

## QUALITATIVE STREAMER THEORY

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A simplified calculation of streamer development is proposed, based on the model of an ideally conducting plasma produced on the boundaries by electrons moving in and out of a plasma. The main conclusions are that the streamer propagation velocity is approximately proportional to its length and the streamer thickness is proportional to the square root of the length; this is in good agreement with experiment<sup>[4]</sup>. The plasma density and the field strength  $E'$  in the streamer are also estimated. It is shown that  $E'$  is much smaller than the applied field, thus confirming the assumption that the plasma has ideal conductivity.

**A**N electron moving in a gas in a homogeneous electric field  $E_0$  and ionizing the gas, produces a cascade of electrons, whose dimension  $r_0$  is determined by the mobility and diffusion of the electrons, so that

$$N = e^{\alpha z}, \quad r_0 = 2\sqrt{Dz / KE_0}, \quad (1)$$

where  $N$  is the number of electrons in the cascade,  $\alpha$  is the first Townsend coefficient (the probability of ionization per unit length),  $z$  is the path traversed by the initial electron,  $D$  is the diffusion coefficient, and  $K$  is the electron mobility.

At gas pressures on the order of atmospheric, at discharge-gap dimensions on the order of  $\sim 1$  cm or more, and at intensities  $E_0$  such that gas breakdown is produced,  $K$  is practically independent of  $E_0$  in a wide range of variation of  $E_0$ ,  $D$  increases slowly, and  $\alpha$  increases very rapidly with increasing  $E_0$ . In air under these conditions, a 10% change of  $E_0$  changes  $\alpha$  by more than a factor of 2.

The electron cascade leaves behind it a trail of ions, the majority of which are adjacent to the cascade over a length  $\sim \alpha^{-1}$ . A positive ion, reaching the cathode with a probability  $\gamma$ , produces an electron (potential ion-electron emission) if the gas ionization potential exceeds double the work function. At pressures on the order of atmospheric,  $\gamma$  is usually very small (in air, with a copper cathode,  $\sim 10^{-4}$ ). If the condition

$$\gamma e^{\alpha d} \geq 1, \quad (2)$$

is satisfied, where  $d$  is the length of the discharge gap, then on the average each passing cascade of electrons gives rise to a new cascade, and Townsend breakdown of the gas sets in. The time of development of such a breakdown in of the order of the time of motion of the ions from the anode to the cathode ( $t \sim 10^{-4}$  sec for  $d = 1$  cm in air).

Photons with an energy exceeding the gas ionization energy cannot go in practice outside of the confines of the cascade, owing to the large absorption coefficient ( $\sim 10^3$  cm<sup>-1</sup> in air). The same pertains also to photons connected with transitions of excited atoms to the ground state (their absorption coefficient in air is  $10^6$  cm<sup>-1</sup> in the center of the line). Thus, the photoionization of the cathode (with the exception of special cathodes with small work functions) cannot play any role in the breakdown of a gas at high pressures. Another possible process is the detachment of the

electrons from the cathode by the excited atoms (metastable and those excited at the first level), but the time of their motion to the cathode is too long<sup>[1]</sup>.

Nevertheless, a very insignificant fraction of the photons connected with the transitions of the excited atoms to the ground level can emerge far beyond the limits of the cascade, owing to the wings of the spectral line. This part of the radiation is absorbed in the gas in accordance with the law  $\sim r^{-7/2}$ , where  $r$  is the distance from the source. Owing to the ion-molecular chemical reaction



which was considered in detail in<sup>[2]</sup>, such an indirect photoionization of the gas occurs far from the cascade<sup>[3]</sup>, and it replaces in the gas breakdown the mechanism of the ion-electron emission in the case of a slight overvoltage. In this case the breakdown develops during a time on the order of several times the time of flight  $t$  of the electron from the cathode to the anode ( $t \sim 10^{-7}$  sec at  $d = 1$  cm in air). Owing to the low intensity of the considered photoionization of the gas, the character of the breakdown changes qualitatively. The number of electrons in the cascade becomes so large, that the space-charge field becomes comparable with the applied field  $E_0$ . Regions with enhanced field and with strongly increased values of  $\alpha$  appear for and behind the cascade, and they are "broken down" independently. On the anode side, the leading front of the electrons breaks through forward, strongly ionizing the gas and continuing the high-conductivity region of quasineutral plasma. On the cathode side, the few of photoionization electrons that enter into the region of the strong field strongly ionize the gas and leave behind them a region of a highly conducting quasineutral plasma.

Thus, a conducting channel grows from the cascade on both sides, along the applied field, and carries on its ends a region of field  $E$  with ever increasing strength. The boundary of the conducting region moves in the direction of the resultant field on the positive end and against the field on the negative end, with velocity  $KE$ , until the discharge gap is short circuited. Such a formation is called a streamer, and the breakdown associated with it is called streamer breakdown. Actually, at high pressures the Townsend discharge usually goes over into a streamer discharge as a re-

sult of the overvoltage accompanying the appearance of the space charge.

### 1. MODEL OF IDEALLY CONDUCTING STREAMER WITH SHARP BOUNDARY

Assume that we have an ideally conducting and consequently equipotential region of the form of an ellipsoid of revolution that is elongated along the field  $E_0$ . Then the field intensity  $E_a$  at the ends of this region is approximately equal to

$$E_a = E_0 a / R \ln \left( \frac{2}{c} \sqrt{\frac{a}{R}} \right), \quad e = 2.718 \dots \quad (4)$$

where  $a$  is half the streamer length and  $R$  is the radius of curvature of its ends. In the limit, if the streamer were to have the form of two spheres with radius  $R$  connected by an infinitesimally thin filament of length  $2a$  ( $a \gg R$ ), then we would get  $E_a = E_0 a / R$ .

If the streamer is indeed elliptical, then the field intensity on its surface, and consequently also the velocity of the points of this surface, are proportional to the cosine of the angle between the normal to the surface and the direction of the field  $E_0$ . They are obviously normal to the surface. But in the case of parallel displacement of the end of the streamer along the field  $E_0$  the normal component of the velocity of the points of the surface is also proportional to the cosine of this angle. Consequently, from the equation of motion of the surface  $\Phi(z, \rho) = \text{const}$

$$\dot{z}_1 = \pm K E_z, \quad \rho_1 = \pm K E_\rho, \quad (5)$$

(where  $\Phi$  is the potential of the ellipsoid,  $z$  and  $\rho$  are cylindrical coordinates, and the plus and minus signs pertain to the anode and cathode ends, respectively) it follows that the two halves of the ellipsoid of revolution will move along the field in opposite directions. This means that if the field at the ends of the streamer is close to ellipsoidal, then the radius of curvature of its ends does not change during the development process and is approximately equal to the radius of the cascade from which it was produced. When  $a \gg R$ , the rate of growth of the streamer, according to (5) and (4), is

$$|\dot{a}| = K E_0 a / R \ln \left( \frac{2}{c} \sqrt{\frac{a}{R}} \right), \quad (6)$$

i.e., it is approximately proportional to its length, and the dependence of its length on the time is given by the formula

$$a \approx \exp\{(K E_0 t / R + C)^{1/2}\}, \quad (7)$$

where  $C$  is a constant that depends on the initial conditions.

The exact solution of the problem of the "true" figure of the streamer is very complicated. The surface of the streamer is determined by the equation

$$\left( \frac{\partial \rho_1}{\partial t} \right)_z = \dot{\rho}_1 - \frac{\partial \rho_1}{\partial z_1} \dot{z}_1 = \pm K \left( E_\rho - \frac{\partial \rho_1}{\partial z_1} E_z \right) = \pm K \frac{E_z^2 + E_\rho^2}{E_\rho}, \quad (8)$$

since

$$\frac{\partial \rho_1}{\partial z_1} = - \frac{\partial \Phi}{\partial z} \Big/ \frac{\partial \Phi}{\partial \rho} = - \frac{E_z}{E_\rho}$$

for a surface with a constant potential  $\Phi$ . Further,  $\partial \rho / \partial a = a^{-1} \partial \rho / \partial t$ , and

$$\left( \frac{\partial \rho}{\partial a} \right)_z = \frac{E_\rho^2 + E_z^2}{E_a E_\rho}. \quad (9)$$

The field components themselves are determined, for a specified external field  $E_0$ , from the solution of the Laplace equation with boundary conditions: potential  $\Phi = \text{const}$  on the surface of the streamer and  $\nabla \Phi \sim E_0$  at large distances from the streamer.

It should also be noted that the solutions for the equation of the surface are unstable against small-scale perturbations. Experiment has shown, however, that in the initial stage the streamer develops as a stable formation, and begin to bend and branch only later. The temporal stability of the streamer is apparently connected with the finite density of the plasma, and consequently with its finite conductivity and finite thickness of the "surface charge," and also with diffusion. When the thickness of the "surface charge" becomes much smaller than the radius of curvature in the case of ideal conductivity, the streamer becomes unstable against branching and bending, or against the growth of a thinner and faster streamer from its end. Therefore, until a more accurate theory is developed, it is advantageous (in order to estimate the streamer thickness) to calculate at least approximately the shape of its surface.

It follows from (9) that with increasing  $a$  the value of  $\rho$  increases everywhere except in the symmetry plane  $z = 0$ , since the potential on this entire plane is the same as on the surface, and the field intensity at the point of intersection of the streamer surface and the streamer plane is identically zero. To estimate this growth we can write in place of (9)

$$\frac{\partial \ln \rho}{\partial a} = \frac{E_\rho^2 + E_z^2}{E_a E_\rho} \frac{1}{\rho} \quad (9a)$$

and substitute in the right side of (9a) the values of  $E_\rho$  and  $E_z$  for an ellipse having a major axis equal to  $a$ . The right side of (9a) then depends only on one parameter of the ellipsoid of revolution—on  $a$ . Expression (9a) is used in place of (9) in order to obtain a result expressed in terms of elementary functions.

Assuming that the streamer was initially a sphere with radius  $R$ , and that  $R$  remains constant on the ends of the streamer as the streamer develops ( $\rho = 0$ ,  $z = \pm a$ ), and subsequently the surface differs little from an ellipsoid of revolution, we obtain from (9a)

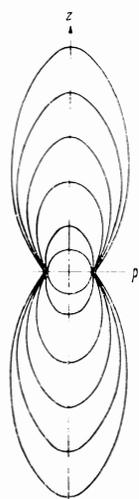
$$\rho|_{z < R} = \rho_0 \exp \left( \int_R^a \frac{E_z^2}{E_a E_\rho} \frac{da}{\rho} \right) = (R + z) \sqrt{\frac{a - z}{a + z}}; \quad (10)$$

$$\begin{aligned} \rho|_{z > R} &= 2 \sqrt{Rz} \sqrt{\frac{a - z}{a + z}} = \\ &= \sqrt{aR} \sqrt{1 - \frac{z^2}{a^2}} \sqrt{\frac{4z/a}{(1 + z/a)^2}}. \end{aligned} \quad (10a)$$

The greater part of the obtained surface hardly differs from the ellipsoid of revolution  $\rho = \sqrt{aR(1 - z^2/a^2)}$ . Successive cross sections of the obtained surfaces are shown in the figure.

A similar calculation for

$$\frac{\partial \rho^2}{\partial a} = 2 \frac{E_\rho^2 + E_z^2}{E_a E_\rho} \rho \quad (9b)$$



Development of the streamer in accordance with the considered model.

gives a very similar result. A noticeable difference is obtained only in the vicinity of the "neck" (see the figure). In this case

$$\rho_{z < R} = \left( R^2 + 2Rz \ln \frac{a}{R} - z^2 \right)^{1/2}, \quad \rho_{z > R} = \left( 2Rz \ln \frac{a}{z} \right)^{1/2}.$$

Thus, we find that the thickness of the streamer increases in proportion to the square root of its length:

$$\rho_{max} \approx 0.83 \sqrt{aR} \quad (z = 0.415 a) \quad (11)$$

and consequently it can assume large values. However, the eye or a photographic camera with a film of insufficient sensitivity will detect the total luminosity at the maximum with thickness  $\sim R$ , and the total thickness of the streamer (11) will be recorded only if the photographic film is sufficiently sensitive<sup>[4]</sup>. In conclusion we note that if the speeds of the positive or negative end of the streamers are not equal, then the neck of the streamer becomes smeared out or disappears completely.

## 2. ALLOWANCE FOR THE FINITE CONDUCTIVITY IN THE STREAMER

If the field intensity inside in the streamer is  $E'$ , then the field intensity on its end will be

$$E_a = (E_0 - \bar{E}') a / R \ln \left( \frac{2}{e} \sqrt{\frac{a}{R}} \right) + E'$$

where  $\bar{E}'$  is the average value of  $E'$ , and the surface charge is  $\sigma = (E - E') / 4\pi$ .

For simplicity we assume that  $E' \ll E_0$  and neglect  $E'$  compared with  $E_0$ , and all the more compared with  $E_a$ . We have  $E' = j / \lambda$ , where  $j$  is the current density and  $\lambda = \epsilon n K$  is the conductivity ( $n$  and  $\epsilon$  are the electron concentration and charge, respectively). Furthermore

$$j = \frac{\partial \sigma}{\partial t} = \frac{1}{4\pi} \frac{dE_a}{dt};$$

$dE_a/dt$  is the change of the field on the end of the streamer per unit time,  $dE_a/dt = E_a \dot{a}$ .

Thus, in accordance with (6), we have

$$j = \frac{K}{4\pi a} E_a^2 = KE_0^2 a / 4\pi R^2 \ln^2 \left( \frac{2}{e} \sqrt{\frac{a}{R}} \right), \quad (12)$$

$$E' = \frac{j}{\lambda} = \frac{E_a^2}{4\pi n \epsilon a}. \quad (13)$$

It can be shown that when the streamer propagates, the energy released in a unit volume for ionization, excitation, etc. is  $E^2/8\pi$ . Assume that the fraction due to ionization is  $\theta E^2/8\pi$ . The value of  $\theta$  is known if the Townsend ionization coefficient  $\alpha$  is known. In the absence of space charge, the fraction of the energy going to ionization is  $\alpha U/E$ , where  $U_i$  is the ionization potential; it changes from  $\sim 5 \times 10^{-3}$  at  $E = 3 \times 10^4$  V/cm to  $\sim 0.2$  for  $E = 3 \times 10^5$  V/cm and then decreases somewhat with increasing  $E$ . In the presence of a streamer, the electrons move in different fields, and the value of  $\theta$  is determined from the formula

$$\theta = \frac{8\pi U_i \int_0^E \alpha(E) K E dE}{E^2 K E} = \frac{2U_i \alpha(E)}{B + 2E}. \quad (14)$$

The value of  $B$  is determined from the relation  $\alpha \approx A e^{-B/E}$ . For air at atmospheric pressure  $B \approx 200$  kV/cm. From this we find from (14) that  $\theta \approx 0.1$  and changes strongly with changing  $E$ , and since  $\theta E^2/8\pi = n U_i \epsilon$ , it follows that

$$E' = 2U_i / \theta a. \quad (15)$$

For air, even when  $a = 0.1$  cm, we get  $E' = 3$  kV/cm, amounting to 10% of the breakdown field intensity. Thus, the larger  $a$ , of the better the condition  $E' \ll E_0$  is satisfied at the end of the streamer. However, according to (13), the current density increases in proportion to  $a$ , but at each point on the streamer  $\lambda$  remains constant in first approximation, leading ultimately to a field intensity comparable with the applied field, and to development of ionization inside the streamer.

At a sufficiently large streamer length, when  $n$  reaches a value on the order of 1% of the gas concentration, Spitzer conductivity<sup>[5]</sup>, which does not depend on the further increase of the concentration  $n$ , sets in. If the streamer were to remain stable, then the Spitzer conductivity in air would set in at a  $\approx 10$  cm. If the streamer becomes unstable at a  $< 10$  cm, then such a conductivity can occur earlier.

The presence of finite conductivity apparently is the main cause of the temporal stability of the streamer. In fact, the growth of a thinner streamer from the head of the main streamer is accompanied by a sharp increase of the current density, whereas the conductivity cannot follow the growth of the current. This causes a drop in field intensity at the head of the thin new streamer, and hinders its further development.

<sup>1</sup>J. M. Meek and J. D. Craggs, *Electrical Breakdown of Gases*, Oxford, 1953.

<sup>2</sup>E. D. Lozanskiĭ, *Dokl. Akad. Nauk SSSR* 183, 315 (1968) [*Sov. Phys.-Dokl.* 13, 1134 (1969)].

<sup>3</sup>E. D. Lozanskiĭ, *Zh. Tekh. Fiz.* 38, 1563 (1968) [*Sov. Phys.-Tech. Phys.* 13, 1269 (1969)].

<sup>4</sup>V. A. Davidenko, B. A. Dolgoshein, and S. V. Somov, *Zh. Eksp. Teor. Fiz.* 55, 435 (1968) [*Sov. Phys.-JETP* 28, 227 (1969)].

<sup>5</sup>L. Spitzer, *Physics of Fully Ionized Gases*, Interscience, 1956.