QUASILINEAR APPROXIMATION FOR A COLLISIONAL MAGNETOACTIVE PLASMA: CERENKOV EFFECT

V. V. LOGVINOV and G. Z. MACHABELI

Submitted June 17, 1968

Zh. Eksp. Teor. Fiz. 55, 1953–1963 (November, 1968)

Using a quasilinear approximation with collisions taken into account we have obtained a kinetic equation that describes the "slow" distribution function in a magnetoactive plasma. This equation is suitable for the analysis of interactions between particles and various kinds of high-frequency electromagnetic waves in a plasma. Using this equation for the case of Cerenkov resonance we have computed the energy and momentum balance equations for the individual plasma components in the case for which the distribution function for the components is Maxwellian with a temperature anisotropy and a directed velocity. For the stationary state we have determined the electron heating and the electron currents due to the nonlinear effect of various kinds of high-frequency waves. It is shown within the framework of the quasilinear approximation, in which the wave energy is small, that for wave propagation along the magnetic field transverse waves do not produce a directed velocity or heating of the electrons. On the other hand, longitudinal waves lead to the production of a current along the wave vector, a temperature anisotropy for the electrons (the longitudinal component of the electron pressure is greater than the transverse pressure), and heating of the electrons with respect to the ions. In the case of wave propagation in the plane perpendicular to the magnetic field the transverse waves can produce a significantly larger nonlinear current than the longitudinal waves that propagate along the magnetic field; longitudinal waves that propagate perpendicularly to the magnetic field generally do not produce an electron current or electron heating in this approximation. For the case of oblique wave propagation (with respect to the magnetic field) the contribution of the transverse component of the waves to the nonlinear current along the magnetic field can also exceed significantly the corresponding contribution due to the longitudinal waves. Finally, making use of a model collision integral which takes account of the temperature anisotropy of the electrons, we find the longitudinal (with respect to the magnetic field) component of the electron distribution function.

1. INTRODUCTION

KLIMONTOVICH[1] has obtained an equation for the quasilinear approximation that takes account of both the self-consistent rapidly varying field in a plasma as well as the correlation phenomena associated with dissipative effects for a plasma in the absence of a magnetic field. A feature of these equations, in contrast with the usual equations for quasilinear theory, is the fact that the particle collisions are taken into account in the equation for the "slow" distribution function not only by means of the collision integral, but also directly by an effective collision frequency ν_a; this simulates the rapidly varying part of the collision integral and appears in the quasilinear term, which describes the effect of plasma waves on the slow distribution function.

These equations have been used[2] for the analysis of the stationary state of a plasma in the presence of high-frequency plasma waves. In the present work we consider similar problems for the case of a magnetoactive plasma.

In Sec. 2 we derive an equation for the slow distribution function f_a in the quasilinear approximation for a spatially uniform magnetoactive plasma in which we do not make use of the assumption that the function f_a is independent of the polar angle φ in velocity space (the z-axis is taken along the external magnetic field $\mathbf{H}_0$). In particular, this procedure allows us to determine the current in the xy plane that arises as a result of the nonlinear entrainment of particles by waves.

In Sec. 3 we analyze the momentum and energy balance equations for a plasma in the stationary state in the case in which the interaction between particles and waves occurs under the condition of Cerenkov resonance (for particle with velocities $v_z \sim \omega/k_z$). In this case the function f_a is assumed to be a Maxwellian with anisotropies in the directed velocity and the temperatures of the components. We determine the currents and heating of the electrons as a result of the interaction with various waves.

In Sec. 4, making use of a model collision integral S_e for the anisotropy in the electron temperature we find the electron distribution function f_e subject to the limitation that it be represented by an expression of the form $f_e = f_e^z(v_z) f_e^{\perp}(v_\perp^2)$.

Various problems that arise in the quasilinear theory of a magnetoactive plasma (primarily the determination of wave damping) have been investigated by many authors, starting with the well-known work by Vedenov, Velikhov, and Sagdeev (cf. for example [3]). A general feature of all this work is the fact that the slow function f_a has been assumed to be independent of φ and collisions were either neglected or taken into account only by means of a collision integral which was not completely consistent, especially in the investigation of steady-state conditions. These limitations do not appear in the present work. In particular, taking account of the dependence of f_a on φ allows us to make a more detailed determination of the contribution due to terms that correspond to various kinds of wave-particle interactions

(Cerenkov resonance and cyclotron resonance).

2. DERIVATION OF THE EQUATIONS

The point of departure will be the kinetic equations for the slow and fast distribution functions in a spatially uniform plasma located in a constant magnetic field $\mathbf{H}_0$:

$$\frac{\partial f_a^0}{\partial t}+\left\langle e_a\mathbf{E}\frac{\partial f_a^1}{\partial \mathbf{p}}\right\rangle+\left\langle \frac{e_a}{c}[\mathbf{vH}]\frac{\partial f_a^1}{\partial \mathbf{p}}\right\rangle+\frac{e_a}{c}[\mathbf{vH}_0]\frac{\partial f_a^0}{\partial \mathbf{p}}=S_a^0, \quad (1)^*$$

$$\frac{\partial f_a^1}{\partial t}+\mathbf{v}\frac{\partial f_a^1}{\partial \mathbf{q}}+e_a\left(\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right)\frac{\partial f_a^0}{\partial \mathbf{p}}+\frac{e_a}{c}[\mathbf{vH}_0]\frac{\partial f_a^1}{\partial \mathbf{p}}=S_a^1. \quad (2)$$

Here, $\mathbf{E}$ and $\mathbf{H}$ are the rapidly varying fields in the plasma while S_a^0 and S_a^1 are the slow and fast parts of the collision integral for species a; the angle brackets denote averages over the rapid variations.

If we assume that

$$\Omega_a\ll\omega_a, \quad \Omega_a=\frac{e_aH_0}{m_ac}, \quad \omega_a=\left(\frac{4\pi n_ae_a^2}{m_a}\right)^{1/2}, \quad (3)$$

then the external magnetic field will not have an effect on the collisions. We also assume that dissipation effects are determined by the shortwave part of the correlation spectrum in the plasma (the wavelengths of the oscillations are much larger than the Debye radius). Under these conditions, S_a^0 for an electron ion plasma can be taken in the Landau form, that is to say, we need not consider polarization in the collision integral.

In accordance with [1] the quantity S_a^1 is written

$$S_a^1=-\nu_af_a^1, \quad (4)$$

where ν_a simulates the contribution of collisions in rapidly varying processes and consequently, in the dielectric constant.[1)]

We shall be interested in waves whose phase velocities are much higher than the electron thermal velocity. This means that in Eqs. (1) and (2) we can neglect terms containing $\mathbf{H}$ and need only consider the electric field associated with the waves.[2)]

The solution of Eq. (2) is found by means of the familiar method of characteristics (cf. for example, [1]). Using the canonical equations of motion we find the relation between the coordinates and momenta $\mathbf{p}$ and $\mathbf{q}$ at time t and their values $\mathbf{P}_a$ and $\mathbf{Q}_a$ at time t':

$$\mathbf{p}=(\mathbf{P}_a\mathbf{h})\mathbf{h}+[[\mathbf{hP}_a]\mathbf{h}]\cos\Omega_a(t-t')+[\mathbf{P}_a\mathbf{h}]\sin\Omega_a(t-t'),$$

$$\mathbf{q}=\mathbf{R}_a+\frac{1}{m_a}(\mathbf{P}_a\mathbf{h})\mathbf{h}(t-t')+\frac{1}{m_a\Omega_a}[[\mathbf{hP}_a]\mathbf{h}]\sin\Omega_a(t-t')$$
$$+\frac{1}{m_a\Omega_a}[\mathbf{P}_a\mathbf{h}](1-\cos\Omega_a(t-t')), \quad \mathbf{h}=\frac{\mathbf{H}_0}{|\mathbf{H}_0|}. \quad (5)$$

Solving (2) we write the Fourier transform of f_a^1 taking account of (5):

$$f_a^1(\omega,\mathbf{k})=-\int\left(e_a\mathbf{E}(\omega,\mathbf{k})\frac{\partial f_a^0}{\partial\mathbf{p}}\right)_{\mathbf{p}\to\mathbf{P}_a}\exp\Big\{-\nu_a\tau+i\Big[\omega\tau-k_zv_z\tau$$
$$-\frac{k_\perp v_\perp}{\Omega_a}[\sin(\varphi-\theta+\Omega_a\tau)-\sin(\varphi-\theta)]\Big]\Big\}d\tau. \quad (6)$$

Equation (6) is now written in cylindrical coordinates with the z-axis along $\mathbf{H}_0$:

$$v_x=v_\perp\cos\varphi, \quad v_y=v_\perp\sin\varphi, \quad v_z=v_z,$$
$$k_x=k_\perp\cos\theta, \quad k_y=k_\perp\sin\theta, \quad k_z=k_z.$$

Using the relation

$$e^{i\alpha\sin(\varphi-\theta)}=\sum_{n=-\infty}^{\infty}J_n(\alpha)e^{in(\varphi-\theta)},$$

where $J_n(\alpha)$ is the Bessel function and integrating with respect to τ, we have[3)]

$$f_a^1(\omega,\mathbf{k})=-i\frac{e_a}{m_a}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}J_m\left(\frac{k_\perp v_\perp}{\Omega_a}\right)J_n\left(\frac{k_\perp v_\perp}{\Omega_a}\right)e^{i(m-n)(\varphi-\theta)}\cdot$$
$$\cdot\left\{\frac{\partial f_a^0}{\partial v_z}\frac{E_z(\omega,\mathbf{k})}{\omega-k_zv_z-n\Omega_a+i\nu_a}+\frac{1}{2}\frac{\partial f_a^0}{\partial v_\perp}E_\perp(\omega,\mathbf{k})\cdot\right. \quad (7)$$
$$\left.\cdot\left[\frac{e^{i(\varphi-\psi)}}{\omega-k_zv_z+\Omega_a-n\Omega_a+i\nu_a}+\frac{e^{-i(\varphi-\psi)}}{\omega-k_zv_z-\Omega_a-n\Omega_a+i\nu_a}\right]\right\},$$

where ψ is the angle between the x-axis and $\mathbf{E}_\perp$.

Now, substituting Eq. (7) in Eq. (1) and averaging over the fast variable we obtain the following equation for the slow distribution function for a magnetoactive plasma in the quasilinear approximation (the subscript 0 is omitted below):

$$\frac{\partial f_a}{\partial t}-\mathrm{Re}\,i\frac{e_a^2}{2m_a^2}\sum_{m,\,n=-\infty}^{\infty}\hat{L}J_m\left(\frac{k_\perp v_\perp}{\Omega_a}\right)J_n\left(\frac{k_\perp v_\perp}{\Omega_a}\right)e^{i(m-n)(\varphi-\theta)}\cdot$$
$$\times\left\{\frac{\partial f_a}{\partial v_z}\frac{E_z}{\omega-k_zv_z-n\Omega_a+i\nu_a}+\frac{1}{2}\frac{\partial f_a}{\partial v_\perp}E_\perp\left[\frac{e^{i(\varphi-\psi)}}{\omega-k_zv_z+\Omega_a-n\Omega_a+i\nu_a}\right.\right.$$
$$\left.\left.+\frac{e^{-i(\varphi-\psi)}}{\omega-k_zv_z-\Omega_a-n\Omega_a+i\nu_a}\right]\right\}-\Omega_a\frac{\partial f_a}{\partial\varphi}=S_a, \quad (8)$$
$$\hat{L}=E_z\frac{\partial}{\partial v_z}+\cos(\varphi-\psi)E_\perp\frac{\partial}{\partial v_\perp}-\sin(\varphi-\psi)\frac{E_\perp}{v_\perp}\frac{\partial}{\partial\varphi}.$$

Here, the quantities $E_i(\omega,\mathbf{k})$ have the meaning of the corresponding projections of the electric field of the wave in the plasma while ω and $\mathbf{k}$ are related by the dispersion equation of the linear theory.

Equation (8) is most general for high-frequency waves and can serve as a point of departure for the solution of a wide class of problems in the quasilinear approximation (various wave branches, longitudinal and transverse, oblique propagation; various wave plasma interactions—Cerenkov resonance and cyclotron resonance). Below, as an example, we consider the case of the Cerenkov resonance in which the external magnetic field is so large that the condition

$$k_\perp v_a^\perp\ll\Omega_a, \quad (9)$$

is satisfied, where $V_a^\perp$ is the perpendicular component of the thermal velocity of species a with respect to the magnetic field.

3. MOMENTUM AND ENERGY BALANCE EQUATIONS FOR CERENKOV RESONANCE

In the summation over n we consider the terms corresponding to the Cerenkov resonance in the interaction of particles with the wave. For this purpose we need

---

1)It can be shown that for high-frequency fields the quantity ν_e coincides with the effective electron-ion collision frequency.

2)For low-frequency (for example, magnetohydrodynamic) waves terms containing H must be retained.

*[vH] ≡ v × H.

3)In Eq. (7) we have not taken account of the derivative of f_a^0 with respect to φ because this term makes a small contribution in the balance equations.

only take $n = 0, \pm 1$. Furthermore, evaluating the expansion of $J_n(k_\perp v_\perp/\Omega_a)$, in the light of (9) we find

$$\frac{\partial f_a}{\partial t} - \frac{e_a^2}{2m_a^2}\hat{L}\frac{1}{(\omega - k_z v_z)^2 + \nu_a^2} \times \left\{\left[\nu_a - (\omega - k_z v_z)\frac{k_\perp v_\perp}{\Omega_a}\sin(\varphi - \theta)\right]E_z\frac{\partial f_a}{\partial v_z} + \frac{1}{2}(\omega - k_z v_z)\frac{k_\perp v_\perp}{\Omega_a}\sin(\psi - \theta)E_\perp\frac{\partial f_a}{\partial v_\perp}\right\} - \Omega_a\frac{\partial f_a}{\partial \varphi} = S_a. \quad (10)$$

We now consider the momentum and energy balance equations assuming that the distribution function f_a is Maxwellian with anisotropic directed velocities and temperatures:

$$f_a(t, \mathbf{v}) = \frac{n_a}{(\pi)^{3/2}(v_a^\perp)^2 v_a^z}\exp\left\{-\frac{(\mathbf{v}_\perp - \mathbf{u}_a^\perp(t))^2}{(v_a^\perp)^2} - \frac{(v_z - u_a^z(t))^2}{(v_a^z)^2}\right\}$$

$$v_a^\perp = (2T_a^\perp(t)\,/\,m_a)^{1/2}, \quad v_a^z = (2T_a^z(t)\,/\,m_a)^{1/2}. \quad (11)$$

We shall also assume that the directed velocities u_a are much smaller than the thermal velocities v_a so that the square of this ratio can be neglected compared with unity.

We first consider the momentum balance equation for the species a. Multiplying Eq. (10) by $m_a v_j$ (j = x, y, z) and integrating over velocity we have

$$n_a m_a\frac{du_{aj}}{dt} + \frac{n_a e_a^2}{2m_a}E_j\left\{E_z\left(\beta_1 - \beta_2\frac{1}{\Omega_a}[\mathbf{k}\mathbf{u}_a]_z\right) - \beta_3\frac{1}{\Omega_a}[\mathbf{k}\mathbf{E}]_z\right\} + n_a m_a\Omega_a(u_{ax}\delta_{yj} - u_{ay}\delta_{xj}) = \int m_a v_j S_a d\mathbf{v}, \quad (12)$$

where

$$\beta_1 = \frac{2\sqrt{\pi}}{k_z(v_a^z)^2}\{y_a\,\mathrm{Im}\,w(z_a) - x_a\,\mathrm{Re}\,w(z_a)\},$$

$$\beta_2 = \frac{2}{k_z(v_a^z)^2}\{1 - \sqrt{\pi}[x_a\,\mathrm{Im}\,w(z_a) + y_a\,\mathrm{Re}\,w(z_a)]\},$$

$$\beta_3 = \frac{\sqrt{\pi}}{k_z v_a^z}\mathrm{Im}\,w(z_a), \quad z_a = x_a + iy_a = \frac{\omega - k_z u_{az}}{k_z v_a^z} + i\frac{\nu_a}{k_z v_a^z},$$

$$w(z) = e^{-z^2}\left\{1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right\}.$$

The integrals that appear on the right side of Eq. (12) can be computed by the method used by Kogan,[4] and are of the following form:

$$\int m_a v_j S_a d\mathbf{v} = -n_a\sum_b\frac{m_a(u_a - u_b)_j}{\tau_{ab}^u}\Phi_j(\alpha), \quad (13)$$

where τ_{ab}^u is the relaxation time for the directed velocities associated with collisions of particles of species a with particles of species b:

$$\tau_{ab}^u = \frac{3}{4\sqrt{2\pi}}\frac{m_a m_b}{(e_a e_b)^2 n_b\Lambda}\left(\frac{T_a^\perp}{m_a} + \frac{T_b^\perp}{m_b}\right)\left(\frac{T_a^z}{m_a} + \frac{T_b^z}{m_b}\right)^{1/2}\frac{m_a}{m_a + m_b}$$

and where the dimensionless functions $\Phi_j(\alpha)$ characterize the anisotropy:

$$\Phi_x(\alpha) = \Phi_y(\alpha) = \frac{3}{4}\int_0^1\frac{(1 + x^2) + \alpha x^2(x^2 - 3)}{(1 - \alpha x^2)^2}dx,$$

$$\Phi_z(\alpha) = \frac{3}{2}(1 - \alpha)\int_0^1(1 - x^2)\frac{1 + \alpha x^2}{(1 - \alpha x^2)^2}dx,$$

and the anisotropy parameter is given by

$$\alpha = \frac{(v_a^z)^2 + (v_b^z)^2 - (v_a^\perp)^2 - (v_b^\perp)^2}{(v_a^z)^2 + (v_b^z)^2}$$

If the anisotropy is small ($\alpha \to 0$)

$$\Phi_x(\alpha) = \Phi_y(\alpha) \approx 1 + {}^1/_5\alpha, \quad \Phi_z(\alpha) \approx 1 - {}^2/_5\alpha,$$

and if $\alpha = 0$, Eq. (13) coincides with the expression obtained earlier for an isotropic plasma (cf. for example, [2]).

In the derivation of the energy balance equation we take account of the anisotropy in the temperatures parallel to and transverse to the magnetic field. Multiplying Eq. (10) by $m_a v_z^2/2$ and by $m_a v_\perp^2/2$ and integrating we obtain the following expressions for the longitudinal and transverse energies respectively:

$$n_a\frac{1}{2}\frac{dT_a^z}{dt} + \frac{n_a e_a^2}{2m_a}E_z\left\{E_z\left(\beta_4 - \beta_5\frac{1}{\Omega_a}[\mathbf{k}\mathbf{u}_a]_z\right) - \beta_6\frac{1}{\Omega_a}[\mathbf{k}\mathbf{E}]_z\right\} = \int\frac{m_a v_z^2}{2}S_a\,d\mathbf{v}, \quad (14)$$

$$n_a\frac{dT_a^\perp}{dt} + \frac{n_a e_a^2}{2m_a}\left\{E_z\left[\beta_1(u_{ax}E_x + u_{ay}E_y) + \beta_2\frac{(v_a^\perp)^2}{2\Omega_a}[\mathbf{k}\mathbf{E}]_z\right] - \beta_3\frac{3}{2\Omega_a}(u_{ax}E_x + u_{ay}E_y)[\mathbf{k}\mathbf{E}]_z\right\} = \int\frac{m_a v_\perp^2}{2}S_a\,d\mathbf{v}, \quad (15)$$

where

$$\beta_4 = \frac{2\nu_a}{k_z^2(v_a^z)^2}\left\{\sqrt{\pi}\left[(2x_a + \eta_a)\,\mathrm{Im}\,w(z_a) - \frac{x_a^2 - y_a^2 + x_a\eta_a}{y_a}\mathrm{Re}\,w(z_a)\right] - 1\right\},$$

$$\beta_5 = \frac{2\omega}{k_z^2(v_a^z)^2}\left\{1 - \sqrt{\pi}\left[\left(y_a + \frac{x_a y_a}{x_a + \eta_a}\right)\mathrm{Re}\,w(z_a) + \left(x_a - \frac{y_a^2}{x_a + \eta_a}\right)\mathrm{Im}\,w(z_a)\right]\right\},$$

$$\beta_6 = \frac{1}{k_z}\left\{\sqrt{\pi}[y_a\,\mathrm{Re}\,w(z_a) + (x_a + \eta_a)\,\mathrm{Im}\,w(z_a)] - 1\right., \quad \eta_a = \frac{u_{az}}{v_a^z}.$$

The expressions on the right sides of Eqs. (14) and (15) are given by

$$\int\frac{m_a v_z^2}{2}S_a\,d\mathbf{v} = -n_a\sum_b\left\{\frac{1}{2}\frac{T_a^z - T_b^z}{\tau_{ab}^T}\Phi_1(\alpha) + \frac{m_a(v_a^z)^2}{\tau_{ab}^u}\frac{(v_a^\perp)^2 + (v_b^\perp)^2}{(v_a^z)^2 + (v_b^z)^2}\Phi_2(\alpha)\right\}, \quad (16)$$

$$\int\frac{m_a v_\perp^2}{2}S_a\,d\mathbf{v} = -n_a\sum_b\left\{\frac{T_a^\perp - T_b^\perp}{\tau_{ab}^T}\Phi_3(\alpha) - \frac{m_a(v_a^\perp)^2}{\tau_{ab}^u}\Phi_2(\alpha)\right\}, \quad (17)$$

where τ_{ab}^T is the relaxation time for the temperature associated with collisions between particles of species a and particles of species b:

$$\tau_{ab}^T = \tau_{ab}^u\frac{m_a + m_b}{2m_a}$$

and the dimensionless functions:

$$\Phi_1(\alpha) = \frac{3}{2}(1 - \alpha)\int_0^1\frac{1 - x^2}{1 - \alpha x^2}dx, \quad \Phi_2(\alpha) = \frac{3}{2}\alpha\int_0^1\frac{x^2(1 - x^2)}{(1 - \alpha x^2)^2}dx,$$

$$\Phi_3(\alpha) = \frac{3}{4}\int_0^1\frac{1 + x^2}{1 - \alpha x^2}dx.$$

If the anisotropy is small

$$\Phi_1(\alpha) \approx 1 - {}^4/_5\alpha, \quad \Phi_2(\alpha) \approx {}^1/_5\alpha, \quad \Phi_3(\alpha) \approx 1 + {}^2/_5\alpha.$$

When $\alpha = 0$ Eqs. (16) and (17) become the familiar expression for an isotropic plasma and when (a = b) (single-component system) these results coincide with those obtained by Kogan.[4] We also note that in the summation over a the conservation relations for momentum and energy for the entire plasma are satisfied, as they should be. These relations are satisfied automatically for any degree of anisotropy.

We now consider the stationary state ($d/dt = 0$) assuming that the waves in the plasma are specified. We shall limit our analysis to the case of high-frequency fields in an electron–ion plasma, in which case $|z_e| \gg 1$. In this case the effect of the waves on the ions can be neglected (in particular, $u_i = 0$).

It has been shown in [2] that longitudinal high-frequency waves cause an entrainment of the electrons in the direction of the wave vector and this causes the electron temperature to increase with respect to the ion temperature. In order to obtain the appropriate quantitative relations in a magnetoactive plasma it is convenient to consider the following particular cases individually.

1) $k_\perp = 0$. In this case, as is well known, the waves in the plasma can be divided into a) pure longitudinal waves ($E_\perp = 0$) and b) ordinary and extraordinary transverse waves ($E_z = 0$);

2) $k_z = 0$. In this case we can distinguish (cf. for example, [5]) a) pure transverse linearly polarized waves for which $E_\perp = 0$, b) transverse linear polarized waves in which the electric vector is perpendicular to the magnetic field, and c) longitudinal waves (for which the refractive index approaches ∞).

We shall consider these cases in order.

1) $k_\perp = 0$. Using the asymptotic expansion of $w(z_e)$ for $|z_e| \gg 1$ for the case of a small anisotropy, we find that within the framework of the quasilinear approximation the transverse waves do not produce a directed velocity and do not heat the electrons. On the other hand, for the longitudinal waves, from Eqs. (12)–(17) we find (the subscript e on the directed velocity will be omitted below)

$$u_z = \frac{k_z}{\omega}\frac{e^2E_z^2\nu_e\tau_{ei}^u}{m_e^2\omega^2}\left(1-\frac{2}{5}\alpha\right)^{-1}, \tag{18}$$

$$T_e^z = T_i^z + \frac{e^2E_z^2\nu_e\tau_{ei}^T}{m_e\omega^2}\left(1-\frac{4}{5}\alpha\right)^{-1}$$

$$2\left(1+\frac{1}{\sqrt{2}}\right)\frac{m_i}{m_e}T_e^\perp\cdot\frac{1}{5}\alpha\left(1-\frac{4}{5}\alpha\right)^{-1}, \tag{19}$$

$$T_e^\perp = T_i^\perp + \left(1+\frac{1}{\sqrt{2}}\right)\frac{m_i}{m_e}T_e^\perp\frac{1}{5}\alpha\left(1+\frac{2}{5}\alpha\right)^{-1}, \quad \alpha = \frac{T_e^z - T_e^\perp}{T^z}. \tag{20}$$

When $\alpha = 0$ the relation in (18) and (19) become the corresponding relations for an isotropic plasma.[2] Assuming that the ions are isotropic ($T_i^z = T_i^\perp = T_i$) and using Eqs. (19) and (20) we can find the relation between T_e^z and $T_e^\perp$. Subtracting (20) from (19) and solving approximately the resulting quadratic equation we find the following expression for

$$T_e^z = T_e^\perp + \frac{5\sqrt{2}}{6(1+\sqrt{2})}\frac{e^2E_z^2\nu_e\tau_{ei}^u}{m_e\omega^2}, \tag{21}$$

that is to say, the longitudinal waves in the direction of the magnetic field cause an anisotropy in the electron temperature, this anisotropy being due to the transfer of energy to the electron primarily in the direction of the wave vector. However, this anisotropy is small since it is determined by the small parameter used in the quasilinear approximation $e^2E_z^2/m_eT_e\omega^2 \ll 1$, whereas $\nu_e\tau_{ei}^u = 1$.

Now, substituting (21) in (20) we can find the relation between the electron and ion temperatures (compare this with [21]):

$$T^\perp{}_e = T_i + \frac{m_i}{m_e}\frac{e^2E_z^2\nu_e\tau_{ei}^u}{6m_e\omega^2}, \tag{22}$$

that is to say, the anisotropy in the electron temperature is small compared with the difference between the electron and ion temperatures.

2) $k_z = 0$. It follows from Eqs. (12)–(17) that for the case of the pure transverse ordinary wave, in which the electric vector is parallel to the magnetic field, in the quasilinear approximation $u_x = u_y = u_z = 0$. The temperatures are given by the same expressions as for the longitudinal waves treated in 1). Thus the difference between the longitudinal waves and the transverse waves, for which $\mathbf{E} \parallel \mathbf{H}_0$, lies in the fact that the longitudinal waves produce a constant nonlinear current in the plasma whereas the transverse waves do not.

If $E_z = 0$ we find the following expressions for the currents and temperatures, where we have taken $k_\perp = k_x$ and $k_y = 0$ for definiteness:

$$u_x = \frac{k_x}{\omega}\frac{e^2E_y\tau_{ei}^u}{2m_e\Omega_e}\frac{E_x + E_y(\tau_{ei}^u\Omega_e)}{1+(\tau_{ei}^u\Omega_e)^2} \tag{23}$$

$$u_y = \frac{k_x}{\omega}\frac{e^2E_y\tau_{ei}^u}{2m_e\Omega_e}\frac{E_y - E_x(\tau_{ei}^u\Omega_e)}{1+(\tau_{ei}^u\Omega_e)^2}, \quad u_z = 0, \quad T_e^z = T_e^\perp = T_i. \tag{24}$$

For the case of longitudinal waves, which can be distinguished if the refractive index approaches ∞, we have $E_y = 0$ so that there are no currents in the plasma. For the transverse waves ($E_x = 0$) the current along the x-axis is significantly greater than the current along the y-axis since $\tau_{ei}^u\Omega_e \gg 1$:

$$u_x = \frac{k_x}{\omega}\frac{e^2E_y^2}{2m_e\Omega_e^2} = u_y(\tau_{ei}^u\Omega_e). \tag{25}$$

The characteristic feature of this current is the fact that it is appreciably greater than the analogous nonlinear current for different values of the amplitudes and wavelengths; this nonlinear current is produced by the longitudinal waves along the magnetic field [compare with (18)] since $\omega^2 \gg \Omega_e^r$. The basis for this behavior lies in the fact that an active role is played by the magnetic field in the formation of this current and this leads to drift motion of the electrons along the x-axis.

Similar considerations can be used to determine the corresponding expressions for the case of oblique wave propagations with respect to the magnetic field. In this case, as will be shown in Sec. 4, the current produced along the z-axis by the transverse component (with respect to the wave vector) can be much greater than the corresponding current produced by the longitudinal waves.

4. DISTRIBUTION FUNCTION

In the general case the solution of the system of nonlinear integro-partial-differential equations for the function f_a as given in (8) is extremely difficult if not impossible. For this reason we shall find it desirable to examine the qualitative features of the distribution function in the quasilinear approximation, being guided by the following considerations.

1. The electron distribution function f_e will be

sought in the stationary state and the ion distribution function f_i will be assumed to be Maxwellian with an isotropic temperature T_i.

2. The collision integral S_e will be taken as a model collision integral, taking account of the anisotropy in the electron temperature $T_e^z \neq T_e^\perp$.

3. It will be assumed that the distribution function f_e is independent of the angle φ (in so doing we eliminate currents in the plane perpendicular to the magnetic field); this function is written in the form $f_e(\mathbf{v}) = f_e^z(v_z) \times f_e^\perp(v_\perp^2)$ where $f_e^\perp$ is a Maxwellian distribution function characterized by an effective temperature $T_e^\perp$. This choice of the function f_e is appropriate for the stationary problem in the sense that the plateau on the distribution function, as will be evident from Eq. (8), can only be formed along the magnetic field (if the latter is large enough).

The model collision integral is written in the Fokker-Planck form:

$$S_e = S_{ee} + S_{ei} = \nu_{ee}{}^k \frac{\partial}{\partial v_j}\left\{\frac{T_e{}^k}{m_e}\frac{\partial f_e}{\partial v_j} + v_j f_e\right\} + \nu_{ei}\frac{\partial}{\partial v_j}\left\{\frac{T_i}{m_e}\frac{\partial f_e}{\partial v_j} + v_j f_e\right\}, \tag{26}$$

where the subscript k characterizes the anisotropy in the electron temperature and assumes the following values in the summation over j: when $j = z$, $k = x = y = \perp$; when $j = x, y$, $k = z$ and by definition

$$T_e{}^k = \int m_e v_k{}^2 f_e \, d\mathbf{v}.$$

The quantities $\nu_{ee}^\perp$, ν_{ee}^z and ν_{ei} can be found by comparison with the exact values of the integrals that appear on the right sides of the balance equations (16) and (17):

$$\nu_{ee}{}^\perp = \frac{2}{5}\frac{T_e{}^\perp}{T_e{}^z}\left(\frac{1}{\tau_{ee}{}^u} + \frac{1}{\tau_{ei}{}^u}\right), \quad \nu_{ee}{}^z = \frac{1}{2}\nu_{ee}{}^\perp, \quad \nu_{ei} = \frac{1}{2}\frac{1}{\tau_{ei}{}^T}$$

In the form given in (26) the integral S_e satisfies energy conservation and vanishes if f_e is a Maxwellian distribution function with $T_e^z = T_e^\perp = T_i$.

Substituting f_e in (8), writing the model integral S_e in cylindrical coordinates and integrating over $\mathbf{v}_\perp$, we obtain the following solution for f_e^z:

$$\ln f_e{}^z = -\frac{m_e}{T_i}\int \left\{v_z\left(1 + \frac{\nu_{ee}{}^\perp}{\nu_{ei}}\right) - \frac{e^2 E_\perp E_z k_\perp \sin(\psi - \theta)(\omega - k_z v_z)}{2 m_e \nu_{ei}\Omega_e[(\omega - k_z v_z)^2 + \nu_e{}^2]}\right\} \cdot \left\{1 + \frac{\nu_{ee}{}^\perp}{\nu_{ei}}\frac{T_e{}^\perp}{T_i} + \frac{e^2 E_z{}^2 \nu_e}{2 m_e \nu_{ei} T_i[(\omega - k_z v_z)^2 + \nu_e{}^2]}\right\}^{-1} dv_z. \tag{27}$$

The subsequent computation of this integral does not represent any fundamental difficulties and it is not necessary to present the final complicated expression. For purposes of illustration we shall consider certain particular cases.

a) For longitudinal waves ($\psi = 0$) and when $k_\perp = 0$ or $E_\perp = 0$ the second term in the numerator of the integrand vanishes and f_e^z is of the same form as in [2] except for the anisotropy in the electron temperature. When $k_z \to 0$, we find that f_e^z is Maxwellian with an effective temperature T_e^z that coincides with (19).

b) When $k_z \to 0$, if there is a transverse wave component in the plasma (with respect to the wave vector) and $k_\perp$, $E_\perp$, $E_z \neq 0$, using (27) we obtain the following directed velocity along the z-axis:

$$u_z \cong \frac{e^2 E_z E_\perp k_\perp \sin(\psi - \theta)}{2 m_e \omega \Omega_e \nu_{ee}{}^\perp}, \tag{28}$$

which, to within a numerical factor, coincides with the corresponding expression for u_z from the balance equation (12). This determines the contribution of the transverse wave components (with respect to **k**) to the current along the magnetic field in a magnetoactive plasma. We note that when $k_\perp E_\perp \sim k_z E_z$ and $\sin(\psi - \theta) \sim 1$, this current is $\omega^2/\nu_e\Omega_e$ times larger than the current produced by the longitudinal waves which propagate along the magnetic field [cf. (18)].

The expressions obtained in Secs. 3 and 4 indicate the possibility of a quasilinear approximation taking account of collisions in the determination of the nonlinear current flow and electron heating in the stationary state for a specified (quasilinear) energy level for the electromagnetic waves in a plasma. The question of wave damping has not been considered in the present work; however, the calculation of the appropriate damping rates taking account of collisions does not present any fundamental difficulties. Collisions lead to a damping of the waves so that the maintenance of a stationary level of rapidly varying fields in a plasma is possible only in the presence of external sources (beams, external electric field and so on). If the external sources are specified, the quasilinear approximation can be used to determine the wave amplitudes (cf. for example [2]).

We also note that the method described here can be used to calculate nonlinear effects that arise in cyclotron resonances and in the case of small phase velocities.

In conclusion, the authors wish to express their gratitude to Yu. L. Klimontovich for guidance and interest in the work.

[1] Yu. L. Klimontovich, Statistical teoriya neravnovesnykh protsessov v plazme (Statistical Theory of Nonequilibrium Processes in a Plasma), Suppl. to English language edition, Sec. 19.

[2] Yu. L. Klimontovich and V. V. Logvinov, PMTF (Appl. Math. and Theoret. Phys.) 2, 35 (1967).

[3] A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, Usp. Fiz. Nauk 73, 701 (1961) [Sov. Phys.-Usp. 4, 332 (1961)]; V. L. Sizonenko and K. N. Stepanov, Zh. Eksp. Teor. Fiz. 49, 1197 (1965) [Sov. Phys.-JETP 22, 832 (1966)]; J. Rowlands, V. D. Shapiro, and V. I. Shevchenko, Zh. Eksp. Teor. Fiz. 50, 979 (1966) [Sov. Phys.-JETP 23, 651 (1966)]; J. Rowlands, V. L. Sizonenko, and K. N. Stepanov, Zh. Eksp. Teor. Fiz. 50, 994 (1966) [Sov. Phys.-JETP 23, 661 (1966)]; V. L. Sizonenko and K. N. Stepanov, Zh. Eksp. Teor. Fiz. 51, 858 (1966) [Sov. Phys.-JETP 24, 572 (1967)].

[4] V. I. Kogan, Plasma Physics and the Problem of Controlled Thermonuclear Reactions, Pergamon Press, New York, 1959, vol. 1.

[5] V. D. Shafranov, Reviews of Plasma Physics, Consultants Bureau, New York, 1967, vol. 3.

Translated by H. Lashinsky
216