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# FLUCTUATIONS OF RADIATION BUILDUP IN GAS LASERS

E. V. BAKLANOV, S. G. RAUTIAN, B. I. TROSHIN, and V. P. CHEBOTAEV

Semiconductor Physics Institute, Siberian Division, USSR Academy of Sciences; Novosibirsk State Institute of Measures and Measuring Instruments

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Statistical phenomena occurring after the gas laser gain is turned on rapidly are studied theoretically and experimentally. The shape of the transient curve ("track") of the generation power is determined in the quasiclassical approximation. When the threshold is exceeded by a sufficient amount, the randomness of the track is described by a single parameter, the "priming" number of photons $n_0$. This primer is made up of spontaneous radiation emitted before as well as after the gain is turned on. The photon-number distribution function W(n, t) is calculated. W(n, t) is measured experimentally, and the theory and experiment are compared without "adjustment" parameters. The experiments confirm the theory.

## 1. INTRODUCTION

INTEREST has recently increased in the study of statistical characteristics of laser emission. This topic is the subject of a number of theoretical and experimental papers[1-5]. In this paper we investigate the statistical properties of radiation during the transient occurring when the gain is sharply changed from a subthreshold to an above-threshold value. In this case the time of establishment of the steady state experiences appreciable fluctuations. Pariser and Marshall[6], for example, reported the existence of this phenomenon and indicated that the transient-time fluctuations are connected with the random character of the initial conditions at the instant when the gain (or the Q of the resonator) is turned on. We shall show below, however, that the same consequences result not only from the "initial primer," but also the spontaneous noise which is radiated by the active medium after the gain is turned on. Depending on the numerical values of a number of parameters, either of the two causes of fluctuations may be decisive.

Arecchi[5] measured the distribution functions of the photons in the transient process and attempted to compare the experimental data with the results of theoretical calculations. However, the values of the parameters entering in the theory were not measured by Arecchi[5] independently, but were chosen so as to obtain best agreement between the experimental and theoretical curves. Such a comparison is all the more unconvincing since he took into account only one of the two causes of the aforementioned fluctuations. Therefore the question of the relation between theory and experiment remains open.

We have performed a statistical analysis of the transients in a gas laser. In the theoretical analysis we used a quasiclassical approach and the well-developed formalism of statistical radiophysics (Sec. 2). As shown by experiment and theoretical estimates such a basis is perfectly adequate for most gas lasers. The third and fourth sections contain a description of the setup, the experimental data, and their discussion.

## 2. THEORY

We start from the following equation for the field E (single-mode regime):

$$\ddot{E} + \frac{\omega}{Q}\dot{E} + \omega^2 E = 4\pi\omega^2 P, \tag{2.1}$$

where $\omega$ and Q are the natural frequency and the Q of the selected oscillation mode, P is the polarization due to the active particles. For simplicity, we consider a case when the transition frequency $\omega_{mn}$ of the active particles coincides with $\omega$ (the case $\omega_{mn} \neq \omega$ will be discussed in the end of this section). We change over to complex notation

$$E(t) = \mathrm{Re}\{\mathscr{E}(t)e^{-i\omega t}\}, \quad P(t) = \mathrm{Re}\{\mathscr{P}(t)e^{-i\omega t}\}, \tag{2.2}$$

assuming the amplitudes $\mathscr{E}(t)$ and $\mathscr{P}(t)$ to be slowly varying functions of the time. We then get from (2.1) the abbreviated equation

$$\dot{\mathscr{E}} + \frac{\omega}{2Q}\mathscr{E} = i2\pi\omega\mathscr{P}(t). \tag{2.3}$$

Following Lamb[7], we represent the polarization $\mathscr{P}(t)$ in the form of a sum of an induced term and a stochastic term[1)]

$$\mathscr{P} = \mathscr{P}_i + \mathscr{P}_s. \tag{2.4}$$

The induced part $\mathscr{P}_i$, with allowance for the first correction for the saturation effect, can be represented in the form

$$\mathscr{P}_i = -i\chi\mathscr{E}, \quad \chi = \chi_0 - \chi_1|\mathscr{E}|^2. \tag{2.5}$$

We measure the field amplitude in units of $\sqrt{8\pi\hbar\omega/V}$. Then $|\mathscr{E}|^2$ coincides with the number of photons in the volume V of the resonator. Substituting (2.4) and (2.5) in (2.3), we find

$$\dot{\mathscr{E}} - {}^1/_2(\alpha - \beta|\mathscr{E}|^2)\mathscr{E} = f(t), \tag{2.6}$$

where

$$f(t) = i\sqrt{V[8\pi\hbar\omega]^{-1}}\cdot 2\pi\omega\mathscr{P}_s(t). \tag{2.7}$$

1) Our notation differs somewhat from that in [7] – we use a complex notation without separating the real amplitude and phase.

Here $\alpha$ is the difference between the gain and the loss and $\beta$ is the saturation parameter. The function f(t) specifies the rate of growth of the field as a result of the spontaneous emission of the active medium. The spectral power density of this "noise" coincides with the luminescence-line contour. In our case the width of the luminescence line is much larger than $\omega/Q$, i.e., the noise can be regarded as $\delta$-correlated[2)]:

$$\langle f^*(t)f(t')\rangle = I\delta(t-t'). \quad (2.8)$$

It will be made clear later (see the discussion of (2.11)) that the coefficient I is none other than the rate of entry of the photons into the mode.

Let us consider first the regime below the generation threshold, neglecting the term $\beta|\mathcal{E}|^2$:

$$\dot{\mathcal{E}} + {}^1/_2\alpha_1\mathcal{E} = f(t). \quad (2.9)$$

The quantity $\alpha_1^{-1}$ determines the characteristic time of the transients, decreased, compared with $Q/\omega$, as a result of the gain of the active medium (regeneration). The stationary value of the field

$$\mathcal{E}_1(t) = \int_{-\infty}^{t} f(t')\exp\left\{-\frac{\alpha_1}{2}(t-t')\right\}dt' \quad (2.10)$$

has a correlation function

$$\langle\mathcal{E}_1^*(t)\mathcal{E}_1(t+\tau)\rangle = \frac{I}{\alpha_1}e^{-\alpha_1|\tau|}. \quad (2.11)$$

Since the function f(t) is $\delta$-correlated, the phase of the field $\mathcal{E}_1$ is distributed uniformly in the interval $[0; 2\pi]$, and the following distribution holds for $|\mathcal{E}_1|^2 \equiv n$[8]:

$$W(n) = \frac{1}{\bar{n}_1}\exp\left\{-\frac{n}{\bar{n}_1}\right\}, \quad \bar{n}_1 = \langle|\mathcal{E}_1|^2\rangle = \frac{I}{\alpha_1}. \quad (2.12)$$

The quantity $\bar{n}_1$ is determined, as usual, from the correlation function at $\tau = 0$. Under stationary conditions the average number of photons in the resonator is conserved, i.e., the rates of loss and creation of photons should be the same. Since $\alpha_1\bar{n}_1$ is the rate of photons escape, then, in accordance with (2.12), the coefficient I has the meaning of the number of photons entering in the mode per unit time.

On going over from (2.6) to (2.9), we discarded the saturation term $\beta|\mathcal{E}|^2$. From (2.6) and (2.12) we see that this is valid if

$$\alpha_1^2 \gg \beta I. \quad (2.13)$$

We now turn to the analysis of the transient produced in the generator if at the instant of time t = 0 the coefficient $\alpha$ changes jumpwise from a certain negative value $-\alpha_1$ to a positive value $\alpha_2$. The determination of $\mathcal{E}(t)$ for $t > 0$ calls for allowance for saturation. However, at sufficiently short times directly following the instant of the switching of the gain, we can confine ourselves to the linear approximation

$$\dot{\mathcal{E}} - {}^1/_2\alpha_2\mathcal{E} = f(t). \quad (2.14)$$

The solution of this equation can be written in the form of two terms: the solution of the homogeneous equation, determined by the initial condition at t = 0, and the solution of the inhomogeneous equation

$$\mathcal{E}(t) = [\mathcal{E}_1(0) + \mathcal{E}_2(t)]\exp\{{}^1/_2\,\alpha_2 t\},$$
$$\mathcal{E}_2(t) = \int_0^t f(t')\exp\{-{}^1/_2\,\alpha_2 t'\}dt', \quad (2.15)$$

and $\mathcal{E}_1(0)$ is given by expression (2.10) with t = 0. Formula (2.15) describes an exponential growth of generation, with the "priming" field $\mathcal{E}_1 + \mathcal{E}_2$ naturally being broken into two parts. The "primer" $\mathcal{E}_1$ is obviously the value of the field that has accumulated in the resonator at the instant when the gain is turned on. The term $\mathcal{E}_2$ describes the field growth due to the spontaneous emission after the gain is turned on. The correlation function for the field is

$$\langle\mathcal{E}^*(t)\mathcal{E}(t+\tau)\rangle = \left[\frac{I_1}{\alpha_1} + \frac{I_2}{\alpha_2}(1-e^{-\alpha_2 t})\right]e^{\alpha_2 t-\alpha_2|\tau|}, \quad (2.16)$$

where $I_1$ and $I_2$ are the values of the parameter I before and after the gain is turned on, and generally speaking differ from each other. The distribution for the number of photons $n = |\mathcal{E}|^2$ is given by formula (2.12), and the average number of photons, in accord with (2.16), is

$$\bar{n}(t) = \left[\frac{I_1}{\alpha_1} + \frac{I_2}{\alpha_2}(1-e^{-\alpha_2 t})\right]e^{\alpha_2 t}. \quad (2.17)$$

Unlike the first primer $I_1/\alpha_1$, the contribution of the second priming field depends on the time. However, when $t > 1/\alpha_2$, we can discard $\exp(-\alpha_2 t)$, and we obtain in lieu of (2.17)

$$\bar{n}(t) = \bar{n}_0 e^{\alpha_2 t}, \qquad \bar{n}_0 = I_1/\alpha_1 + I_2/\alpha_2. \quad (2.18)$$

Formula (2.18) can obviously be obtained from the homogeneous equation (2.14), if we redefine the initial conditions, replacing $I_1/\alpha_1$ by the sum $I_1/\alpha_1 + I_2/\alpha_2 = \bar{n}_0$. Consequently, when $t > 1/\alpha_2$, we can discard f(t) in Eq. (2.14), and the influence of the noise radiated when $t > 0$ reduces to an effective increase of the initial primer by an amount $I_2/\alpha_2$.

Thus, the initial period of the development of the generation is represented physically in the following manner: at the instant (t = 0) when the above-threshold gain is turned on, there are accumulated in the resonator $n_1 = |\mathcal{E}|^2$ photons with the mean value of $I_1/\alpha_1$. After the gain is turned on, this primer leads to an exponential growth of the field energy (at a rate $\alpha_2$). The noise at $t > 0$ effectively increases the primer, and the resonator accumulates these photons only during a time $1/\alpha_2$. Physically this is connected with the fact that the noise radiated after $t \gtrsim 1/\alpha_2$ plays a much smaller role than the noise radiated during the time $0 < t < 1/\alpha_2$, since the latter had time to become amplified by a factor $\exp(\alpha_2 t) \gg 1$ times.

If $\alpha_2 \gg \alpha_1$, then the principal role is played by the priming field produced prior to the turning on of the gain (at $t < 0$). In the opposite limiting case $\alpha_2 \ll \alpha_1$ the primer is the field spontaneously radiated at $t > 0$. In the general case, both primers add up with weights proportional to the effective accumulation times $1/\alpha_1$ and $1/\alpha_2$.

The linear part of the transient, which we considered above, takes place up to fields satisfying the condition $\beta|\mathcal{E}|^2 \ll \alpha_2$. Since $\alpha_2/\beta = n_\infty$ is the average number of photons in the stationary generation regime, the

[2)]This is essentially the condition for the applicability of the quasistatic approximation (2.5).

linearity condition implies, by virtue of (2.18),

$$[I_1/\alpha_1 + I_2/\alpha_2]e^{\alpha_2 t} \ll n_\infty.$$

We note that $n_\infty$ as a rule exceeds the priming intensity $I_1/\alpha_1 + I_2/\alpha_2$ by many orders of magnitude:

$$n_\infty \gg \bar{n}_0 = I_1/\alpha_1 + I_2/\alpha_2. \quad (2.19)$$

Therefore there exists a time t

$$1/\alpha_2 \lesssim t \ll \frac{1}{\alpha_2}\ln\frac{n_\infty}{n_0}, \quad (2.20)$$

such that the linear approximation is still valid, but the second primer has already been fully formed and it is consequently possible to disregard the succeeding spontaneous emission.

We now turn to an analysis of the nonlinear period of development of the generation, when it is necessary to take the saturation effect into account. We assume in this case that the condition (2.20) is satisfied. Then, in place of (2.6), we can consider the homogeneous nonlinear equation

$$\dot{\mathscr{E}} - {}^1/_2[\alpha_2 - \beta|\mathscr{E}|^2]\mathscr{E} = 0. \quad (2.21)$$

From (2.21) follows an equation for the numbers of photons n, which are the only ones of interest to us from now on:

$$\dot{n} - [\alpha_2 - \beta n]n = 0. \quad (2.22)$$

Accordingly, the initial value $n(0) = n_0$ is a random quantity with a distribution

$$W(n_0) = \frac{1}{\bar{n}_0}\exp\left\{-\frac{n_0}{\bar{n}_0}\right\}, \quad \bar{n}_0 = \frac{I_1}{\alpha_1} + \frac{I_2}{\alpha_2}. \quad (2.23)$$

From (2.22) we readily obtain

$$n(t) = n_0\left[1 + \frac{n_0}{n_\infty}(e^{\alpha_2 t} - 1)\right]^{-1} e^{\alpha_2 t}, \quad n_\infty = \frac{\alpha_2}{\beta}. \quad (2.24)$$

When $t \gtrsim 1/\alpha_2$, the only time when the employed approach is valid, we can rewrite (2.24) in the form

$$n(t) = \frac{n_0}{e^{-\alpha_2 t} + n_0/n_\infty}. \quad (2.25)$$

If we fix the value of $n_0$, then the function n(t) determines one of the possible curves ("tracks") of generation growth. The characteristic features of the tracks, which follow directly from (2.25) and are illustrated by the oscillograms of Fig. 4a, are as follows: after the instant of switching, the generation power increases relatively slowly for a certain time of the order of $\alpha_2^{-1}\ln(n_\infty/\bar{n}_0)$. Then, for a time $\sim 1/\alpha_2$, a rapid increase of n(t) takes place and the stationary value, which is practically the same for all tracks is reached. In accordance with the assumption (2.19), we have $n_\infty \gg n_0$. Therefore the first period is several times longer than the second, and the entire n(t) curve has the form of a smooth-out "step." Since $n_0$ is a random quantity, the tracks obtained after repeated switching will be different. The statistics of the tracks is obviously determined mainly by the distribution (2.23) of the priming numbers of photons.

The distribution of the photon numbers n at the instant of time t, can be readily calculated from (2.23), and (2.25):

$$W(n) = \frac{1}{\bar{n}_0}\left(1 - \frac{n}{n_\infty}\right)^{-2}\exp\left\{-\alpha_2 t - \frac{n}{\bar{n}_0}\left(1 - \frac{n}{\bar{n}_0}\right)^{-1} e^{-\alpha_2 t}\right\}. \quad (2.26)$$

If the number of repeated switchings is large, $W(n)\Delta n$ determines the density of the tracks whose intensity lies at the instant t in the interval $(n, n + \Delta n)$. The mean value and the dispersion are given by the formulas

$$\langle n\rangle = n_\infty\{1 + ae^a\,\mathrm{Ei}(-a)\},$$
$$\sigma_n^2 = (a+2)n_\infty\langle n\rangle - n_\infty^2 - \langle n\rangle^2; \quad (2.27)$$

$$a = \frac{n_\infty}{\bar{n}_0}e^{-\alpha_2 t}, \quad \mathrm{Ei}(-a) = -\int_a^\infty e^{-y}\frac{dy}{y}. \quad (2.28)$$

The experiment described below consisted of fixing the instant of time when the generation power reached a certain value $n < n_\infty$. Therefore, for an interpretation of these experiments, it is necessary to know another function, namely the probability of appearance of a specified number of photons n in a time interval $(t, t + \Delta t)$:

$$W(t)\Delta t = \frac{n_\infty}{\bar{n}_0}\eta\exp\left\{-\alpha_2 t - \frac{n_\infty}{\bar{n}_0}\eta e^{-\alpha_2 t}\right\}\alpha_2\Delta t, \quad (2.29)$$
$$\eta = n/(n_\infty - n).$$

The probability (t) reaches its maximum value

$$W_m = \alpha_2/e \quad (2.30)$$

when

$$t_m = \frac{1}{\alpha_2}\ln\left[\frac{n_\infty}{\bar{n}_0}\eta\right]. \quad (2.31)$$

Using the parameters $W_m$ and $t_m$, we can rewrite (2.29) in the following simple form:

$$W(t) = W_m\exp\{-\alpha_2(t - t_m) + [1 - e^{-\alpha_2(t - t_m)}]\}. \quad (2.32)$$

For the average time and for the mean-square deviation from the mean we can readily obtain the expressions

$$\bar{t} = \int_0^\infty tW(t)\,dt = \frac{1}{\alpha_2}C + t_m,$$
$$\sigma_t^2 = \overline{(t - \bar{t})^2} = \int_0^\infty (t - \bar{t})^2 W(t)\,dt = \frac{\pi^2}{6\alpha_2^2}, \quad (2.33)$$

C = 0.577 is Euler's constant. Since $n_\infty\eta/n_0 \gg 1$, it follows that t and $t_m$ differ relatively little.

We now discuss the region of applicability of the developed theory. The assumption that the frequency of the atomic transition coincides with the frequency of the resonator is not a substantial limitation. As is well known, deviation from resonance makes the coefficients $\alpha$ and $\beta$ in (2.6) complex. In this case the analysis is perfectly analogous. In particular, the obtained expressions for the photon-number distributions remain in force if we take $\alpha_1$, $\alpha_2$, and $\beta$ to mean their real parts.

The most important limitation is the classical description of the field, which is justified when the following inequality is satisfied

$$\bar{n}_0 = I_1/\alpha_1 + I_2/\alpha_2 \gg 1. \quad (2.34)$$

This condition can be recast in a different form. We denote by N and $\Delta N$ respectively the population of the upper level and the modulus of the difference of the populations. Since the probabilities of the spontaneous and stimulated emission in the mode, calculated per atom, are related as 1 : n, we have

$$\frac{I}{N}=\frac{4\pi\omega\chi_0 n}{\Delta N}\cdot\frac{1}{n}$$

and we can rewrite (2.34) with the aid of (2.7) in the form

$$\bar{n}_0=\left(\frac{\omega}{Q\alpha_1}-1\right)\frac{N_1}{\Delta N_1}+\left(\frac{\omega}{Q\alpha_2}+1\right)\frac{N_2}{\Delta N_2}\geqslant 1. \qquad (2.35)$$

Thus, the priming number of photons $\bar{n}_0$ can be expressed exclusively in terms of the ratios $N_{1,2}/\Delta N_{1,2}$, the excess of the gain over the threshold value $\alpha_2 Q/\omega$, and an analogous quantity for the subthreshold regime $\alpha_1 Q/\omega$. Since the expansion (2.5) for $\chi$ is valid only if $\omega/Q \gg \alpha_2$ and we always have $N > \Delta N$, the inequality (2.35) can be readily satisfied. In particular, in the experiments described below, $\alpha_1, \alpha_2 \sim 10^{-2}\omega/Q$, and both priming fields can be described classically.

In conclusion we note that the results of the present section can be obtained by using a different approach, namely by using the Fokker-Planck equation for W(n, t). Applying the standard method to the stochastic equation (2.6) (see, for example,[8,9]), we can find that W(n, t) satisfies the equation

$$\frac{\partial W}{\partial t}=\frac{\partial}{\partial n}[(\alpha-\beta n)nW]+I\frac{\partial}{\partial n}\left[n\frac{\partial W}{\partial n}\right]. \qquad (2.36)$$

It is also easy to show that Eq. (2.36) is the limiting case of the Scully-Lamb quantum equation[10] with $n \gg 1$. This corresponds fully to the classical nature of Eq. (2.36).

The approach based on (2.36) is somewhat less lucid but perfectly natural, since it operates with the function W(n, t) which is of direct interest to us. In addition, this formalism simplifies the solution of a number of concrete problems. Let us consider, for example, the stationary regime of generation and calculate the photon distribution[3)]. Assuming $\partial W/\partial t = 0$, we can find the stationary $W(n, t) \equiv W(n)$:

$$W(n)=C\exp\left\{-\left[n-\frac{\alpha}{\beta}\right]^2 2\sigma^2\right\}, \qquad (2.37)$$

$$\sigma^2=\frac{I}{\beta},\quad C=\sqrt{\frac{2}{\pi}}\frac{1}{\sigma}[1+\Phi(z)]^{-1},\quad z=\frac{\alpha}{\sqrt{2I\beta}}$$

where $\Phi(z)$ is the probability integral[4)]. Unlike the normal distribution, the function (2.37) is meaningful only when $n > 0$, i.e., it is a Gaussian curve "cut off" at the point $n = 0$. The distribution (2.37) describes in a unified manner three characteristic stationary regimes: subthreshold ($\alpha < 0$), $\alpha = 0$, and above-threshold ($\alpha > 0$, Fig. 1).

The expressions for the mean value $\langle n\rangle$ and the dispersion are

$$\langle n\rangle=\frac{\alpha}{\beta}\left\{1+\frac{\exp(-z^2)}{\sqrt{\pi}z[1+\Phi(z)]}\right\}, \qquad (2.38)$$

$$\langle[n-\langle n\rangle]^2\rangle=\frac{I}{\beta}-\left[\langle n\rangle-\frac{\alpha}{\beta}\right]\langle n\rangle.$$

If $z = \alpha/\sqrt{2}\beta\sigma \gg 1$, then the distance from the maximum of the distribution function to zero is much larger than the dispersion (see Fig. 1), and W(n) can be regarded as practically coinciding with a normal distribution.

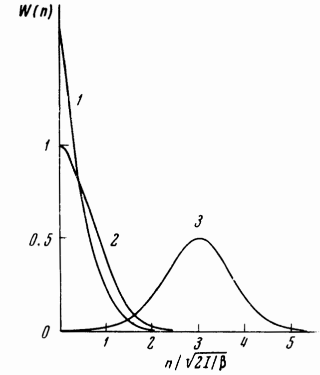


FIG. 1. Stationary distribution of photons W(n) (in units $\sqrt{2\beta/\pi I}$): 1–z = –½, 2–z = 0, 3–z = 3.

We emphasize that the inequality $z \gg 1$, coincides with the condition that the second primer $I_2/\alpha_2$ is much smaller than $n_\infty = \alpha_2/\beta_2$. Thus, one of the conditions for the applicability of the transient picture analyzed above (see (2.19)) receives a new natural interpretation—the stationary generation power should be much larger than the dispersion of the stationary fluctuations.

## 3. EXPERIMENT

In the experiments described below we investigated the transient following the switching on of the gain above the threshold value, and the ensuing statistical phenomena. The experiments were performed with a neon-helium laser, $\lambda = 0.63\ \mu$, constructed in accordance with the circuit of Fig. 2. A rectangular voltage pulse was applied to the supply circuit of the laser tube from a square-wave generator SWG, thereby changing the gain jumpwise (Fig. 3). The length of the pulse was $2\times10^{-4}$ sec; the time of gain switching $\Delta\tau$ was $\sim1.5\times10^{-6}$ sec. The transient growth of radiation intensity was registered with an FÉU-2 photomultiplier and was observed on an oscilloscope screen. Figure 4 shows the corresponding oscillograms in the form of tracks. Each track corresponds to a transient following one switching. During the time of the exposure, approximately 30 switchings were made. Each individual track has the form of a smooth curve without appreciable traces of noise, but the growth time of the intensity has a random character. It is clear therefore that only the spontaneous noise accumulated prior to the switching of the gain and emitted at the very start of the transient is of appreciable significance.

The role of this period of generation development is illustrated by the oscillogram of Fig. 4b, obtained at a much larger oscilloscope gain than Fig. 4a, Figure 4b shows the field fluctuations prior to the switching of the above-threshold gain, and the subsequent growth of the generation power, which reveals certain traces of noise. This period, however, is a relatively small fraction of the total time of generation development. On the other hand, it is seen from Figs. 4a and b that the generation power greatly exceeds the "priming" power. This means that the "second primer" can be formed only before the saturation effect comes into play (see the discussion of formulas (2.17)–(2.20)).

3)Expression (2.26) does not contain this distribution, since the spontaneous noise in the presence of saturation was not taken into account in its derivation.

4)We note that (2.37) coincides with the stationary distribution of the oscillation energy in the Thomson vacuum-tube generator (see, for example [9], Sec. 24).

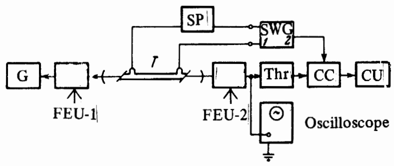


FIG. 2. Diagram of setup. SWG—generator of rectangular pulses, Thr—threshold unit, CC—coincidence circut, G—galvanometer, SP—supply pulse, CU—counter unit, T—laser tube.

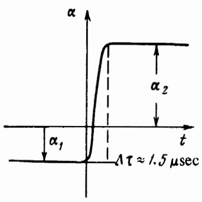


FIG. 3. Plot of difference between the gain and loss as a function of the time.

These circumstances were precisely the assumptions made in the theory of Sec. 2, so that it can be assumed that the general picture of the phenomenon has been confirmed.

The setup (Fig. 2) makes it possible to measure the probability $W(t)\Delta t$ of appearance of a given number of photons n in the time interval $(t, t + \Delta t)$. To this end, the signal from the FÉU-2 photomultiplier is applied to the threshold unit THz. After the signal reaches a definite (adjustable) level, Thz delivers a standard rectangular pulse which is fed to the coincidence circuit CC. CC receives also a second pulse (from another output of the SWG), delayed relative to the time of switching of the gain in the laser tube by a certain time t. The delay time t can be varied continuously. If both pulses arrive at CC simultaneously, then the CC produces a signal which is recorded by a counter. Thus, the entire circuit operates only when the generation intensity reaches at the instant of time t a definite value specified by the threshold unit. Of course, both the standard and the delayed pulse have a finite width, and the coincidence circuit operates in a certain interval $(t, t + \Delta t)$. The value of $\Delta t$ in formula (2.29) for $W(t)\Delta t$ was measured experimentally and found to be $\Delta t = 0.7 \times 10^{-6}$ sec.

The measurements were made in the following manner. A definite value of the delay time and a definite threshold were set up, and the laser was tuned to the center of the line. The laser was turned on $10^4$ times (discharge-supply pulse repetition frequency 1 kHz, measurement time 10 sec). The number of coincidences registered by the counting unit divided by $10^4$ gave the value of $W(t)\Delta t$. This quantity was compared with the theoretical expression (2.29).

The formula for $W(t)\Delta t$ contains, besides $\Delta t$, the parameters $n/n_\infty$, $\bar{n}_0/n_\infty$, and $\alpha_2$. A complete verification of the theory presupposes independent measurement of these quantities. The parameter $n/n_\infty$ was determined directly from the oscillogram, by comparing the intensity of the laser emission at the instant of operation of the threshold unit with the intensity in the stationary regime.

The parameter $\bar{n}_0/n_\infty$ contains two terms—the priming numbers of the photons accumulated before and after the switching of the gain. Obviously, only the first part can be measured directly. This was done with photomultiplier FÉU-1 and galvanometer G (Fig. 2), with the aid of which we measured the average power of the radiation prior to the instant of the gain switching ($\bar{n}_1$) and in the stationary generation regime ($n_\infty$). A set of diaphragms prevented spontaneous emission in the lateral modes from reaching the photomultiplier FÉU-1. An experimental verification has shown that the parasitic emission was not more than 6% of the spontaneous-emission power in one selected mode. The axial-mode selection was necessitated by the following: as seen from formula (2.12), the spontaneous-emission power in the mode is inversely proportional to $\alpha_1 = \omega/Q - 4\pi\omega\chi_0$, where $\chi_0 \propto \exp[-(\omega - \omega_{mn})^2/\alpha(k\bar{v})^2]$ is the susceptibility, i.e., the parameter $\alpha_1$ is larger for nonresonant modes than at $\omega = \omega_{mn}$. At the numerical values of the parameters used in the measurements ($\omega/Q = 10^7$ sec$^{-1}$, $\omega - \omega_{mn} = 2.5 \times 10^9$ sec$^{-1}$, $k\bar{v} = 10^{10}$ sec$^{-1}$, $\alpha_1 \sim 10^5$ sec$^{-1}$), the dependence of $\chi_0$ on $\omega$ led to a strong "suppression" of the modes located off the line center (by a factor of 5–10).

The parameters $\alpha_1$ and $\alpha_2$ were determined from the oscillograms. In accordance with (2.22), $\alpha_2$ was calculated from the growth rate n(t) on the leading front of the transient, and $\alpha_1$ from the trailing edge (cessation of generation, see Fig. 4). In addition, we measured the resonator detuning at which the generation stopped. From this detuning and from the known resonator loss, we calculated $\alpha_2$ at the center of the line. Both measurements of $\alpha_2$ gave practically identical results. The accuracy with which the parameters $n/n_\infty$, $\bar{n}_1/n_\infty$, and $\alpha_2$ were measured was about 10%.

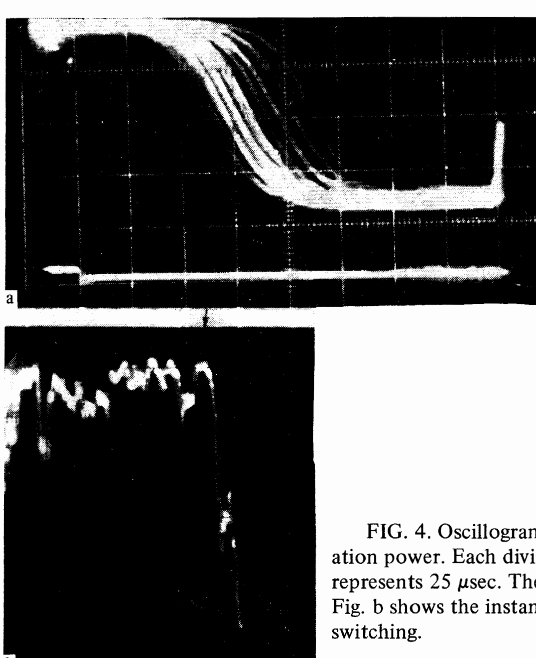

FIG. 4. Oscillograms of generation power. Each division in Fig. 1 represents 25 μsec. The arrow on Fig. b shows the instant of switching.

## 4. MEASUREMENT RESULTS AND THEIR DISCUSSION

The measurements of $W(t)\Delta t$ in accordance with the described procedure were made for the following four groups of parameter values:

a) $\alpha_2 = 1.6\cdot10^5$ sec$^{-1}$, $n/\bar{n}_1(1-n/n_\infty)^{-1} = 0.95\cdot10^3$, $\alpha_1 = 0.3\cdot10^5$, $n/n_\infty = 0.05$;

b) $\alpha_2 = 1.6\cdot10^5$ sec$^{-1}$, $n/\bar{n}_1(1-n/n_\infty)^{-1} = 0.43\cdot10^3$, $\alpha_1 = 0.13\cdot10^5$, $n/n_\infty = 0.06$;

c) $\alpha_2 = 1.0\cdot10^5$ sec$^{-1}$, $n/\bar{n}_0(1-n/n_\infty)^{-1} = 1.0\cdot10^4$, $t_m = 92\cdot10^{-6}$ sec, $n/n_\infty = 0.09$;

d) $\alpha_2 = 1.43\cdot10^5$ sec$^{-1}$, $n/\bar{n}_0(1-n/n_\infty)^{-1} = 0.55\cdot10^4$, $t_m = 60\cdot10^{-6}$ sec, $n/n_\infty = 0.05$.

The results of the measurements of $W(t)\Delta t$ are represented by the points on Fig. 5 in a logarithmic scale. The solid curves correspond to calculation by means of formula (2.29), using the indicated values of the parameters. In cases (a) and (b) we have $\alpha_2 \gg \alpha_1$, and in plotting the theoretical curves we used as the priming number of photons the measured value of $\bar{n}_1$. In cases (c) and (d), $\alpha_2 < \alpha_1$ and the bulk of the "primer" should be formed, according to Sec. 2, after the gain is turned on. In full agreement with this, the theoretical curves calculated under the assumption $\bar{n}_0 = \bar{n}_1$ differ greatly from the experimental data. In particular, the maxima of the curves would be located at $t > 98\times10^{-6}$ sec and $t > 67\times10^{-6}$ sec in lieu of the experimental values $t_m = 92\times10^{-6}$ sec and $60\times10^{-6}$ sec. This discrepancy shows clearly the decisive role of the second primer $n_2$ when $\alpha_1 > \alpha_2$. Since a direct measurement of the total primer is impossible in cases (c) and (d), the theoretical curves on Figs. 5c and d were plotted using the position of the maximum $t_m$ as an adjustment parameter (see formulas (2.31) and (2.32).

Under the measurement conditions we have $\bar{n}_0 \sim 10^3$ and $n_\infty/\bar{n}_0 \sim 10^4$. Therefore the criteria (2.13), (2.19), (2.34) of the applicability of the theory proposed in Sec. 2 are satisfied, and it is legitimate to compare the experimental data with this theory.

The good agreement between the experimental and theoretical data, shown in Fig. 5, indicates first of all that the fluctuations of the transients are due precisely to the spontaneous noise. This is corroborated by the fact that in cases (a) and (b) we have performed an

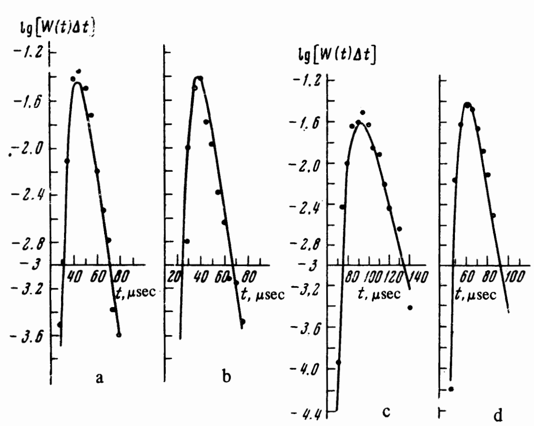


FIG. 5. Plots of log $[W(t)\Delta t]$.

"absolute" comparison (unlike in[5]), all the parameters were measured experimentally, and none was used as an adjustment parameter, In cases (c) and (d), there was one adjustment parameter ($t_m$), and the results of the measurements under these conditions were considered as a proof of the important role played by the "second primer." which is accumulated already after the start of generation development. The plots a, b and c, d of Fig. 5 do not differ qualitatively, which confirms the theoretical conclusion in Sec. 2 that the first and second primers are additive (see (2.18) and its discussion).

Inasmuch as the functions $W(t)$ and $W(n_0)$ can be uniquely recalculated one in terms of the other, the data of Fig. 5 show that the distribution with respect to the number of photons in the subthreshold region, and at the very start of the generation development, is described by (2.23) (the left side of the plot of $\ln[W(t)\Delta t]$, including the region of the maximum, is sensitive to the form of $W(n_0)$).

Thus, the performed experiments have shown that the classical approach to the analysis of fluctuation phenomena in lasers is valid. The quantum effects, which require a different analysis (see, for example,[10]), come into play apparently when $\bar{n}_0 = \bar{n}_1 + \bar{n}_2 \sim 1$. In lasers, on the other hand, the situation $n_\infty \gg n_0 \gg 1$ is typical, so that quantum effects are more readily an exception than the rule.

In conclusion we note that the employed experimental method of investigating fluctuations is a promising one. In essence, the laser serves in these experiments as a unique amplifier, which amplifies the intensity of the fluctuations by several orders of magnitude during the time of the transient. The unique relation between the amplified and initial fluctuations makes this method reliable and convenient.


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