STABILITY OF A MAGNETIZED PLASMA WITH A MONOENERGETIC COMPONENT

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We continue an investigation of the stability of a magnetized plasma that contains a small number of fast monoenergetic ions. This work represents a continuation of the investigation in[1-3]. It is shown that a plasma that contains an arbitrarily small number of fast ions is unstable against the excitation of Alfven waves. At the same time, the existence of the fast monoenergetic ions does not have any significant effect on stability with respect to drift waves.

THE investigation of the stability of a magnetized plasma containing a small number of fast and essentially monoenergetic ions is intimately related with the investigation of plasma stability in the case in which thermonuclear reactions occur. The point here is that if the reaction ions possess energies that do not exceed tens of kilo-electron volts, the distribution function for the products of the thermonuclear reactions can be essentially monoenergetic (cf.[1] where an explicit expression is given for the distribution functions for particles that are produced in a weakly inhomogeneous thermonuclear plasma). Furthermore, in regimes characteristic of thermonuclear reactions the most important instabilities are those which occur in a time much shorter than the characteristic relaxation times. Hence, in an investigation of the stability of a thermonuclear plasma the time variation of the distribution function for the thermonuclear products can be neglected. It can also be assumed that the number of charged particles resulting from the thermonuclear reactions is small.

The stability of a plasma containing a small number of monoenergetic ions with respect to the excitation of various wave branches has been investigated in[1-3]. In[1] an investigation was made of the stability of a weakly inhomogeneous thermonuclear plasma with respect to instabilities characterized by frequencies of order ω_{pi}. Here we shall first consider the effect of an essentially monoenergetic component on the stability of an inhomogeneous plasma with respect to excitation of oscillations at the drift frequencies of the primary ions.

Assuming that the following conditions are satisfied:

$$k\rho_{He} \ll 1 \ll k\rho_{Hi} \ll k/k_z, \quad \omega \gg k_z v_{Te}, \tag{1}$$

where $\rho_H = v_T/\omega_H$ and v_T is the mean-square velocity, we can write the following dispersion equation:

$$1+\frac{\omega_{pe}^2}{\omega_{He}^2}-\frac{k_z^2\omega_{pe}^2}{k^2\omega^2}+\frac{1}{k^2d_i^2}\left(1-\frac{\omega_{*i}}{\omega}\right)\left(1-\sum_{l=-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\,k\rho_{Hi}}\frac{\omega}{\omega-l\omega_{*i}}\right)$$
$$+\frac{2}{k^2d'^2}\left[\Psi'(0)-\frac{\omega_*'}{\omega}+\sum_l\frac{\omega_*'}{2}\frac{A_l}{\omega-l\omega_*'}\right]=0, \tag{2}$$

where

$$A_l=\int_0^\infty dw J_l^2(k_\perp\rho_H'\sqrt{w})\left(\frac{2\omega}{\omega_*'}\frac{\partial\Psi'}{\partial w}+\Psi'\right), \tag{3}$$

in which Ψ' is the distribution function for the fast ions over the transverse energy $w = v_\perp^2/u^2$ normalized to unity,

$$\omega_{*i}=-\frac{k_x v_{Ti}^2}{\omega_{Hi}n_i}\frac{dn_i}{dy},$$
$$\omega_*'=-\frac{k_x u^2}{\omega_H'n'}\frac{dn'}{dy}, \quad d_i=\frac{v_{Ti}}{\omega_{pi}}, \quad d'=\frac{u}{\omega_p'};$$

and the primes denote quantities that characterize the fast monoenergetic ions.

If $\omega \neq l\omega_{Hi}$, taking account of the small magnitude of the ratio n'/n we can easily obtain the zeroth approximation for the solution of Eq. (2):

$$\omega^{(0)}=\frac{\omega_{*i}}{1+k^2d_i^2(1+\omega_{pe}^2/\omega_{He}^2)}. \tag{4}$$

Making use of Eq. (4) we can simplify the expression for A_l:

$$A_l\approx\frac{\Gamma}{2\pi k_\perp\rho_H'}, \quad \Gamma\approx\int_0^\infty dw\frac{\Psi'}{\sqrt{w}}. \tag{5}$$

In Eq. (2) terms that arise because of the existence of the small number of fast ions are found to be important when $\omega^{(0)} \approx l\omega_H'$. For reasons of simplicity we shall consider the case in which $l\omega_H' < \omega_{Hi}$. Then, writing ω in the form $\omega^{(0)} + i\gamma$ (where $\gamma \ll \omega$) we have

$$\gamma=\pm l\omega_H'\frac{d}{d'}\sqrt{\frac{\omega_*'}{\omega_*}\frac{\Gamma}{2k_\perp\rho_H'}}\sim\pm l\omega_H'\sqrt{\frac{n'}{n}\frac{1}{k_\perp\rho_H'}}. \tag{6}$$

Since we have assumed that k_z is small, damping on the electrons and primary ions is exponentially small and the drift cyclotron instability will be important for an arbitrarily small number of fast ions. It should be noted, however, that for the assumptions made in (1) there also exists the unusual drift instability due to the primary ions $\omega^{(0)} = l\omega_{Hi}$; the growth rate for this instability is $\sim\sqrt{nu/n'v_{Ti}}$ times greater than the growth rate obtained above. Hence, the presence of a mixture of fast ions has essentially no effect on the stability of a plasma if the condition in (1) is satisfied.

The stability of a uniform plasma containing monoenergetic ions with respect to the excitation of ion-cyclotron waves and fast magnetoacoustic waves has been considered by Korablev and Rudakov.[2,3] We now wish to investigate the stability of a uniform plasma at the Alfven frequencies ($\omega = k_z v_A$, where $v_A = H_0/\sqrt{4\pi n_i m_i}$. The fact that the fast ions are not perfectly mono-energetic can be neglected, that is to say, we shall assume that

$$f'(v)=\frac{n'}{4\pi u^2}\delta(v-u). \qquad (7)$$

For the Alfven waves, for which

$$\frac{\omega_{pi}^2}{\omega_{Hi}^2}\gg 1,\quad k_z v_{Ti}\ll\omega\ll k_z v_{Te},\quad \frac{k_x}{k_z}\frac{\omega}{\omega_H}\ll 1,\quad k_\perp = k_x,$$

the following dispersion relation holds:

$$\omega^2 = k_z^2 v_A^2\Big[1 - i\sqrt{\pi}k_\perp^2\rho_{Hi}^2\frac{\omega}{k_z v_{Te}} - i\frac{\pi}{2}\frac{n'm_i}{nm'}\frac{\omega_{Hi}^2}{\omega k_z u}\sum\frac{l^2}{z_l}\frac{\partial J_l^2}{\partial z_l}\Big],\quad |l|<\Big|\frac{k_z u}{\omega_H'}\Big|, \qquad (8)$$

where

$$z_l=\frac{k_x u}{\omega_H'}\sqrt{1-\frac{l^2\omega_H'^2}{k_z^2u^2}},$$

and $J_l(z_l)$ is the Bessel function of order l. The last term in Eq. (8), which arises because of the mono-energetic ions, vanishes when $|k_z| < \omega_H/u$. Writing $\omega = k_z v_A + i\gamma$ and $\gamma \ll k_z v_A$ we have

$$\gamma = -\frac{\sqrt{\pi}}{2}\frac{k_z v_A^2}{v_{Te}}k_x^2\rho_{Hi}^2 - \frac{\pi}{4}\frac{n'm_i}{nm'}\frac{\omega_{Hi}^2}{k_z u}\sum\frac{l^2}{z_l}\frac{\partial J_l^2}{\partial z_l}. \qquad (9)$$

The expression in (9) becomes particularly simple when $|k_z| < 2\omega_H'/u$ (we recall that the condition $|k_z| > \omega_H'/u$ is always satisfied). In this case, in the summation that appears in (9) the only nonvanishing terms are those for which $|l| = 1$. It is evident that the instability will arise if the density of fast ions exceeds the critical value

$$n_{\text{кр}}' = \frac{m_e}{m_i}\left(\frac{v_A}{u}\right)^2 n_i \qquad (10)$$

and if the wave vectors fall in one of the following regions:

$$p_{1,\nu}' < \frac{k_x u}{\omega_H'}\sqrt{1-\frac{\omega_H'^2}{k_z^2u^2}} < p_{1,\nu+1}, \qquad (11)$$

where $p_{1,\nu}$ and $p_{1,\nu}'$ are the ν-th roots respectively of the functions $J_1(z_1)$ and $J_1'(z_1)$. The maximum growth rate is of order

$$\gamma \sim (n'/n)\omega_H'. \qquad (12)$$

We can now estimate the critical density of mono-energetic ions. If $v_A \sim 10^8$ cm/sec and $u \sim 10^9$ cm/sec, then $n_{cr}'/n \sim 10^{-5}$. Thus, the threshold for this instability is found to be significantly lower than that for the instabilities investigated in[1-3]. Furthermore, if the plasma electrons are cold, that is to say, if $\omega \gg k_z v_{Te}$ (we recall that Eq. (8) has been obtained in the opposite limit) then the damping due to the electrons and the ions will be exponentially small and the Alfven waves will be excited for an arbitrarily small number of monoenergetic ions.

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