THE PRODUCTION OF LEPTON PAIRS BY HIGH-ENERGY NEUTRINOS IN THE FIELD OF A STRONG ELECTROMAGNETIC WAVE

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The production of e^-e^+ and $\mu^-\mu^+$ pairs in the scattering of neutrinos in the field of a plane monochromatic electromagnetic wave is considered. Formulas are derived for the probabilities of these processes for the case in which the product of the energy of the initial lepton, in units of the rest energy of the lepton, times the field strength in the wave, in units of the critical field strength for the lepton, is much larger than unity. The entire treatment is conducted in the limit opposite to that in which perturbation theory in terms of the external electromagnetic field can be applied.

1. INTRODUCTION

THE theory of the universal weak interaction proposed by Gell-Mann and Feynman[1] predicts direct scattering of neutrinos by electrons and muons. Effects considered previously for the investigation of this phenomenon are the elastic scattering of neutrinos by electrons,[1] the deceleration production of lepton pairs in the scattering of neutrinos by nuclei,[2-4] and the deceleration production of neutrino-antineutrino pairs in the scattering of electrons in the fields of nuclei.[5]

Since the cross sections for processes caused by the weak interaction increase with increasing energy of the initial particles, it is natural to consider the processes mentioned at high energies. But despite the fact that at initial particle energies of the order of 10 GeV the cross sections are fairly large ($\approx 10^{-40}$ cm^2) the numbers of recoil electrons or of lepton pairs produced per unit time in these processes are still below the level that is observable. Further increase of the numbers of events in the processes we have mentioned calls for larger energies and intensities of the initial neutrinos than are available at the present time, and also for an increase in the interaction volume.

The development of lasers giving large electromagnetic field strengths allows a different approach to the problem of the direct lepton-neutrino scattering. In the field of a laser beam the processes

$$\nu_e \to \nu_e + e^- + e^+, \qquad (1)$$

$$\nu_\mu \to \nu_\mu + \mu^- + \mu^+, \qquad (2)$$

are allowed, and they will be studied in the present paper.

Baĭer and Katkov[6] have previously considered the process

$$e^- \to e^- + \nu_e + \bar{\nu}_e \qquad (3)$$

in a constant magnetic field. This process, however, is subject to such strong competition from the electrodynamic process

$$e^- \to e^- + \gamma, \qquad (4)$$

that the intensities of the two processes become comparable only at initial electron energies of the order of 10^{32} eV if the magnetic field strength is of the order of 10^4 Oe.[6]

Our processes (1) and (2) are not subject to competition. Moreover, in the case of the plane electromagnetic wave of a laser the present attainable field strength in units of the critical field m^2/e, where m is the mass of the electron, is larger than the corresponding quantity in the case of a constant magnetic field by a factor of 10^2 to 10^3.

As will be shown below, the probabilities of processes (1) and (2) increase with increasing initial neutrino energy and increasing field strength in the wave. Therefore in these processes the number of events per unit volume and time can be increased not only by raising the energy and intensity of the neutron beam, as is the case for the processes considered in[1-4], but also by increasing the field strength.

The probabilities of processes (1) and (2) will be derived in the second section. In the third section we shall discuss the domains of applicability of the formulas, as to the field strength in the wave, and give estimates of the probabilities of these processes for the energies and intensities of neutrino beams from reactors and accelerators.

2. THE TOTAL PROBABILITIES OF PROCESSES (1) AND (2)

We shall describe the weak interaction of neutrinos with leptons in the framework of the four-fermion theory, starting from the V – A version. The interaction Hamiltonian is

$$H = \frac{G}{\sqrt{2}}(\bar{\psi}_l\gamma_\mu(1+\gamma_5)\psi_\nu)(\bar{\psi}_\nu\gamma_\mu(1+\gamma_5)\psi_l), \qquad (5)$$

where ψ_l is the field operator of the electron or muon, ψ_ν is the corresponding neutrino field operator, and G is the universal weak interaction constant; in the system of units $\hbar = c = 1$ we have $G = 10^{-5}/m_p^2$, where m_p is the mass of the proton. We set $G_V = G_A = G$.

Using the Nikishov-Ritus method developed in[7,8], we can write the matrix element for process (1) in the form

$$M=-\frac{G}{2\sqrt{E_\nu}}\sum_{s=-\infty}^{+\infty}\Big\{\bar u(p_1)\Big[A_0\gamma_\mu+e\Big(\frac{\hat a\hat k}{2qk}\gamma_\mu-\gamma_\mu\frac{\hat k\hat a}{2q'k}\Big)A_1$$
$$+e^2a^2\frac{\hat k}{2(qk)(q'k)}k_\mu A_2\Big](1+\gamma_5)v(-p_2)\Big\}$$
$$\times\{\bar u(q_2)\gamma_\mu(1+\gamma_5)u(q_1)\}(2\pi)^4\delta(sk+q_1-q-q'-q_2). \quad (6)$$

Here p_1 and p_2 are the four-momenta of the electron and positron, and q_1 and q_2 are the four-momenta of the initial and final neutrinos; the four-potential of the plane electromagnetic wave is of the form $A_\mu = a_\mu e^{-ikX}$, where k is the wave four-vector satisfying the condition $k^2 = 0$[1)] and E_ν is the energy of the initial neutrino. The expressions for the functions A_0, A_1, and A_2 can be obtained for n = 0, 1, 2 from the general formula

$$A_n(s,\alpha,\beta)=\frac{1}{2\pi}\int_{-\pi}^{+\pi}e^{f(\varphi)}\cos^n\varphi d\varphi, \quad (7)$$

in which

$$f(\varphi)=i\alpha\sin\varphi-i\beta\sin2\varphi+is\varphi. \quad (8)$$

The parameters α and β are as follows;

$$\alpha=-e\Big\{\frac{qa}{qk}+\frac{q'a}{q'k}\Big\},\quad \beta=-\frac{e^2a^2}{8}\Big\{\frac{1}{qk}+\frac{1}{q'k}\Big\}, \quad (9)$$

where $e^2/4\pi = 1/137$ and

$$q_\mu=p_{1\mu}-\frac{e^2a^2}{4(p_1k)}k_\mu,\quad q_\mu'=p_{2\mu}-\frac{e^2a^2}{4(p_2k)}k_\mu. \quad (10)$$

In Eqs. (6)–(8) φ = kX, and s is the number of photons with momentum k that are absorbed from the wave or emitted into the wave.

In what follows we shall carry out all of the calculations for process (1). To obtain the probability for process (2) from that for process (1) we must make the replacement $m \to m_\mu$, where m_μ is the mass of the muon.

Summing the square of the matrix element (6) over the polarizations of the final and initial particles and using the fact that the functions A_n are real, which follows from (7) and (8), we get the differential probability per unit volume and time in the form

$$dW=\frac{G^2n}{2^6\pi^5E_\nu}\sum_{s=-\infty}^{+\infty}\delta(sk+q_1-q-q'-q_2)T_{\mu\nu}R_{\mu\nu}d\tau_3, \quad (11)$$

where

$$d\tau_3=\frac{d\mathbf{q}_2}{q_{20}}\frac{d\mathbf{q}}{q_0}\frac{d\mathbf{q}'}{q_0'}, \quad (12)$$

the tensor $R_{\mu\nu}$ is of the form

$$R_{\mu\nu}=q_{2\mu}q_{1\nu}+q_{2\nu}q_{1\mu}-(q_1q_2)g_{\mu\nu}+i\varepsilon_{\alpha\beta\mu\nu}q_{2\alpha}q_{1\beta}, \quad (13)$$

and n is the number density of incident neutrinos.

The tensor $T_{\mu\nu}$ in Eq. (11) can be written in the following way:

$$T_{\mu\nu}=4A_0^2b_{\mu\nu}^{(1)}+4eA_0A_1b_{\mu\nu}^{(2)}+2e^2(2A_1^2b_{\mu\nu}^{(3)}+A_0A_2b_{\mu\nu}^{(4)})$$
$$+2e^3A_1A_2b_{\mu\nu}^{(5)}+e^4A_2^2b_{\mu\nu}^{(6)}, \quad (14)$$

where the tensors $b_{\mu\nu}^{(1)}$–$b_{\mu\nu}^{(6)}$ are given in the Appendix.

In the Nikishov-Ritus method one introduces the parameter $x = e|a|/m$[7]; if $x \ll 1$ it is possible to take the external electromagnetic field into account by means of perturbation theory, and $x \gg 1$ is the opposite case. Since $x = mB/\omega B_0$, where ω is the frequency of the field, $B = \omega|\mathbf{a}|$ is the field-strength amplitude, and $B_0 = m^2/e$, for $B \approx 10^{-5}B_0$ and $\omega \approx 10^{15}$ sec^{-1} we have $x \approx 10$. Consequently, for conditions such as one now has in laser technique, we can consider the asymptotic probability of process (1) for $x \gg 1$.

Let us introduce the new variables

$$\chi=\frac{qk}{m^2}x,\quad \chi'=\frac{q'k}{m^2}x,\quad \varkappa=\frac{q_1k}{m^2}x,$$
$$\cos\psi=\frac{\alpha}{8\beta},\quad \cos\xi=\frac{e}{8\beta}\Big\{\frac{qa}{qk}-\frac{q'a}{q'k}\Big\}. \quad (15)$$

It is convenient to make the further calculations in a so-called "special" reference system, which we choose in the following way: axis 1 is directed along $\mathbf{q}_1$ and axis 3 along $\mathbf{k}$, the vector $\mathbf{a}$ is directed along axis 1 and the magnetic field along axis 2, and $a_0 = 0$. Then in the "special" reference system

$$\cos\psi=\frac{\chi q_x'+\chi' q_x}{mx(\chi+\chi')},\quad \cos\xi=\frac{\chi q_x'-\chi' q_x}{mx(\chi+\chi')}. \quad (16)$$

Let us introduce two more variables in this reference system:

$$\tau=\frac{\chi q_y'+\chi' q_y}{m(\chi+\chi')},\quad \eta=\frac{\chi q_y'-\chi' q_y}{m(\chi+\chi')}. \quad (17)$$

Using Eqs. (15)–(17), we can write s for $x \gg 1$ in the following form:

$$s=\frac{x^3(\chi+\chi')}{4\chi\chi'}\Big\{1+2\cos^2\xi+\frac{\varkappa(\chi+\chi')^2}{2\chi\chi'(\varkappa-\chi-\chi')}\Big(\cos\psi$$
$$-\frac{\chi-\chi'}{\chi+\chi'}\cos\xi\Big)^2+\frac{2}{x^2}\Big[\eta^2+\frac{\varkappa(\chi+\chi')^2}{4\chi\chi'(\varkappa-\chi-\chi')}\Big(\tau-\frac{\chi-\chi'}{\chi+\chi'}\eta\Big)^2\Big]\Big\}. \quad (18)$$

Since it follows from (18) that $s \sim x^3$, using (15)–(17) for $x \gg 1$ we can write

$$\sum_{s=-\infty}^{+\infty}\delta(sk+q_1-q-q'-q_2)d\tau_3\to\frac{m^2x^3(\chi+\chi')^4}{4\chi^3\chi'^3(\varkappa-\chi-\chi')}$$
$$\times\sin\psi\sin\xi\,d\chi\,d\chi'\,d\psi\,d\xi\,d\tau\,d\eta. \quad (19)$$

Making use of the conservation laws in the "special" reference system, which are here of the form

$$\varkappa=\lambda+\chi+\chi',\quad E_\nu=q_{2x}+q_x+q_x',\quad 0=q_{2y}+q_y+q_y',$$

where $\lambda = eB(q_{20} - q_{2z})/m^3$, we can put the probability for process (1) in the following form:

$$W=\frac{G^2m^2n}{2^8\pi^5E_\nu}\int_{\chi_{min}}^{\varkappa-\chi'_{min}}d\chi\int_{\chi'_{min}}^{\varkappa-\chi}d\chi'\int_{-\pi/2}^{+\pi/2}d\psi\sin\psi$$
$$\times\int_0^\pi d\xi\sin\xi\int_{-\infty}^{+\infty}d\tau\int_{-\infty}^{+\infty}d\eta\frac{x^3(\chi+\chi')^4}{\chi^3\chi'^3(\varkappa-\chi-\chi')}w. \quad (20)$$

Because it is cumbersome the function w, which depends on the variables of integration in (20), is not given here.

In Eq. (20) let us consider the asymptotic behavior for $x \gg 1$. From (15) and the conservation laws it is not hard to show that $\chi_{min} \sim \kappa^{-1}$, $\chi'_{min} \sim \kappa^{-1}$, where the proportionality constants are of the order of unity. Using these relations, Eqs. (9), (15), (18), (20), and also the relations (A.2) and the asymptotic behavior of the function A_0 for $x \gg 1$, which were derived in[7],

[1)]We use the Feynman metric $ab = a_0b_0 - \mathbf{a}\cdot\mathbf{b}$, with $a = (a_0, \mathbf{a})$.

and keeping only terms in $\kappa^2 \ln \kappa$, we have the following expression for the probability of process (1) per unit volume and time for $x \gg 1$ and $\kappa \ll 1$:

$$W = \frac{29}{27} \frac{G^2 m^6}{2^9 \pi^4} \frac{n}{E_\nu} \varkappa^2 \ln \varkappa. \tag{21}$$

Using the formula for κ in (15) and introducing the electromagnetic field tensor $F_{\sigma\rho}$, we can write in an arbitrary reference system

$$\varkappa = \frac{e}{m^3} \sqrt{(F_{\sigma\rho} q_{1\rho})^2}. \tag{22}$$

It follows from (22) that the probability of process (1), written in the form (21), is relativistically invariant and gauge invariant.

Making the replacement $m \to m_\mu$, we get from (21) the probability of process (2). It is not hard to see that when we use (22) the expression for process (2) differs from (21) by the replacements $m \to m_\mu$ and $\kappa \to \kappa_\mu$, where $\kappa_\mu = e[(F_{\sigma\rho} q_{1\rho})^2]^{1/2}/m_\mu^3$. But the condition $\kappa_\mu \gg 1$, required for the asymptotic formula to hold for the probability in the case of process (2), is realized at field strengths $(m_\mu/m)^3$ times larger than for the case of process (1), if the quantity E_ν is the same for both processes and for process (1) $\kappa \gg 1$.

3. DISCUSSION OF THE RESULTS

Using Eqs. (21) and (22) and going over to the "special" reference system and to the ordinary system of units, we can write for the probability of process (1)

$$W = 2.2 \cdot 10^{-15} \left(\frac{m}{m_p}\right)^5 \frac{I_\nu}{\lambda\!\!\!^{-}_p} \left(\frac{B}{B_0}\right)^2 \left(\frac{E_\nu}{mc^2}\right) \ln\left(\frac{E_\nu}{mc^2} \frac{B}{B_0}\right), \tag{23}$$

where I_ν is the flux density in the neutrino beam and $\lambda\!\!\!^{-}_p$ is the Compton wavelength of the proton.

The corresponding quantity for process (2) is obtained from (23) by the replacements $m \to m_\mu$ and $B_0 \to B_0^{(\mu)}$, where $B_0^{(\mu)} = (m_\mu/m)^2 B_0$. It can be seen from (23) that the probability of process (1) increases with increasing field strength. But there is an upper bound on the values of B, because the one-particle treatment of a charged lepton in the field of the plane electromagnetic wave, as used above, is valid only for $B \lesssim B_0$ for the electron and for $B \lesssim B_0^{(\mu)}$ for the muon.

Let us consider process (1) for electronic neutrinos from a reactor. In this case $I_\nu \sim 10^{13}$ neutrinos/cm²sec with $E_\nu \sim 5$ MeV and $\kappa \gg 1$ give $B \gtrsim 0.5 B_0$, while for such values of the field strength $x \approx 10^6$. Then with $B \approx 0.5 B_0$ and the given E_ν and I_ν we get for the probability of process (1) $W \sim 2 \cdot 10^{-4}$ events/cm³sec. In the case of the muon process we use the data of [9] for the Serpukhov accelerator at 70 GeV: $I_\nu \sim 10^4$ neutrinos/cm²sec with $E_\nu \sim 5$ GeV. Then the condition $\kappa_\mu \gg 1$ is satisfied with $B \gtrsim 0.1 B_0^{(\mu)}$, and in this case $x \approx 10^7$. Using the given values of E_ν and I_ν, we get for the probability of process (2) with $B \approx 0.1 B_0^{(\mu)}$ the value $W \sim 10^{-2}$ events/cm³ sec.

Since the interaction volume in these processes is of the order of 1 cm³, these estimates show that the numbers of lepton pairs produced per unit time in both process (1) and process (2) are sufficiently large. The field strengths, however, that are required for this and that correspond to the conditions for applicability of the asymptotic formulas for the probabilities, are many orders of magnitude larger than those now available.

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APPENDIX

The tensors $b_{\mu\nu}^{(1)}$--$b_{\mu\nu}^{(6)}$ which appear in Eq. (14) are given by the following expressions:

$$b_{\mu\nu}^{(1)} = p_{1\mu} p_{2\nu} + p_{1\nu} p_{2\mu} - (m^2 + p_1 p_2) g_{\mu\nu} + i\varepsilon_{\rho\sigma\mu\nu} p_{1\rho} p_{2\sigma}; \tag{A.1}$$

$$\begin{aligned} b_{\mu\nu}^{(2)} = {} & (p_1 - p_2)_\mu a_\nu + (p_1 - p_2)_\nu a_\mu + k_\mu \left(\frac{qa}{qk} p_{2\nu} - \frac{q'a}{q'k} p_{1\nu}\right) \\ & + k_\nu \left(\frac{qa}{qk} p_{2\mu} - \frac{q'a}{q'k} p_{1\mu}\right) - g_{\mu\nu} (qk + q'k) \left(\frac{qa}{qk} - \frac{q'a}{q'k}\right) \\ & + i\varepsilon_{\rho\sigma\mu\nu} \left\{ (p_1 + p_2)_\rho a_\sigma - k_\sigma \left(\frac{q'a}{q'k} p_{1\rho} + \frac{qa}{qk} p_{2\rho}\right) \right\}; \end{aligned} \tag{A.2}$$

$$\begin{aligned} b_{\mu\nu}^{(3)} = {} & \left(a^2 \frac{m^2 + p_1 p_2}{(qk)(q'k)} - 2 \frac{(qa)(q'a)}{(qk)(q'k)}\right) k_\mu k_\nu \\ & + \left(\frac{q'a}{q'k} + \frac{qa}{qk}\right) (a_\mu k_\nu + a_\nu k_\mu) - 2 a_\mu a_\nu - a^2 \frac{(qk + q'k)}{2(qk)(q'k)} \\ & \times \{k_\mu (p_2 + p_1)_\nu + k_\nu (p_2 + p_1)_\mu + i\varepsilon_{\rho\sigma\mu\nu} k_\sigma (p_1 - p_2)_\rho\} \\ & + a^2 \frac{(qk + q'k)^2}{2(qk)(q'k)} g_{\mu\nu} + i\varepsilon_{\rho\sigma\mu\omega} k_\sigma \frac{1}{(qk)(q'k)} \{a_\omega \\ & \times [k_\nu ((qa) p_{2\rho} - (q'a) p_{1\rho}) - a_\nu ((qk) p_{2\rho} - (q'k) p_{1\rho})] + k_\nu p_{1\rho} p_{2\omega}\}; \end{aligned} \tag{A.3}$$

$$\begin{aligned} b_{\mu\nu}^{(4)} = {} & \frac{a^2}{(qk)(q'k)} \{(q'k)(p_{1\mu} k_\nu + p_{1\nu} k_\mu) + (qk)(p_{2\mu} k_\nu + p_{2\nu} k_\mu) \\ & - 2 k_\mu k_\nu (m^2 + p_1 p_2) + i(\varepsilon_{\rho\sigma\omega\nu} k_\mu - \varepsilon_{\rho\sigma\omega\mu} k_\nu) k_\omega p_{1\rho} p_{2\sigma}\}; \end{aligned} \tag{A.4}$$

$$\begin{aligned} b_{\mu\nu}^{(5)} = {} & \frac{a^2}{(qk)(q'k)} \{2(q'a - qa) k_\mu k_\nu + (qk - q'k)(k_\mu a_\nu + a_\mu k_\nu) \\ & + i(\varepsilon_{\sigma\omega\rho\mu} k_\nu - \varepsilon_{\sigma\omega\rho\nu} k_\mu) a_\omega k_\sigma (p_2 + p_1)_\rho\}; \end{aligned} \tag{A.5}$$

$$b_{\mu\nu}^{(6)} = \frac{a^4}{(qk)^2 (q'k)^2} (2(qk)(q'k) + i\varepsilon_{\rho\sigma\omega\alpha} p_{1\rho} p_{2\sigma} k_\omega k_\alpha) k_\mu k_\nu. \tag{A.6}$$

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