverse (m) and longitudinal (q) indices in a resonator free of the active medium can be represented in the form

$$E_{mq} = C_{mq} X_m Z_q, \qquad (2a)$$

$$X_m = H_m(2s\sqrt{M'(1-i)}) \exp\{-(M'-1)s^2\} \exp\{iM's^2\}, \qquad (2b)$$

$$Z_q = \sin\frac{\pi q}{2l} z, \qquad (2c)$$

where

$$H_m(t) = (-1)^m e^{-t^2/2} \frac{d^m}{dt^m} e^{-t^2/2}$$

is an m-th order Hermitian polynomial, $s = x/a$ and $M' = \sqrt{a^2/2l\lambda}$ is a parameter for variable-transmission mirrors similar to the M parameter for a flat resonator with uniform mirror transmission[4], λ is the emission wavelength, and C_{mq} is a constant for a given mode. Expressions (2a – 2c) are approximate and valid for $M' \gg 1$. We assume that this condition is satisfied.

The chosen rule (1) of transmission distribution over the mirror surface can be used to determine analytically the field structure in the far field for various modes. Equation (2b) allows us to determine the transverse current distribution at the mirror in such a resonator and to compute the electromagnetic field at the far field by the standard method, using Green's functions and introducing the auxiliary electric Hertz vector[5].

The spherical coordinate system (ρ, φ, θ) can be used to obtain the following expressions for the far field emission intensity distribution in the azimuth (angle θ) for the first three lower transverse modes (regarding $M' \gg 1$):

$$f_0(\theta) = A_0 e^{-\beta}, \quad f_1(\theta) = A_1\beta e^{-\beta}, \quad f_2(\theta) = A_2(2\beta^2 - \beta + 0.25)e^{-\beta}. \qquad (3)$$

Here $\beta = (k^2a^2/4M')\sin\theta$; f_0, f_1, f_2 are functions characterizing the spatial intensity distribution of the corresponding mode; A_0, A_1, A_2 are constants; and $k = 2\pi/\lambda$. The angular divergence of the emission of the first three modes can be readily obtained from (3) at 0.5 of maximum intensity:

$$\theta_0 = 1.1\frac{\sqrt{M'}}{\pi}\frac{\lambda}{a}, \quad \theta_1 = 3.3\frac{\sqrt{M'}}{\pi}\frac{\lambda}{a}, \quad \theta_2 = 4.3\frac{\sqrt{M'}}{\pi}\frac{\lambda}{a}. \qquad (4)$$

It follows from (3) that the far field mode structures in a resonator with mirror transmission that is variable over the cross section are qualitatively the same as in spherical resonators.

We now consider the problem of the number and intensity of the generated transverse modes. The energy distribution of various modes in such a resonator has the form

$$u_{mq} = B_m X_m^2 Z_q^2 u_{mq}^{max},$$

where $B_m = 1/\sqrt{2\pi}m!$ is determined from the normalization of the energy density of a given mode integrated over the resonator volume, and u_q^{mx} is the amplitude value of the energy density.

We consider that the laser resonator is completely filled with a four-level active medium whose lower level is frozen out and whose luminescence line is homogeneously broadened. Then according to[6] the equations for population inversion N and total number of quanta n_{kq} in a given mode have the following form in the stationary case:

$$\eta_1 B_p u_p N_0 - N/\tau - B_g N \sum_{k,q} g_{kq} u_{kq} = 0,$$

$$B_g g_{kq} \int_{-\infty}^{+\infty}\int_0^{2l} N u_{kq}\, dx\, dz - \gamma_{kq} n_{kq} = 0,$$

$$n_{kq} = \frac{1}{h\nu_g}\int_{-\infty}^{+\infty}\int_0^{2l} u_{kq}\, dx\, dz, \qquad (5)$$

where $\eta_1 B_p u_p N_0$ is a term related to pumping (see[6]), B_g is the Einstein coefficient for stimulated emission, τ is the spontaneous relaxation time of the upper working level, γ_{kg} are resonator losses for a given mode, and $h\nu_g$ is the energy of an emission quantum.

The exact expression for field distribution along the resonator axis is somewhat different from (2c), but we can show that (2c) is valid with sufficient accuracy for several lower transverse modes in our approximation ($M' \gg 1$). Consequently for those lower modes the field distribution along the resonator axis is the same as in resonators with ordinary flat mirrors. Therefore, as shown in[1,6,7], we may consider the competition of modes with the same longitudinal index q and with different transverse indices k. Here

$$g_{kq} = g_k = 1.$$

The system (5) can be transformed into

$$n\left[\int_{-\infty}^{+\infty}\int_0^{2l} X_k^2(s) Z_q^2(z)\, ds\, dz\right]^{-1}$$

$$\times \int_{-\infty}^{+\infty}\int_0^{2l} X_k^2(s) Z_q^2(z)\left[1 + \sum_m A_m X_m^2(s) Z_q^2(z)\right]^{-1} ds\, dz = \frac{\gamma_k}{\gamma_0}. \qquad (6)$$

Here $A_m = B_g B_m u_{mq}^{max}$, γ_0 are losses of the first mode ($k = 0$), $n = u_p/u_{p\,thr}$ excess power over threshold for the first mode, and $u_{p\,thr} = \gamma_0/B_g h\nu_g \eta_1 B_p N_0 \tau$ is the threshold pump power for the first mode. The summation in the denominator of the integrand is performed for all transverse modes excited at the given pump level.

Considering that the left-hand side of (6) is less than the right-hand side for modes that are not excited under the given conditions, the system of equations of the type (6) for all excited transverse modes can be used to determine the number of generated modes and their intensity distribution as a function of excess pump power over threshold.

Solving (6) for two lower transverse modes ($k = 0, 1$) at a moderate pump level ($n < 1.4$), we obtain after transformation

$$A_0 = 1.89\frac{n-1}{n}, \quad n_1 = 2\frac{\gamma_1}{\gamma_0} - 1, \qquad (7)$$

where n_1 is the excess pump power over threshold for the second mode ($k = 1$), γ_1 are the second mode losses, and A_0 is a quantity proportional to the energy density of the lower transverse mode.

Similarly solving (6) for the three lower modes ($k = 0, 1, 2$) we obtain

$$A_0 = 0.95\frac{(n - 3 + 2\gamma_1/\gamma_0)}{n},$$

$$A_1 = 3.8\frac{(n + 1 - 2\gamma_1/\gamma_0)}{n},$$

$$n_2 = 3.33\frac{\gamma_2}{\gamma_0} - 2.67\frac{\gamma_1}{\gamma_0} + 0.33, \qquad (8)$$

where n_2 is the excess pump power over threshold giving rise to the third mode ($k = 2$), and γ_2 are losses of the third mode.

The energy losses of each mode in the laser resonator can be determined in the form[6]

$$\gamma_k = D'\left(\ln\frac{1}{R_k} + G_{k\,\mathrm{diff}} + G_{\mathrm{na}}\right). \qquad (9)$$

Here D' is the coefficient of proportionality ($\sim 10^8$) and $G_{k\,\mathrm{diff}}$ are diffraction losses of the k-th mode (in a double pass). In our case the losses according to[3] are equal to

$$G_{k\,\mathrm{diff}} = (k+1)/M', \quad k = 0, 1, 2; \qquad (10)$$

G_{na} are non-active energy losses in an active specimen (in a double pass), and R_k is the reflection coefficient of the resonator mirror for the kth mode determined by

$$R_k = \int R(x) X_k^2(x)\, dx \Big/ \int X_k^2(x)\, dx, \qquad (11)$$

where $R(x)$ is the law of distribution of the reflection coefficient over the mirror surface. The integration in (11) is performed over the mirror surface (over x in our case).

Using (1) and (2) according to (11) for the first three modes we find

$$R_0 = R_{max}\left(1 - \frac{1}{M'}\right)^{1/2}, \quad R_1 = R_{max}\left(1 - \frac{1}{M'}\right)^{3/2},$$

$$R_2 = R_{max}\frac{2.5(M'-1)^2}{2.5M'^2 + 0.5}\left(1 - \frac{1}{M'}\right)^{1/2}. \qquad (12)$$

Figure 1 shows the number of excited modes as a function of pump power excess over threshold computed from (7)—(12) for the case of a normal distribution of the resonator mirror transmission coefficient, where the transmission at the mirror center is $\tau = 10\%$ and at the edges $\tau = 40\%$ ($a = 6.0$ cm). The dimension of the mirrors (in the direction of the x axis) $D = 5.5$ cm, the distance between the mirrors $2l = 30$ cm, and $G_{na} = 0$. The figure also shows the dependence of the number of excited modes for an analogous flat resonator having mirrors with uniform reflectivity ($R = 90\%$) computed from equations obtained in[1]. It is clear that a laser with resonator mirror transmission that is variable over the cross section provides a fairly effective selection of transverse modes.

We also note that a laser with variable transmission

Table I.

I/I_{max}	Angular divergence of emission (seconds of arc)											
	Uniform transmission mirrors, τ = 22%				Variable transmission mirrors, τ = 8 – 40%				Variable transmission mirrors, τ = 10 – 70%			
	Experiment	Mode 1	Mode 2	Mode 3	Experiment	Mode 1	Mode 2	Mode 3	Experiment	Mode 1	Mode 2	Mode 3
	resonator length $2l$ = 30 cm											
0.5	95—100	4	11	15	70—80	15	45	59	50—60	19	57	74
0.1	150—160				145—150				145—150			
0.03	200—210				170—175				170—180			
	resonator length $2l$ = 100 cm											
0.5	60	4	11	15	48—55	11	33	43	35—40	14	42	55
0.1	150—180				130—140				80—90			
0.03	180—200				140—160				90—110			

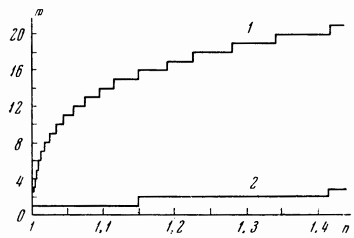

FIG. 1. Number of excited transverse modes as a function of excess pump power over threshold (resonator length $2l$ = 30 cm, D = 5.5 cm). 1 – flat mirrors with transmission τ = 10%; 2 – flat mirrors with normal distribution of transmission coefficient amounting to τ = 10% at the mirror center and τ = 40% at the edges (a = 6.0 cm).

mirrors has a somewhat higher pump power threshold required to excite the lower transverse mode. This increase is determined by the expression

$$\alpha = \frac{\gamma_{0\,\text{variable}}}{\gamma_{0\,\text{uniform}}} \approx \frac{1/M' + G_{na} - \ln[R_{max}(1 - 1/M')^{1/2}]}{G_{na} - \ln R}.$$

The increase of the pump power threshold is small when $M' \gg 1$.

3. EXPERIMENTAL RESULTS

The experiments were carried out with a generator having an active speciment of homogeneous neodymium glass (Nd_2O_3 content of 4%) rectangular in cross section. One of the resonator mirrors had a variable transmission coefficient that increased from the center to the edge of the mirror (along the x axis, Fig. 2). The envelope of the transmission coefficient distribution over the mirror (along the x axis) was close to the normal distribution. Two variants of variable transmission mirrors were used in the experiments; the first had 8% transmission at the mirror center and 40% at the edges, and the second had 10% transmission at the center and 70% at the edges.

The dimensions of mirrors and of the active specimen were 5.5 cm along the x axis and 0.8 cm along the y axis. The pumping system used in the experiments provided for a uniform distribution of population inversion over the cross section of the specimen along the x axis[1)][8]. The second mirror of the laser reso-

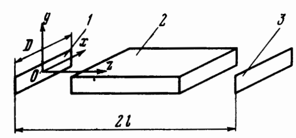

FIG. 2. Diagram of the laser. 1 – mirror with transmission variable along the x axis; 2 – active element; 3 – mirror with transmission τ = 0.5%.

nator had a uniform transmission of τ = 0.5% over the cross section. The distance between the mirrors was 30 and 100 cm in the experiments.

To determine the angular divergence, laser emission was photographed in the focal plane of an f = 163 cm lens and the image was measured with the MF-4 microphotometer. A diffraction image of the slit[9] was printed on each film frame for this purpose. The angular divergence of laser emission for resonators with variable and uniform mirror transmission is given in Table I (pump excess over threshold n = 3 to 4). The Table also presents data on the angular divergence of the first three lower transverse modes for mirrors with cross section variable transmission computed according to (4), and for mirrors with cross section uniform transmission (at 0.5 of the maximum intensity I_{max}).

According to the experimental data given in Table I, the integrated (over the entire generation pulse) angular divergence of a laser with variable transmission mirrors is lower than the corresponding angular

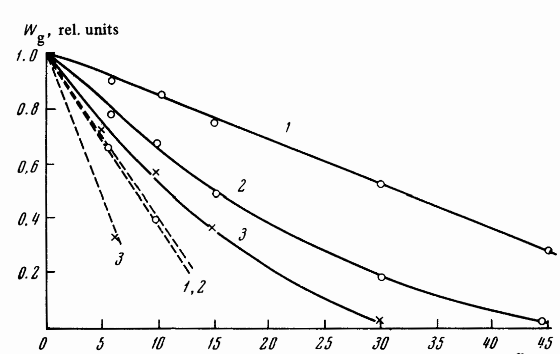

FIG. 3. Generation energy as a function of the angle of departure from parallelism (α'') of the resonator mirrors. 1, 2 – flat mirrors with normal distribution of the transmission coefficient amounting to τ = 8% at the center and τ = 40% at the edge (curve 1), and τ = 10% at the center and τ = 70% at the edge (curve 2); 3 – flat mirrors with a transmission τ = 22% (solid curves: $2l$ = 30 cm, D = 5.5 cm; dashed curves: $2l$ = 100 cm, D = 5.5 cm).

[1)]The distribution of population inversion was not uniform along the y axis: it was larger at the edges than in the center. The corresponding data are given in [8].

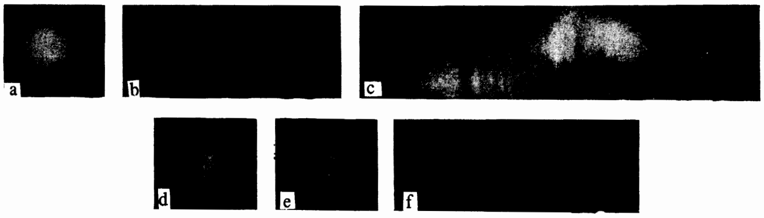

FIG. 4. The effect of the angle of departure from parallelism of the resonator mirrors on far field distribution ($2l$ = 100 cm, D = 5.5 cm). a, b, c – flat mirrors with transmission τ = 10%; d, e, f – flat mirrors with normal distribution of transmission over mirror surface (τ = 10% at the center, τ = 70% at the edges). Misalignment is absent in a and d, 1″–2″ misalignment in b and e, 5″–6″ misalignment in c and f.

Table II.

Departure from parallelism, sec. of arc	Angular divergence of emission at 0.5 of I_{max}, sec. of arc	
	Uniform transmission mirrors, τ = 22%	Variable transmission mirrors, τ = 10 – 70%
0	60	35—40
1—2	200—250	45—50
5—6	300—360	130—140
10—12	≫1000	420

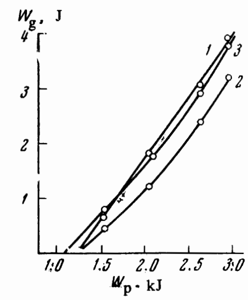

FIG. 5. Generation energy as a function of pumping energy ($2l$ = 30 cm, D = 5.5 cm). 1, 2–flat mirrors with normal distribution of transmission (τ = 8% at the center, τ = 40% at the edge for curve 1; τ = 10% at the center and τ = 70% at the edge for curve 2); 3 – flat mirrors with a transmission τ = 22%.

divergence of a laser with flat mirrors having uniform transmission.

However if we compare the angular divergence of individual modes that have the same transverse indices in these resonators, it is always higher in the case of variable transmission mirrors. The observed decrease in the total angular divergence of emission can be explained by the considerable selection of the number of generated modes in resonators having cross section variable mirror transmission. A comparison of theory with experiment shows that lasers with cross section variable transmission mirrors generate two to four transverse modes, while uniform transmission mirrors under analogous conditions have 18–20 transverse modes.

We studied the effect of departure from parallelism of the mirrors on the energy and angular characteristics of emission from lasers with flat mirrors having transmission either variable or constant over the cross section. The resulting data are given in Fig. 3 and Table II. Figure 4 shows photographs of the far field for the above resonators with various degrees of departure from parallelism of the mirrors.

The obtained data show that lasers with variable transmission mirrors are much less sensitive to resonator mirror alignment both with respect to the generation energy and to angular divergence than lasers with ordinary flat mirrors. This can be due to the fact that the fields of the individual transverse modes are more "squeezed" towards the resonator axis and are less perturbed by mirror misalignment.

The generation energy of lasers with flat mirrors having cross section variable transmission is practically the same as in lasers with ordinary flat mirrors of the same dimensions, as shown in Fig. 5.

Consequently the use of cross section variable transmission mirrors in lasers decreases the total angular divergence without noticeable reduction of the generation energy. Furthermore such mirrors are much less sensitive to mirror alignment than the ordinary flat mirrors, a feature that is particularly valuable with large apertures. We also note that lasers with such mirrors can easily generate a single transverse mode with a marked pumping energy excess over threshold.

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133