## CONCERNING THE PROBLEM OF REGISTRATION OF ULTRASHORT LIGHT PULSES

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It is shown that measurements based on the interaction of delayed light beams, which are widely used to determine the duration of light pulses, do not make it possible to establish uniquely the time variation of the investigated radiation. Calculations are presented, from which it follows that when the measurement accuracy is increased and the results are suitably reduced, the experiments can yield information concerning the instantaneous radiation power. For an experimental proof that the main part of the energy of laser radiation is concentrated in isolated picosecond pulses it is necessary to have a measurement accuracy on the order of  $10^{-3}$ .

**L** T is universally recognized by now that certain lasers emit, in the mode-self-synchronization regime, in the form of very short pulses, on the order of  $10^{-12} \sec^{[1-3]}$ . However, there is no direct proof of this fact. Direct measurements of such short durations have not been performed by anyone as yet, and all the ideas that some system of ultrashort pulses has been obtained is based on indirect data.

We shall show here that the indirect measurements performed  $in^{[2,3]}$  in order to prove the presence of ultrashort pulses actually do not make it possible to reconstruct uniquely the time variation of the investigated radiation<sup>1)</sup>. To prove experimentally that the bulk of the radiation energy is concentrated in ultrashort pulses it is necessary to alter the measurement procedure radically.

1. As is well known,  $\operatorname{Armstrong}^{[2]}$  and Giordmaine et al.<sup>[3]</sup> performed the following measurements. The initial light beam was transformed into two beams with controlled path difference, and the response of a nonlinear medium to the action of the two beams was measured. The effects registered in<sup>[2,3]</sup> are connected in the following manner with the time characteristics of the investigated radiation. We denote the intensity of the initial light beam by I(t), the path difference of the two beams by  $\tau$ , and the observation time by T'. Then the quantity measured in<sup>[2]</sup> can be represented, apart from a proportionality factor, in the form

$$\frac{1}{T'} \int_{0}^{T'} I(t) I(t+\tau) dt.$$
 (1)

The quantity measured in<sup>[3]</sup> is proportional to

$$\frac{1}{T'}\int_{0}^{T'} [I(t)I(t) + 2I(t)I(t+\tau)] dt.$$
 (2)

Explanations of the integral quantities (1) and (2) are contained in<sup>[4]</sup>. We recall that in the experiments of<sup>[1-3]</sup> the intensity I(t) was a quasiperiodic function, whose period was determined by the length of the laser resonator. We shall henceforth assume for con-

venience that I(t) is a periodic function with period T. Inasmuch as  $T' \gg T$ , the integrals (1) and (2) can be expressed in terms of integrals over the period

$$\frac{1}{T'} \int_{0}^{T'} I(t)I(t+\tau)dt = \frac{1}{T} \int_{0}^{T} I(t)I(t+\tau)dt \equiv \psi(\tau), \quad (3)$$
$$\frac{1}{T'} \int_{0}^{T'} [I(t)I(t)+2I(t)I(t+\tau)]dt$$
$$= \frac{1}{T} \int_{0}^{T} [I(t)I(t)+2I(t)I(t+\tau)]dt \equiv \psi(0) + 2\psi(\tau) \equiv \Psi(\tau). \quad (4)$$

The quantity  $\psi(\tau)$ , as seen from its definition, is the correlation function of the intensity I(t), while  $\psi(\tau)$  is connected with the correlation function by a linear relation. The dependence of the functions  $\psi$  and  $\Psi$  on  $\tau$  was measured in the discussed experimental investigations. No absolute measurements were made, and the values of  $\psi(\tau)$  and  $\Psi(\tau)$  were obtained in relative units. We recall also that the authors of  $[^{2,3}]$  confined themselves to a registration of the maximum of the function  $\psi(\tau)$  or  $\Psi(\tau)$  at  $\tau = 0$  and to a measurement of the width of this maximum. In  $[^{2}]$  there is contained, in addition, an estimate of  $\psi(\tau)$  outside the region of the maximum, while in  $[^{3}]$  there is no such estimate. The authors of  $[^{2,3}]$  determined the duration of the radiation pulse from the measured width of the spike.

However, only if it is known beforehand that the radiation is a single pulse is it possible to measure by this method the pulse duration. In the general case, the widths of the spikes of the functions  $\psi(\tau)$  and  $\Psi(\tau)$  give only an idea of the magnitude of the time intervals during which the characteristic changes of intensity take place.

Let us present some examples to illustrate this statement. Let us determine which functions  $\psi(\tau)$  and  $\Psi(\tau)$  correspond to certain cases of the time dependence of the intensity I.

Let the radiation be a single pulse (within a period T) with a zero background. We assume that  $I(t (= 0 \text{ at } |t - t_0| > (\frac{1}{2})\Delta T$ . From the definitions (3) and (4) of the functions  $\psi(\tau)$  and  $\Psi(\tau)$  it follows that  $\psi(\tau) > 0$  when  $|\tau| < \Delta T$ ,  $\psi(\tau) = 0$  when  $|\tau| \ge \Delta T$ ,  $\Psi(\tau) > (\frac{2}{3})\Phi(0)$  when  $|\tau| < \Delta T$ , and  $\Psi(\tau) = (\frac{1}{3})\Psi(0)$  when  $|\tau| \ge \Delta T$ . Plots of  $\psi(\tau)$  and  $\Psi(\tau)$  for a single

<sup>&</sup>lt;sup>1)</sup>The shortcomings of these measurement methods were briefly formulated in our paper [<sup>4</sup>].

pulse are shown in Fig. 1.

As the next example, we consider the function I(t) consisting of two single pulses with different durations  $\Delta_1 T$  and  $\Delta_2 T$  and with different maximum intensities  $I_1$  and  $I_2$  (see Fig. 2). Let the first pulse be much shorter than the second,  $\Delta_1 T \ll \Delta_2 T$ , with the bulk of the energy being concentrated in the second pulse,  $I_1 \Delta_1 T \ll I_2 \Delta_2 T$ , and with

$$I_1^2 \Delta_1 T \gg I_2^2 \Delta_2 T. \tag{5}$$

Calculating the function  $\psi(\tau)$  and using the inequality (5), we find that when  $\Delta_1 T < |\tau| \ll \Delta_2 T$ 

$$\frac{\boldsymbol{\psi}(\boldsymbol{\tau})}{\boldsymbol{\psi}(0)} = \frac{I_2^2 \Delta_2 T}{I_1^2 \Delta_1 T + I_2^2 \Delta_2 T} \ll 1, \quad \frac{\boldsymbol{\Psi}(\boldsymbol{\tau})}{\boldsymbol{\Psi}(0)} - \frac{1}{3} \ll 1.$$

Thus, the plots of  $\psi(\tau)$  and  $\Psi(\tau)$  with allowance for the fact that the quantities  $\psi$  and  $\Psi$  are expressed in relative units, turn out to be close to the plots of  $\psi(\tau)$ and  $\Psi(\tau)$  corresponding to a single pulse (compare Figs. 1 and 2). At the same time, this case differs greatly from the case of a single short pulse, for at the same total energy and the same width of the spike of the function  $\psi(\tau)$ , a much smaller peak power is reached in the short pulse.

As the last example we consider a set of a large number of pulses (per period) of identical shape, width, and intensity:

$$I(t) = \sum_{k=1}^{N} f(t - t_k),$$
  
$$f(t - t_k) = 0 \quad \text{if} \quad |t - t_k| \ge \frac{1}{2} \Delta T.$$

We assume that the distances between the pulses are given by the relation

$$t_k - t_{k-1} = a + kb, \ a/b > N^2, \ b > 2\Delta T$$

It is easy to verify that at such an arrangement of the points  $t_k$ , the distances between arbitrary different pairs of pulses are different. Therefore, for any path difference  $\tau > \Delta T$ , not more than one pair of pulses overlap, i.e.,

$$\psi(\tau) \leqslant \frac{1}{T} \int_{0}^{T} [f(t-t_{i})]^{2} dt \quad \text{if} \quad |\tau| \geq \Delta T.$$
(6)

In addition

$$\psi(0) = N \frac{1}{T} \int_{0}^{T} [f(t-t_1)]^2 dt.$$
(7)

As follows from (6) and (7), the ratio of the background to the maximum for the function  $\psi(\tau)$  does not exceed 1/N. Figure 3 shows plots of the functions  $\psi(\tau)$  and  $\Psi(\tau)$  for radiation consisting of five non-equidistant pulses in the period. It is obvious that at a sufficiently large number of pulses the function corresponding to N pulses, expressed in arbitrary units, is difficult to distinguish from the function  $\psi(\tau)$  corresponding to a single pulse.

It is seen from the foregoing examples that for different time dependences of the intensity, at a limited measurement accuracy, it is possible to register in the scheme of<sup>[2,3]</sup> identical functions  $\psi(\tau)$  or (or  $\Psi(\tau)$ ). Thus, the conclusions drawn in<sup>[2,3]</sup> concerning the temporal character of the radiation are not substantiated.

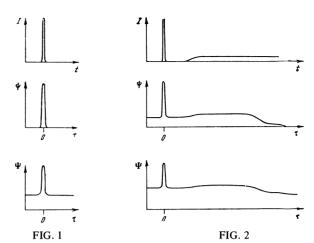


FIG. 1. Variation of the field intensity with time and dependence of the quantities  $\psi$  and  $\Psi$  on the path difference  $\tau$ . Single pulse.

FIG. 2. Time variation of the field intensity and dependence of  $\psi$  and  $\Psi$  on the path difference  $\tau$ . Two pulses of different duration.

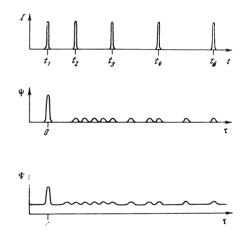


FIG. 3. Variation of the field intensity with time and dependence of the quantities  $\psi$  and  $\Psi$  on the path difference  $\tau$ . Sequence of identical non-equidistant pulses.

2. Let us consider the measurements that are necessary in order to obtain reliable proof of the existence of single ultrashort radiation pulses. As mentioned above, the width of the spike of the function  $\psi(\tau)$  does not contain essential information concerning the time dependence of the intensity I(t). It is therefore necessary to measure some other quantity that reflects more accurately the time behavior of the radiation. Let us consider from this point of view the following quantity:

$$F[I(t)] = \frac{1}{T} \int_{0}^{T} [I(t)]^2 dt \left\| \left[ \frac{1}{T} \int_{0}^{T} I(t) dt \right]^2,$$
 (8)

Denoting by a bar averaging with respect to time, we can represent F[I(t)] in the form

$$F[I(t)] = \bar{I}^2 / (\bar{I})^2.$$
(9)

The introduced quantity (9) can be readily connected with the dispersion. The quantity (9) characterizes the presence of deviation from the mean level in the time variation of the intensity, and consequently the presence of time intervals with increased values of the power. Indeed, for an intensity I(t) that is constant over the entire period, F equals unity. On the other hand, if the radiation represents a single pulse of duration  $\Delta T$ , then  $F = T/\Delta T$ . It is easy to verify that at a given width of the radiation spectrum, equal to  $(\Delta T)^{-1}$ , the quantity F for different functions I(t)ranges from 1 to  $T/\Delta T$ . The maximum of F is reached in the case of complete synchronization of the modes of the entire spectrum, when the intensity is concentrated in a single pulse of duration  $\Delta T$  with zero background.

Measurement of the quantity  $F = \tilde{I}^2/(\tilde{I})^2$  is very desirable. Simultaneous measurement of the quantity F and of the spectral width  $(\Delta T)^{-1}$  can yield proof of complete mode synchronization and of the existence of "true" ultrashort pulses, if  $F = T/\Delta T$  is obtained. On the other hand, if it is observed that  $F < T/\Delta T$ , it is impossible to reconstruct the temporal picture, but nonetheless, the measured value of F gives an idea of the values of the instantaneous power in the investigated radiation.

3. Let us discuss the possible methods of measuring the radiation characteristic of interest to us, namely, the quantity F. Returning to the scheme with delay<sup>[2,3]</sup>, we note that this scheme makes it possible in principle to determine F. The quantity F can be related with the function  $\psi(\tau)$  registered in<sup>[2,3]</sup>. Comparing (8) with (3), we obtain an expression for F in terms of  $\psi(\tau)$ :

$$F = \psi(0) \left/ \frac{1}{T} \int_{0}^{T} \psi(\tau) d\tau \right.$$

However, to determine F it is necessary to measure  $\psi(\tau)$  with very high accuracy, because small errors in the determination of  $\psi(\tau)$  can lead to appreciable errors in F. In fact, let us take two functions  $\psi_1(\tau)$  and  $\psi_2(\tau)$ , of which the first corresponds to a genuine single pulse of duration  $\Delta T$ , and the other differs from it by a small amount

$$\psi_2(\tau) = \psi_1(\tau) + \mu \psi_1(0), \quad \mu \ll 1.$$

We then have

$$F_1 = T / \Delta T, \quad F_2 = (\mu + \Delta T / T)^{-1}.$$
 (10)

It is seen from (10) that the condition  $\mu \ll 1$  is insufficient to make the temporal picture of the radiation in the second case close to that of a single pulse. In order for the temporal characteristics  $F_1$  and  $F_2$  to differ little in these two cases, it is necessary to satisfy the condition  $\mu \ll \Delta T/T$ . As is well known, the problem at present is to measure pulses whose duration is shorter than the period by three orders of magnitude. This means that in the scheme with delay the measurement accuracy should be not lower than 10<sup>-3</sup>. To reach such an accuracy it is apparently necessary to improve greatly the procedure described in<sup>[3]</sup>.

As to the measurement scheme described  $in^{[2]}$ , although the accuracy in it is considerably higher than  $in^{[3]}$ , and apparently larger instantaneous powers were obtained, it is not fully clear from the text of<sup>[2]</sup> whether the results obtained in this manner satisfy the requirement spelled out above.

It should be borne in mind that measurement schemes other than those with delay can also be used. In particular, the quantity F described above can be determined by comparing the efficiencies of converting into a second harmonic the investigated radiation and radiation with known temporal characteristics. This method was used, for example, to measure in<sup>[5]</sup> the duration of pulses emitted by a laser in the induced synchronization regime. If a broadband nonlinear element is used, this method can be employed also to register picosecond pulses.

In conclusion we emphasize once more that the interpretation given in<sup>[2,3,6-9]</sup> for measurements using a delay scheme cannot be regarded as convincing. It is quite possible that this will necessitate a review of certain results based on such measurements, particularly the conclusions of [7-9].

In addition, it is obviously necessary to perform new measurements of the duration of ultrashort light pulses, since the very fact that genuine picosecond pulses have been obtained has not yet been proven.

Note added in proof (5 November 1968). In a recent paper, Weber [<sup>10</sup>] discusses methods of measuring the duration of ultrashort light pulses. Giordmaine et al. [<sup>11</sup>] also dealt with this topic. The authors of both papers have likewise reached the conclusion that measurement of the width of the correlation-function spike is insufficient to determine the duration of a light pulse. The question of the accuracy with which the correlation function outside the spike should be measured is not considered by these authors.

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