GLAUBER CORRECTIONS AND THE INTERACTION BETWEEN HIGH-ENERGY HADRONS AND NUCLEI

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It is shown that the screening effect, which plays an important role in the interaction of hadrons with nuclei, undergoes a marked change for hadron momenta exceeding values of $p_0 = 5$ to 10 BeV/c. For momenta $p < p_0$ the screening is essentially determined by elastic rescattering, whereas for $p > p_0$ it arises mainly from the absorption of particles produced in inelastic collisions. This change in the physics of the screening processes for $p > p_0$ can lead to observable effects in the behavior of total cross sections for light nuclei. At very high energies this change in the character of the screening has the effect that even if the forward cone shrinks in elastic processes due to the motion of the Pomeranchuk pole, the total cross section for the interaction of hadrons with nuclei should be proportional to $A^{2/3}$, where A is the number of nucleons in the nucleus.

1. INTRODUCTION

IN nonrelativistic quantum mechanics the problem of the scattering of a fast particle by a system of weakly bound particles does not present any fundamental difficulties if the cross section for the scattering by a single particle is known. The general picture of the scattering is rather simple in this case: the incoming particle is scattered successively by the individual particles in the target and the resulting scattering is found by averaging over the positions of the scattering particles. If the scattering by an individual particle arises mainly from inelastic processes (the elastic scattering has diffractional character), the situation becomes even simpler: after the first scattering the incident particle leaves the beam, and the particles of the target which after the first scattering lie in the direction of the incident particle do not take part in the scattering. The simplest example for such a process is the scattering of a π meson on a deuteron. As was shown by Glauber,[1] the total cross section σ can be written in the form

$$\sigma = \sigma_1 + \sigma_2 - \frac{\sigma_1\sigma_2}{4\pi}\overline{R^{-2}}, \qquad (1)$$

where σ_1 and σ_2 are the total cross sections for the individual nucleons in the deuteron, $\overline{R^{-2}}$ is the average value of R^{-2}, and R is the distance between the nucleons in the deuteron.

The last term in (1) takes account of the fact that in cases where one nucleon in the deuteron screens the other from the incoming particle, the interaction occurs only with the first nucleon. This term is called the Glauber correction. While the screening effect in the deuteron is a small correction, it becomes predominant in systems containing a sufficiently large number of particles (for example, in heavier nuclei), since in this case there will always be particles which are screened. It is this effect which causes the cross section for scattering by nuclei to be proportional to $A^{2/3}$ and not to A, where A is the number of nucleons in the nucleus.

In the present paper we should like to call attention to the fact that the arguments leading to the $A^{2/3}$ law for nuclear cross sections and to (1) for the cross section for scattering by a deuteron are essentially nonrelativistic. When relativistic effects are taken into account, the nuclear cross sections may begin to change at energies where the cross section for the interaction with the individual nucleons of the nucleus is already almost constant. The point is that at large energies, large longitudinal distances apparently become important, which increase with the energy (cf., e.g., [2]). If this distance becomes of the order of the nuclear radius R, the interaction of the incident particle with nucleons of the nucleus which are located in a tube of cross section $1/\mu^2$ in the direction of the momentum of the incoming particle cannot be transmitted to the following collisions. This can be pictured in the following way: The incoming nucleon can transform, for example, into a nucleon and a π meson in the rest system during a time of order $1/\mu$. In the laboratory system it will be in such a state during a time p/μ^2, in the course of which it traverses a distance of order p/μ^2 ($\hbar = c = 1$). If $R \sim p/\mu^2$, then it can, in this virtual state, interact with all nucleons of the nucleus lying in its path.

This criterion for the violation of the nonrelativistic picture of successive interactions can also be obtained in a different way. For concreteness, let us consider the interaction of a π meson with a deuteron. For the calculation of the total cross section we consider the forward elastic scattering amplitude and regard the radius R of the deuteron as large, $R \gg 1/\mu$.

In zeroth approximation in $1/R$ the scattering amplitude is described by the sum of graphs of Fig. 1, corresponding to a single scattering by the neutron or proton. In the next approximation we must take into account that the particles that are created or scattered, for example, on the proton can subsequently be absorbed by the neutron, so that the total cross section decreases. These processes correspond to Fig. 2.

However, if the energy of the π meson is not very large, only graph a of Fig. 2 makes a contribution, which leads to the Glauber correction (1). This is due

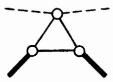

FIG. 1

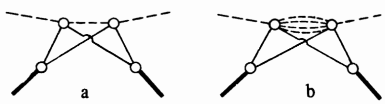

FIG. 2

to the fact that the average momenta of the nucleons in the deuteron are of order $1/R$ and if in the first interaction, the nucleon receives a recoil momentum much larger than $1/R$, the deuteron disintegrates and no reabsorption occurs. At small energies, small momentum transfers can occur only in purely elastic scattering, corresponding to graph a of Fig. 2. As is well known, however, particles can be created with very small momentum transfers under relativistic conditions. If the π meson has momentum p, and the momentum transfer to the nucleon is q, then the mass of the created particle m' is given by

$$m'^2 = (p-q)^2 = \mu^2 + q^2 - 2p_0q_0 + 2|\mathbf{p}|q_z, \quad pq = |\mathbf{p}|q_z. \quad (2)$$

Since $q^2 \sim 1/R^2$ and $q_0 \sim q^2/2m$ in the deuteron, the second and third terms on the right-hand side of (2) can be neglected. The momentum of the π meson necessary for the creation of a particle of mass m' without disintegration of the deuteron is then of order

$$|\mathbf{p}| = \frac{m'^2 - \mu^2}{2q_z} \sim \frac{R}{2}(m'^2 - \mu^2). \quad (3)$$

If m' is of order m, the mass of the π meson, then other graphs begin to make a contribution for momenta

$$|\mathbf{p}| \sim p_0 \sim 5 \div 6 \text{ GeV/C} \quad (4)$$

and the simple theory becomes invalid.

It was shown in [3, 4] that graph a of Fig. 2 leads to an amplitude which decreases rapidly at large energies if the elastic scattering of the π meson on the nucleon is described by a Pomeranchuk pole. In the present paper we show that the graph a of Fig. 2 begins to make a decreasing contribution only for momenta larger than p_0, where the contribution of the other graphs begins to show up. For $|\mathbf{p}| > p_0$ the graph a leads to the Glauber correction (1).

For momenta of order p_0 it is difficult to obtain an exact result. However, for significantly larger energies, when the Pomeranchuk theorem can be regarded as valid and processes corresponding to the exchange of t channel quantum numbers different from those of the vacuum can be neglected, it is possible to obtain the following expression for the total cross section for the interaction of a π meson with a deuteron:

$$\sigma = 2\sigma_N - 2\int dk^2 \rho(4k^2)\frac{d\sigma_N}{dk^2}, \quad (5)$$

where $\rho(k^2)$ is the electromagnetic (charge) form factor of the deuteron; $d\sigma_N/dk^2$ is the sum of the cross sections for all processes which can occur in the interaction of a meson with a nucleon for a given value k^2 of the square of the three-dimensional momentum transfer to the nucleon. Formula (5) remains valid when the π meson is replaced by another particle.

Formula (5) allows one to determine the behavior of the cross section for scattering on nuclei at large energies. Let us begin with the deuteron. We shall use (5) for a qualitative analysis and for not too large energies; as long as the energy is not very large, the momentum of the incoming particle $|\mathbf{p}| < p_0$. The second term in (5) is mainly determined by the elastic scattering (inelastic processes require large momentum transfers, which make a small contribution owing to the deuteron form factor) and coincides with (1). Indeed,

$$\left.\frac{d\sigma}{dk^2}\right|_{k^2=0} \approx \frac{\sigma_N^2}{16\pi}, \qquad \int \rho(4k^2)\,dk^2 = 2\overline{R^{-2}}.$$

With increasing energy inelastic processes set in, and the correction increases, i.e., the cross section for the interaction with the deuteron decreases, when the elastic cross section does not decrease. The elastic scattering cross section does not fall off at energies attainable at present, and therefore the decrease of the cross section for scattering on a deuteron can be observed. If the scattering at high energies is described by a moving Pomeranchuk pole, then the contribution of the elastic scattering must decrease logarithmically because of the shrinking of the cone. This decrease begins, however, only at energies where the average recoil momentum of the nucleon in the elastic scattering becomes smaller than the average momentum of the nucleon in the deuteron. The same must happen with the contribution of the cross section for the production of resonances and for various quasi-elastic processes in which the shrinking of the cone occurs. However, as discussed in detail in [5], no shrinking of the cone occurs in the inelastic processes giving the main contribution to the total cross section; the important momentum transfers are of the order of several μ^2. The contribution from these processes will not fall off with increasing energy, and it can be crudely estimated in the following way. If the total cross section for the inelastic processes is determined by the average momentum of the recoil nucleon $\overline{k^2}$, then a fraction of order $\overline{k_0^2}/\overline{k^2}$ of the processes makes a contribution to the second term of (5), and hence,

$$\sigma = 2\sigma_N - 2(\overline{k_0^2}/\overline{k^2})\sigma_N.$$

Thus the total cross section for the interaction of any hadron with the deuteron can change even in that energy region where the cross sections for the interaction of the deuteron with an individual nucleon are already constant. It should first decrease and then tend to a constant limit different from $2\sigma_N$.

In this connection we note the following. In 1962 it was proposed[6, 7] that if the interaction at large energies is described by the exchange of a Pomeranchuk pole, then the cross section for scattering on nuclei should change from the $A^{2/3}$ law to a linear law with increasing energy, owing to the fact that in such a process the interaction radius and the transparency increase. In particular, it was proposed that σ_d should tend to $2\sigma_N$. As discussed in detail in [5] and shown above, this conclusion is wrong. As judged from formula (5), the reason for the error is obvious. It consists in the assumption that the screening is due to elastic rescattering. As far as more complex nuclei are concerned, the following can be said. A correct calculation of the screening for momenta larger than p_0 is very difficult, even without account of the interaction of the nucleons in the nucleus. This is connected with the circumstance that in the calculation of the cross section one must take account of complicated processes where the particles created on one nucleon are absorbed by several nucleons (Fig. 2, b).

However, qualitatively one may expect that for light nuclei where the total cross section depends on the concrete form of the screening, it will change with increasing energy. For heavy nuclei, where complete screening occurs, the phenomenon described can occur only on account of an interaction with the diffuse nuclear surface, independently of the mechanism of the screening.

2. CONDITIONS FOR THE APPLICABILITY OF THE NONRELATIVISTIC THEORY

Let us consider first the graph of Fig. 1. It corresponds to a contribution to the invariant amplitude of the form

$$F_N = -ig^2\int\frac{d^4p'}{(2\pi)^4}\frac{1}{(m^2-p'^2)[m^2-(p_1-p')^2][m^2-(p'-q)^2]}$$
$$\times\Gamma(p'^2,(p_1-p')^2)f_N((p+p')^2,\ q^2,\ p'^2,(p'-q)^2)\Gamma((p'-q)^2,(p_1-p')^2). \tag{6}$$

The quantity Γ is the vertex part of the deuteron, f_N is the pion–nucleon scattering amplitude, $g^2 = 64\pi M\sqrt{\epsilon m}$, and M is the mass of the deuteron.

Let us set $p' = p_1/2 + k'$. Then the denominators in (6) have the following form in the laboratory system:

$$m^2-p'^2 = \Delta^2 - Mk_0' - k'^2,\quad m^2-(p_1-p')^2 = \Delta^2 + Mk_0' - k'^2,$$
$$m^2-(p'-q)^2 = \Delta^2 - M(k_0'-q_0) + (k'-q)^2, \tag{7}$$

where

$$\Delta^2 = m^2 - M^2/4 = m\varepsilon,\qquad q_0 = \mathbf{q}^2/2M.$$

The main contribution to the integral comes from the region $\mathbf{k}^2 \sim \Delta^2$, $k_0' \sim \Delta^2/M$. If $\mathbf{q}^2 \sim \Delta^2$, the pion–nucleon scattering amplitude f_N can be taken outside the integral for

$$(p+p')^2 = (p+p_1/2)^2 = \mu^2 + m^2 + p_0M = s_1,$$
$$p_1^2 = m^2,\quad (p'-q)^2 = m^2.$$

The error incurred is of order

$$\frac{\Delta^2}{(m+\mu)^2-m^2} \sim \frac{\Delta^2}{2m\mu} \sim \frac{\varepsilon}{2\mu},$$

since the nearest singularities of the amplitude f_N are at $p'^2 = (m+\mu)^2$, $(p'-q)^2 = (m+\mu)^2$. After taking f_N outside the integral, (6) differs from the electromagnetic charge form factor of the deuteron $\rho(q^2)$ by the factor $p_0'/M = 1/2$ and hence,

$$F_N = 2f_N(s_1q^2)\rho(q^2). \tag{8}$$

Let us turn to the graph a of Fig. 2. Its contribution is equal to

$$F_{Mp} = -g^2\int\frac{d^4p'd^4k}{(2\pi)^8}\Gamma(p_1,p')f_n(p,p',k)f_p(p,p_1-p',q-k)$$
$$\times\Gamma(p'+k,p_1-p'+q-k)\{(m^2-p'^2)[m^2-(p'+k)^2][m^2-(p_1-p')^2]$$
$$\times[m^2-(p_1-p'+q-k)^2][\mu^2-(p-k)^2]\}^{-1} \tag{9}$$

For the calculation of the integral (9), we set, as before, $p' = p_1/2 + k'$ and note that for nonrelativistic particles

$$\Gamma(p_1',p_1-p') = \Gamma(\Delta^2 - Mk_0' - k'^2,\ \Delta^2 + Mk_0' - k'^2)$$

is independent of k_0', i.e., is only a function of $\mathbf{k}^2$. This fact is well known, but we repeat the derivation in order to demonstrate its exact nature.

The equation for $\Gamma(\mathbf{k}_0,\mathbf{k})$ can be written in the form

$$\Gamma(k_0',\mathbf{k}') = 1 + \int\frac{d^4k''}{(2\pi)^4 i}\Gamma(k_0'',\mathbf{k}'')$$
$$\times\frac{M(p''^2,(p_1-p'')^2,p_1^2,(p_1-p')^2,p'^2,(p'-p'')^2)}{(\Delta^2-Mk_0''-\mathbf{k}''^2)(\Delta^2+Mk_0''-\mathbf{k}''^2)}, \tag{10}$$

or graphically as in Fig. 3. The amplitude M has no two-particle cuts in p_1. As we shall see below, the integral (9), receives, in contrast to (6), important contributions from values of $\mathbf{k}^2$ up to μ^2, not only from those of order Δ^2. We therefore consider (10) in this region. Since the singularities of M in p'^2, p''^2, $(p_1-p')^2$, and $(p_1-p'')^2$ are located at the value $(m+\mu)^2$ of these variables, we can replace these variables in M by m^2 with an error

$$\frac{p'^2-m^2}{(m+\mu)^2-m^2} \approx \frac{\mu^2}{2m\mu} = \frac{\mu}{2m},$$

The singularities in the momentum transfer $(p'-p'')^2 = (k_0'-k_0'')^2 - (\mathbf{k}'-\mathbf{k}'')^2$ are located at $(p-p')^2 = \mu^2$. As will be seen, the values of k_0', k_0'' of importance in (9) and (10) are of order $\mathbf{k}^2/m \sim \mu^2/m$ and hence, $(p'-p'')^2$ can be replaced by $(\mathbf{k}'-\mathbf{k}'')^2$ with an error μ^2/m^2. The same holds for the singularities in $u = -p_1^2 - (p''-p')^2 + 4m^2$. Thus $M = M[(\mathbf{k}'-\mathbf{k}'')^2]$ in the region considered, and hence, Γ is independent of k_0'. In the same region the amplitudes $f_n[p'^2, (p'+k)^2, (p-k)^2, (p_1+p')^2, \mathbf{k}^2]$ and $f_p[\ldots]$ are independent of k_0' and k_0.

Indeed, the mass variables p'^2, $(p'+k)^2$, $(p_1-p')^2$, $(p_1-p'+q-k)^2$ can be replaced by m^2 with the same error μ/m:

$$(p-k)^2 = \mu^2 + k^2 - 2p_0k_0 + 2\mathbf{pk} \approx \mu^2 - \mathbf{k}^2 + 2\mathbf{pk},$$

if $|\mathbf{p}| \geq \mu$, except for the region of angles $\vartheta \sim \mu/m$. The variable $(p+p')^2$ has the form

$$(p+p')^2 = \mu^2 + m^2 + 2p_0\left(\frac{M}{2}+k_0'\right) - 2\mathbf{pk}' \approx \mu^2 + m^2 + p_0M = s_1$$

and similarly for f_p.

Thus we can carry out the integration over k_0 and k_0' in the integral (9). As a result we have

$$F_{np} = \frac{g^2}{4M^2}\int\frac{d^3k\,d^3k'}{(2\pi)^6}\frac{1}{(\Delta^2+\mathbf{k}'^2)[\Delta^2+(\mathbf{k}-\mathbf{k}'+\mathbf{q}/2)^2][\mu^2-(p-k)^2]}$$
$$\times\Gamma(\mathbf{k}')f_n(s_1,k^2,(p-k)^2)f_p(s_1,(\mathbf{q}-\mathbf{k})^2,(p-k)^2)\Gamma((\mathbf{k}-\mathbf{k}+\mathbf{q}/2)^2). \tag{11}$$

The integration over $\mathbf{k}'$ can be performed by comparing (11) with the expression for the deuteron form factor defined by the integral (6) if we replace in the latter f_n by $1/2$. If we integrate (6) over k_0 after this replace-

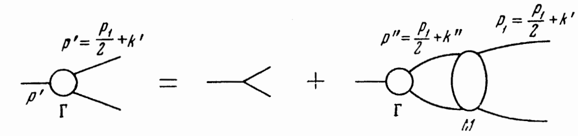

FIG. 3

ment, we obtain

$$\rho(\mathbf{q}^2)=\frac{g}{8M}\int\frac{d^3k'}{(2\pi)^3}\frac{\Gamma(\mathbf{k}')\Gamma(\mathbf{k}'-\mathbf{q}/2)}{[\Delta^2+\mathbf{k}'^2][\Delta^2+(\mathbf{k}'-\mathbf{q}/2)^2]}. \qquad (12)$$

Comparing (11) and (12), we find

$$F_{np}=\frac{2}{M}\int\frac{d^3k}{(2\pi)^3}\rho((2\mathbf{k}-\mathbf{q})^2)\frac{f_n(s_1,\mathbf{k}^2,(p-k)^2)f_p(s_1,(\mathbf{q}-\mathbf{k})^2,(p-k)^2)}{\mu^2-(p-k)^2}. \qquad (13)$$

Let us now turn to the basic question: under what conditions does (13) reduce to the usual expression containing the amplitudes f_n and f_p on the mass shell? For simplicity we set $q = 0$ and assume at first that owing to the presence of $\Gamma(p, p_1)$ in (13), values of $\mathbf{k}^2 \sim 1/R^2$ are important. Formula (13) can be written in the form

$$F_{np}=-\frac{1}{M}\int\frac{d\mathbf{k}^2}{(2\pi)^2}\rho(4\mathbf{k}^2)\int_{-|\mathbf{k}|}^{|\mathbf{k}|}dk_z$$
$$\times\frac{f_n(s_1,0,\mu^2-2|\mathbf{p}|k_z)f_p(s_1,0,\mu^2+2|\mathbf{p}|k_z)}{2|\mathbf{p}|k_z+i\varepsilon}, \qquad (14)$$

where we have used $\mathbf{k}^2 \sim 1/R^2$. It is seen from this that for $2|\mathbf{p}|k_z \sim 2|\mathbf{p}|/R \ll \mu^2$ one can restrict oneself to the pole contribution to the integral (14), and F_{np} is equal to

$$F_{np}=\frac{i\pi}{2Mp}\int\frac{d\mathbf{k}^2}{2\pi^2}f_n(s_1,0,\mu^2)f_p(s_1,0,\mu^2)\rho(4\mathbf{k}^2). \qquad (15)$$

Combining (15) and (8) for $q = 0$, adding the graph for the scattering on the second nucleon, and using the optical theorem, we obtain formula (1), where

$$\frac{2}{R^2}=\int d\mathbf{k}^2\rho(4\mathbf{k}^2). \qquad (16)$$

If $2|\mathbf{p}|/R \gtrsim \mu^2$, the dependence of f_n and f_p on the "mass" of the π meson $(p-k)^2$ becomes important. If f_n and f_p decrease with increasing $(p-k)^2$, then the contribution of graph a of Fig. 2 also decreases. This decrease sets in only for $|\mathbf{p}| \gg R\mu^2$.

Unfortunately, however, even if the condition $|\mathbf{p}| \ll R\mu^2$ holds for the deuteron, the Glauber result is semi-quantitative (even if R were very large, i.e., the binding energy of the deuteron very small). The point is that for $1/R^2 < 4\mathbf{k}^2 < \mu^2$ the deuteron form factor decreases slowly:

$$\rho(4\mathbf{k}^2)=\frac{2\Delta}{|\mathbf{k}|}\arcsin\frac{|\mathbf{k}|}{\sqrt{\mathbf{k}^2+4\Delta^2}} \qquad (17)$$

and hence, $\mathbf{k}^2 \sim \mu^2$ is important in (16) (this has been mentioned earlier). In this case $(p-k)^2 \approx 2|\mathbf{p}|k_z - \mathbf{k}^2 \sim -\mu^2$, i.e., the virtuality of the π meson can become important.

The Glauber result would be exact for a system of the type of the hydrogen molecule, where the probability that the two protons are at a close distance is exponentially small. Let us discuss briefly the contribution from the other graphs, for example the graph of Fig. 4, which includes the interaction of the nucleons in the deuteron between two collisions. If the contribution of the graph of Fig. 4 is calculated nonrelativistically, one can show, as before, that it is equal to zero (all poles are on one side of the real axis). The reason for this is that during the short time necessary for a fast particle to pass from one nucleon to another the slow nucleons

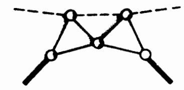

FIG. 4

in the deuteron cannot collide with each other and interact. The account of the retardation or the virtuality leads to a nonvanishing contribution of the graph of Fig. 4 of the order μ/m.

3. SCREENING AT VERY HIGH ENERGIES

Let us consider the graph of Fig. 2, b and write it in the form

$$F_{np}=-g^2\int\frac{d^4k\,d^4k'}{(2\pi)^8}\Gamma(\mathbf{k}'^2)f(s_1,\mathbf{k}^2,(\mathbf{q}-\mathbf{k})^2,\mathbf{q}^2,s')\Gamma(\mathbf{k}-\mathbf{q}/2-\mathbf{k}')$$
$$\times\{(\Delta^2+Mk_0'-\mathbf{k}'^2)(\Delta^2-Mk_0'-\mathbf{k}'^2)[\Delta^2-M(k_0+k_0')-(\mathbf{k}+\mathbf{k}')^2]$$
$$\times[\Delta^2+M(k_0+k_0'-q_0)-(\mathbf{k}+\mathbf{k}'-\mathbf{q})^2]\}^{-1}, \qquad (18)$$

where

$$s_1=(p+p_1/2)^2,\qquad s'=(p-k)^2=\mu^2-2p_0k_0+2|\mathbf{p}|k_z-k^2. \qquad (19)$$

The fact that the amplitude f corresponding to the graph of Fig. 5 is independent of the other variables is seen by taking the scalar products of the vectors p, q, and the vectors $p_1/2 + k' \pm (p_1/2 + k' + k)$, $p_1/2 - k' \pm (p_1/2 - k' - k + q)$, and taking account of the fact that k and k′ vary in the region discussed in the preceding section. In analogy to the previous discussion, we can integrate (18) over k_0 and k_0'. Indeed, if in (18), we introduce the variable s′ instead of k_z, we must substitute for k_z in (18)

$$k_z=\frac{s'-\mu^2+\mathbf{k}_\perp^2}{2|\mathbf{p}|}-k_0\frac{p_0}{|\mathbf{p}|}. \qquad (20)$$

Since k_z enters only in the form $k_z^2 + \mathbf{k}_\perp^2$, $(k_z - q_z)^2 - (\mathbf{k}_\perp - \mathbf{q}_\perp)^2$, we can neglect the term with k_0 even if the first term in (20) is small (in this case it can also be neglected).

After the integration we obtain

$$F_{np}=\frac{2}{M}\int\frac{d^2\mathbf{k}_\perp ds'}{(2\pi)^3 2|\mathbf{p}|}\rho((2\mathbf{k}-\mathbf{q})^2)f(s_1,s',\mathbf{k}^2,(\mathbf{q}-\mathbf{k})^2,\mathbf{q}^2) \qquad (21)$$

for

$$k_z=\frac{s'-\mu^2+\mathbf{k}_\perp^2}{2|\mathbf{p}|},\qquad q_z=\frac{\mathbf{q}^2}{2M}=0.$$

For $q = 0$ we can write, instead of (21),

$$F_{np}=\frac{1}{M}\int\frac{d\mathbf{k}^2}{4\pi^2}\rho(4\mathbf{k}^2)\int_{-2\mathbf{pk}+\mu^2}^{2\mathbf{pk}+\mu^2}\frac{ds'}{2|\mathbf{p}|}f(s_1,s',\mathbf{k}^2). \qquad (21a)$$

The function $f(s_1, s', \mathbf{k}^2)$ has the usual threshold singularities in s′ for positive and negative s′: $s' > s_{10}$, $s' < s_{20}$.

Let us consider the absorptive part δf of the function $f(s_1, s', \mathbf{k}^2)$ in s′. Using the unitarity condition, we

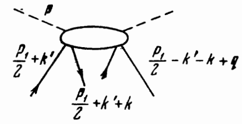

FIG. 5

can write δf for $q = 0$ in the form

$$\delta f=\frac{1}{9}\sum\int f_n(s_1+i\varepsilon, s'+i\varepsilon, k_1,\ldots,k_n)f_n(s_1+i\varepsilon, s'+i\varepsilon, k_1,\ldots,k_n)\,d\Gamma_n, \tag{22}$$

where f_n is the amplitude for the various inelastic processes on the nucleon corresponding to the graph of Fig. 6.

We want to show that for large energies

$$\delta f=-\frac{1}{9}\sum_n\int d\Gamma_n\,|f_n(s_1,s',k_1,\ldots,k_n|^2, \tag{23}$$

i.e., the amplitudes f_n are effectively pure imaginary at high energy. We do this using the assumption that the asymptotic behavior of the processes at large energies is determined by a Pomeranchuk pole and the cuts connected with it. It is very probable that the result is more general, but we have not been able to derive it in another way.

We note that for small **k** the momenta of all created particles $k_1, \ldots, k_n$ are large. Indeed, if the momenta of the particles $0 < |\tilde{k}_{iz}| < \sqrt{s'}$ in the system of the center of mass of the created particles, then in the laboratory system

$$k_{iz}\sim(\tilde{k}_{i0}-\tilde{k}_{iz})\frac{|\mathbf{p}|}{\sqrt{s'}}\gtrsim\frac{m_i^2}{2s'}|\mathbf{p}|\sim\frac{m_i^2}{4k_z}\sim\frac{m_i^2}{4}R\gg m_i.$$

If the inelastic processes at large energies are determined by the Pomeranchuk pole, then the graph of Fig. 6 can under these conditions (large k_{iz}), be replaced by the graph of Fig. 7, and correspondingly, the graph of Fig. 4 by the graph a of Fig. 8.

As shown in [8], the amplitude for the scattering of a reggeon on a particle entering in graph a of Fig. 8 depends only on s' and its absorptive part is determined by the usual unitarity condition. Since the reggeon propagators $[(-s)^\alpha + (s)^\alpha]/\sin\pi\alpha$ are pure imaginary ($\alpha = 1$), the two propagators contribute (-1) and we arrive at (23). The same holds if the Regge pole is replaced by two-reggeon or higher branches (graphs b and c of Fig. 8).

It is clear that owing to (23), the quantity δF is expressed through the differential cross section $d\sigma/d^3k$ for all possible processes occurring on the nucleon with small momentum transfer **k**:

$$\frac{d\sigma}{d^3k}=\frac{1}{8p_0m^2(2\pi)^3}\sum_n\int|f_n|^2d\Gamma_n=-\frac{\delta f}{4p_0m^2(2\pi)^3}. \tag{24}$$

Since δf is real, f is also real in the interval from s_{20} to s_{10}. Therefore the imaginary part of F_{np} is determined by the integral over δf. The contribution from the right cut corresponds to the interaction with one nucleon, and that from the left cut, to the interaction with the other nucleon. Since the cross sections for the interaction with the neutron and the proton are the same, we can write

$$\operatorname{Im}F_{np}=\frac{2}{M}\int\frac{d\mathrm{k}^2}{4\pi^2}\rho(4\mathrm{k}^2)\int\frac{ds'}{2|\mathbf{p}|}\delta f(s_1,s',\mathrm{k}^2) \tag{25}$$

and hence,

$$\sigma_t=2\sigma_N-2\int d\mathrm{k}^2\rho(4\mathrm{k}^2)\frac{d\sigma}{d\mathrm{k}^2}. \tag{26}$$

This result has been discussed in detail in the Introduction.

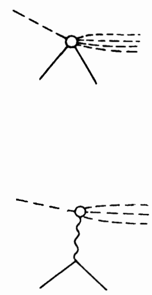

FIG. 7

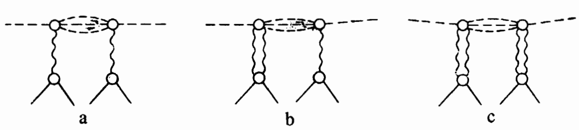

FIG. 8

The accuracy of formula (26) is determined by the value of the effective real part of the amplitude for inelastic processes. If the form factor of the deuteron would fall off rapidly for $q^2 > 1/R^2$ and the radius of the deuteron R were very large, then the corrections to (26) determined by the minimal energy of the created particles m_i^2R would be very small. In reality, however, $k^2 \sim \mu^2$ is important in (26), i.e., effectively $R \sim 1/\mu$ and hence, the accuracy of (26) can be very poor. A real estimate of the accuracy of (26) requires a better knowledge of the properties of inelastic processes than is available at present.

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