CONTRIBUTION TO THE THEORY OF CURRENT FLUCTUATIONS IN SEMICONDUCTORS

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We investigate the fluctuations of the current, of the voltage across the sample, and of the voltage between probes in a semiconductor with carriers of both signs, when the fluctuations are connected with the generation, recombination, and annihilation of random electron-hole pairs on the (ohmic) contacts. We consider the influence of the dimension of the crystal and of the intensity of the external field on the magnitude and spectrum of the fluctuations. It is shown that the hypothesis of M. Lax, namely that the carrier-density fluctuations at different points are not correlated, is equivalent to the method of random currents of L. D. Landau and E. M. Lifshitz.

THERE is an extensive literature[1-6] devoted to the fluctuations of current and voltage in semiconductors with carriers of both signs. We shall show that the dimensions of the crystal have a strong influence on the magnitude and spectrum of the fluctuations.

1. EQUIVALENCE OF THE METHODS OF M. LAX[2,7] AND OF L. D. LANDAU AND E. M. LIFSHITZ[3,8]

We consider a semiconductor crystal of length 2L and cross section S with carriers of both signs and concentrations $n_\mp$ (with $n_- \gtrsim n_+$), connected in series with a resistor R_0. We assume that the generation-recombination processes of the electrons and holes are fully correlated, i.e., the corresponding random sources are $g_+(x, t) = g_-(x, t) = g(x, t)$ and that the carrier lifetimes ν^{-1} are identical.

Thus, the fluctuations are produced by random electron-hole pairs, which are created and annihilated simultaneously, recombining or falling on the contact. We shall consider quasilinear fluctuations $n'_+ = n'_- = n'$,

$$\operatorname{div} \mathbf{j}_-' = \operatorname{div} \mathbf{j}_+' \tag{1}$$

($\mathbf{j}'_\pm$ are the densities of the fluctuation currents).

We locate the origin at the center of the crystal, and assume that the contacts are ohmic:

$$n'(-L, t) = n'(L, t) = 0. \tag{2}$$

The linearized continuity equations in the one-dimensional problem in the presence of an external weak field E_0 are of the form

$$\frac{\partial n'}{\partial t} + \frac{\partial j_\pm'}{\partial x} + \nu n' = g(x, t), \tag{3}$$

$$j_\pm' = \pm \mu_\pm n_\pm E' \pm \mu_\pm E_0 n' - D_\pm \frac{\partial n'}{\partial x} + \tilde{j}_\pm(x, t), \tag{4}$$

$\mu_\pm$ and $D_\pm$ are the mobilities and diffusion coefficients of the holes and electrons, while $\tilde{j}_\pm$ are the random currents.

From (1) we get

$$j_+' - j_-' = \frac{I'(t)}{eS}, \tag{5}$$

I′ is the fluctuation current in the crystal and in the external circuit. We neglect the displacement current, a procedure valid at a frequency $\omega \ll \sigma = \sigma_+ + \sigma_- = en_+\mu_+ + en_-\mu_-$. From (5) we get the fluctuation field

$$E' = \frac{1}{\sigma}\left[\frac{I'}{S} - evn' - e(D_- - D_+)\frac{\partial n'}{\partial x}\right] + \frac{e}{\sigma}(\tilde{j}_- - \tilde{j}_+), \tag{6}$$

$v = (\mu_+ + \mu_-)E_0$. In calculating the potential difference

$$V' = \int_{x_0-d}^{x_0+d} E' dx$$

between probes located at points $x_0 \pm d$, the diffusion term drops out, since it is offset by the "contact" potential difference connected with the concentration difference; this is seen from the fact that when $E_0 = 0$ V′ should have an equilibrium value. Further

$$\frac{\partial n'}{\partial t} - D_a \frac{\partial^2 n'}{\partial x^2} + v_a \frac{\partial n'}{\partial x} + \nu n' = \left(\frac{\partial}{\partial t} + \Lambda\right)n'$$
$$= g - \frac{1}{\sigma}\left[\sigma_- \frac{\partial \tilde{j}_+}{\partial x} + \sigma_+ \frac{\partial \tilde{j}_-}{\partial x}\right] = \tilde{g},$$
$$v_a = \frac{e\mu_+\mu_-}{\sigma}(n_- - n_+)E_0; \quad D_a = \frac{e(\mu_+ D_- n_+ + \mu_- D_+ n_-)}{\sigma}. \tag{7}$$

Under condition (2), the solution of (7) is

$$n'(x, t) = \int_{-\infty}^{t} d\tau \int_{-L}^{L} d\xi G(x, \xi, t - \tau)\tilde{g}(\xi, \tau), \tag{8}$$

where G is the Green's function of the operator $\partial/\partial t + \Lambda(x)$ with boundary conditions (2). In the Fourier representation

$$n_\omega'(x) = \int_{-L}^{L} d\xi G_\omega(x, \xi)\tilde{g}_\omega(\xi),$$

where G_ω is the Green's function of the operator $-i\omega + \Lambda(x)$ with zero boundary conditions. Using the procedure described, for example, in[8], we obtain

$$G_\omega(\xi, \eta) = \frac{1}{2D_a\tilde{p}\operatorname{sh}\tilde{p}L}\begin{cases}[e^{p_1\xi} - e^{-\tilde{p}L+p_2\xi}][e^{\tilde{p}L-p_1\eta} - e^{-p_2\eta}] & \text{for } \xi < \eta \\ [e^{-\tilde{p}L-p_1\eta} - e^{-p_2\eta}][e^{p_1\xi} - e^{\tilde{p}L+p_2\xi}] & \text{for } \xi > \eta,\end{cases} \tag{9}$$

$$p_{1,2} = \frac{v_a}{2D_a} \pm \sqrt{\left(\frac{v_a}{2D_a}\right)^2 + \frac{\nu - i\omega}{D_a}}, \quad \tilde{p} = p_1 - p_2. \tag{10}$$

The fluctuation current I′(t) is determined from Kirchhoff's law

$$\int_{-L}^{L} E' dx + I'R_0 = U_0,$$

U_0 is a random source describing the equilibrium fluctuations of the voltage in the load. The random quantities g(x, t), $\tilde{j}(x, t)$, and $U_0(t)$ are not correlated with one another. According to[3,10], $\overline{U_0(t)U_0(t')} = 2TR_0\delta(t-t')$, and the spectral density is

$$(U_0^2)_\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\tau \overline{U_0(0)U_0(\tau)} e^{-i\omega\tau} = \frac{TR_0}{\pi}.$$

Analogously, using the expression for entropy generation by current flow[3], we have

$$\overline{\tilde{j}_\pm(x,t)\tilde{j}_\pm(x',t')} = \frac{2T\sigma_\pm}{e^2S}\delta(x-x')\delta(t-t').$$

Further, considering the individual generation and recombination processes as independent, in analogy with shot noise, we obtain[4]

$$\overline{g(x,t)g(x',t')} = \frac{1}{S}\frac{2\nu n_+n_-}{n_++n_-}\delta(x-x')\delta(t-t'). \qquad (11)$$

Finally, using $eD_\pm = T\mu_\pm$, we obtain

$$\overline{\tilde{g}(x,t)\tilde{g}(x',t')} = \frac{2n_+n_-}{S(n_++n_-)}\left[\nu\delta(x-x') - D_a\frac{\partial^2\delta(x-x')}{\partial x^2}\right]\delta(t-t')$$
$$= \frac{2n_+n_-}{S(n_++n_-)}[\Lambda(x)+\Lambda(x')]\delta(x-x')\delta(t-t'). \qquad (12)$$

We begin with the calculation of the fluctuations of the number of pairs in the interval $x_0 \pm d$. Using (9) and (12), we get

$$(\Delta N_d^2)_\omega = \frac{\overline{\Delta N_d^2}}{\pi D_a}\operatorname{Re}\frac{1}{\tilde{p}\operatorname{sh}\tilde{p}L}\left\{-\frac{\tilde{p}}{p_1p_2}\operatorname{sh}\tilde{p}L + \frac{1}{2p_1^2d}\operatorname{ch}(\tilde{p}L-2p_1d)\right.$$
$$+\frac{1}{2p_2^2d}\operatorname{ch}(\tilde{p}L+2p_2d) - \frac{1}{2d}\left(\frac{1}{p_1^2}+\frac{1}{p_2^2}\right)\operatorname{ch}\tilde{p}L$$
$$\left.-\frac{2}{p_1p_2d}\operatorname{ch}\tilde{p}x_0\operatorname{sh}p_1d\operatorname{sh}p_2d\right\}. \qquad (13)$$

Analogously

$$(I'^2)_\omega/(I'^2)_{\omega0} = 1 + \frac{\pi e^2\nu^2}{T\sigma}\left(\frac{R}{R+R_0}\right)^2\frac{(\Delta N_L^2)_\omega}{2LS}, \qquad (14)$$

$$(V'^2)_\omega/(V'^2)_{\omega0} = 1 + \frac{1}{TR_d}\left(1-\frac{R_d}{R+R_0}\right)^{-1}\left(\frac{e\nu}{\sigma}\right)^2\frac{n_-n_+}{S(n_-+n_+)}$$
$$\times\left\{\left(\frac{R_d}{R+R_0}\right)^2\operatorname{Re}\int_{-L}^{L}\int d\xi\, d\eta\, G_\omega(\xi,\eta) + \operatorname{Re}\int_{x_0-d}^{x_0+d}\int d\xi\, d\eta G_\omega(\xi,\eta)\right.$$
$$\left.-\frac{R_d}{R+R_0}\operatorname{Re}\int_{x_0-d}^{x_0+d}d\eta\int_{-L}^{L}d\xi[G_\omega(\xi,\eta)+G_{-\omega}(\eta,\xi)]\right\}\cdot \qquad (15)$$

$$(I'^2)_{\omega0} = \frac{T}{\pi(R+R_0)},\quad (V'^2)_{\omega0} = \frac{TR_d}{\pi}\left(1-\frac{R_d}{R+R_0}\right)$$

are the spectral densities of the equilibrium fluctuations of the current and the voltage,

$$R_d = \frac{2d}{\sigma S},\quad R = \frac{2L}{\sigma S},\quad \overline{\Delta N_d^2} = 2dS\frac{n_+n_-}{n_++n_-}$$

From (14) and (15) we see that the nonequilibrium parts $(I'^2)_\omega$ and $(V'^2)_\omega$ are connected with $(\Delta N_L^2)_\omega$ and $(\Delta N_d^2)_\omega$; for $(V'^2)_\omega$, this is true only when $R_0 \gg R$. We shall calculate the spectral density $(\Delta N_d^2)_\omega$, and knowing this quantity we can readily estimate $(I'^2)_\omega$ and $(V'^2)_\omega$. It is best to observe the current fluctuations when $R \gg R_0$, and the voltage fluctuations when $R_0 \gg R$. These are the cases that will be analyzed here.

M. Lax[2,7], proposing that $\overline{n'(x,t)n'(x',t)} \sim \delta(x-x')$, has shown that his assumption is equivalent to the choice of $\overline{\tilde{g}\tilde{g}}$ in the form (12). When random sources are introduced in the initial equations, a correlator of this type is obtained automatically, thus proving directly the equivalence of the Lax hypothesis and the method of random currents of L. D. Landau and E. M. Lifshitz[3,10].

2. CURRENT FLUCTUATIONS

The lifetime of a fluctuation pair is limited by three processes: diffusion towards the contacts (characteristic time $t_D \approx L^2/D_a$), drift towards the contacts ($t_d \approx L/v_a$), and recombination ($t_\nu = \nu^{-1}$). If the smallest of these times is smaller than the fluctuation period $1/\omega$, then the noise is white; in the opposite case it depends on the frequency. The distance over which the fluctuations diffuse within t_ν (i.e., the ambipolar diffusion length) is $L_D = \sqrt{D_a/\nu}$, and the distance over which it drifts (the ambipolar drift length) is $L_D = v_a/\nu$.

Let us consider the limiting cases.

1. Slow drift and long crystal, $L_D \ll L_D \ll L$. The fraction of the fluctuating pairs annihilating as the result of diffusion on the contacts after $t \approx 1/\omega$ is equal to $L^{-1}\sqrt{D_a t}$, and the fraction lost to recombination is νt. If $\omega \ll \max(\nu, D_a/L^2) = \nu$, then the noise is white. When $L^{-1}\sqrt{D_a t} \gg \nu t$, i.e., when $\omega \gg \nu(L+L_D)^2$, the recombination is negligible and the noise is due to diffusion (the latter idea was advanced in[2]). If $L^{-1}\sqrt{D_a t} \ll \nu t\,(\omega \ll \nu(L/L_D)^2)$, then the diffusion is negligible and the noise is determined by recombination. These considerations are confirmed by calculation. When $L_d \ll L_D \ll L$ we have

$$(\Delta N_L^2)_\omega = \frac{\overline{\Delta N_L^2}}{\pi}\left\{\frac{\nu}{\omega^2+\nu^2}\right.$$
$$\left.+\frac{1}{L}\sqrt{\frac{D_a}{2}}\frac{\omega[\sqrt{\omega^2+\nu^2}-\nu]^{1/2} - \nu[\sqrt{\omega^2+\nu^2}+\nu]^{1/2}}{(\omega^2+\nu^2)^{3/2}}\right\}. \qquad (16)$$

If $\omega \ll 2\nu(L/L_D)^2$, the first term predominates, i.e., generation-recombination (GR) noise takes place.

With this, according to (14), the non-equilibrium part of the fluctuations is

$$\delta I_\omega' = \frac{(I'^2)_\omega - (I'^2)_{\omega0}}{(I'^2)_{\omega0}} = \frac{(\mu_++\mu_-)^2E_0^2}{D_-n_-+D_+n_+}\frac{n_-n_+}{n_-+n_+}\frac{\nu}{\omega^2+\nu^2}.$$

The non-equilibrium noise is negligible in a non-intrinsic semiconductor. In an intrinsic semiconductor, where there is no ambipolar drift and the condition $L_d \ll L_D$ is automatically satisfied at any field intensity, we have

$$\delta I_\omega' = \frac{eE_0^2(\mu_++\mu_-)}{2T}\frac{\nu}{\omega^2+\nu^2}$$

which can exceed unity.

If $\omega \gg 2\nu(L/L_D)^2$, then the diffusion term predominates and (we have in mind $|\omega|$ throughout)

$$(\Delta N_L^2)_\omega = \frac{\overline{\Delta N_L^2}}{\pi L}\left(\frac{D_a}{2\omega^3}\right)^{1/2}. \qquad (17)$$

2. Slow drift and short crystal, L_d, $L \ll L_D$. The fluctuation pair has time to diffuse towards the contacts within the time L^2/D_a much smaller than the time ν^{-1}, and therefore the noise is due to diffusion.

At low frequencies $\omega \ll D_a/L^2$ we have

$$(\Delta N_L{}^2)_\omega = \frac{\overline{\Delta N_L{}^2} L^2}{3\pi D_a};$$

$$\delta I_\omega' = \frac{1}{3}\left(\frac{eE_0L}{T}\right)^2 \frac{(\mu_+ + \mu_-)^2}{\mu_+\mu_-} \frac{n_+n_-}{(n_+ + n_-)^2} \tag{18}$$

which can be large for an intrinsic semiconductor. At large frequencies $\omega \gg D_a/L^2$ formula (17) is valid. In items 1 and 2, the field does not influence $(\Delta N_L^2)_\omega$.

3. **Rapid drift,** $L_d \gg L_D$. When $\omega \gg \omega_0 = v_a^2/4D_a$, the drift is negligible (at all values of L). This is due to the fact that the fluctuation pair diffuses within a time $t = 1/\omega$ over a distance $\sqrt{D_a/\omega}$, which is larger than the drift distance v_a/ω, if $\omega \gg \omega_0$.

When $\omega \ll \omega_0$, expanding (13) in powers of L_D/L_d and ω/ω_0, we obtain

$$(\Delta N_L{}^2)_\omega = \frac{\overline{\Delta N_L{}^2}}{\pi}\frac{\nu}{\omega^2+\nu^2}\left\{1 + \frac{v_a}{2L\nu(\omega^2+\nu^2)}\left[(\omega^2-\nu^2) - \exp\left\{-\frac{2L\nu}{v_a}\left(1+\frac{D\omega^2}{v_a{}^2\nu}\right)\right\}\left((\omega^2-\nu^2)\cos\frac{2\omega L}{v_a} + 2\omega\nu\sin\frac{2\omega L}{v_a}\right)\right]\right\}. \tag{19}$$

It is necessary to distinguish between the case $L \gg L_d$, when the noise is of the generation-recombination type (since the fluctuation does not have time to reach the contact), the case

$$L \ll L_D{}^2/L_d \tag{20}$$

when it is possible to neglect recombination in such a short crystal and the diffusion in the case of (20) predominates over the drift and the noise is of the diffusion type, and the case $L_d \gg L \gg L_D^2/L_d$.

4. **The case** $L_d \gg L \gg L_D^2/L_d$. In this case there are four characteristic frequencies

$$\nu,\ \omega_1 = \frac{v_a}{2L},\ \omega_2 = \sqrt{\frac{v_a{}^3}{2D_aL}} \ \text{и} \ \omega_0 = \frac{v_a{}^2}{4D_a};$$

$$\nu \ll \omega_1 \ll \omega_2 \ll \omega_0.$$

Expanding (19) in powers of L/L_d and ω/ω_1, we obtain

$$(\Delta N_L{}^2)_\omega = \frac{\overline{\Delta N_L{}^2} L}{\pi v_a}; \quad \delta I_\omega' \approx \frac{v_aL}{D_a}\frac{\mu_-n_+}{\mu_+n_-}$$

which is much larger than unity.

In this case the noise is white, which is understandable, since the fluctuation has time to vanish on the contacts after $L/v_a \ll \omega^{-1}$. When $\omega_1 \ll \omega \ll \omega_2$, accurate to terms of order L/L_d and ω_1/ω, we get from (19)

$$(\Delta N_L{}^2)_\omega = \frac{\overline{\Delta N_L{}^2}}{\pi}\frac{\nu}{\omega^2}\left[1 + \frac{v_a}{L\nu}\sin^2\frac{\omega L}{v_a}\right]. \tag{21}$$

When ω is not too close to $2\pi n\omega_1$ ($n = 1, 2, \ldots n_{max} \ll \sqrt{v_aL/2\pi^2D_a}$), the unity term can be neglected. $(\Delta N_L^2)_\omega$ oscillates, and its value changes by a larger amount ($L_d/L \gg 1$) in the interval $\pi\omega_1$, experiencing at the same time many oscillations. The noise is modulated like $\sim 1/\omega^2$. The amplitude of the noise, $\approx \overline{\Delta N_L^2} v_a/\pi L\omega^2$, is smaller by a factor $(L\omega/v_a)^2$ than when $\omega \ll \omega_1$, but is larger by $(\omega_2/\omega)^2$ than the equilibrium value. Finally, when $\omega_2 \ll \omega \ll \omega_0$ the addition to the equilibrium noise is small ($\delta I_\omega' < 1$)

$$(\Delta N_L{}^2)_\omega = \frac{\overline{\Delta N_L{}^2}}{\pi}\frac{v_a}{2L\omega^2}.$$

3. FLUCTUATIONS OF VOLTAGE BETWEEN PROBES

The fluctuations of the voltage between the probes do not depend on the presence of ohmic contacts and turn out to be the same as in an infinite sample, provided the distance from the probes to the contacts is equal to several diffusion lengths at $L_D \gg L_d$. When $L_d \gg L_D$, this distance must be not several times L_d, but larger than $L_1 = D_a/v_a \ll L_D$; in this case the diffusion term in (4) is smaller than D_an'/L_1, i.e., is smaller than the drift term v_an'. This follows formally from the fact that when $L_d \gg L_D$ and $L \gg L_1$ (9) coincides with the Green's function for an infinite sample in the segment $(-L, L)$, when η and ξ are separated from $\pm L$ by L_1.

Putting $L \to \infty$, we obtain from (13)

$$(\Delta N_d{}^2)_\omega = \frac{\overline{\Delta N_d{}^2}}{\pi D_a}\operatorname{Re}\frac{1}{(p_1-p_2)}\left[-\frac{p_1-p_2}{p_1p_2} + \frac{1}{2p_1{}^2d}e^{-2p_1d} + \frac{1}{2p_2{}^2d}e^{2p_2d} - \frac{1}{2d}\left(\frac{1}{p_1{}^2}+\frac{1}{p_2{}^2}\right)\right]. \tag{22}$$

Let us consider two cases.

1. **Slow drift** $L_d \ll L_D$. When the probes are close together, $2d \ll L_D$, there are two characteristic frequencies ν and $\omega_3 = D_a/d^2$. If $\omega \gg \omega_3$, then (22) differs from the corresponding result of section 2 in that L is replaced by d and a coefficient 1/2 is added.

When $\nu \ll \omega \ll \omega_3$ we have $(\Delta N_d^2)_\omega = \overline{\Delta N_d^2} d/\pi\sqrt{D_a\omega}$, and when $\omega \ll \nu$ we have $(\Delta N_d^2)_\omega = \overline{\Delta N_d^2} d/\pi\sqrt{D_a\nu}$.

In the case of large distances between the probes, the fluctuations in the region of the diffusion noise differ from those calculated in item 1 of section 2 in that L is replaced by d and a coefficient 2 is added.

2. **Rapid drift and remote probes** $L_d \gg L_D$ **and** $d \gg L_1$. When $d \ll L_d$ and $\omega \ll \omega_0$, the fluctuations are the same as in item 4 of Sec. 2, and when $\omega \gg \omega_0$ they are half as small (with L replaced by d).

3. **Dependence of fluctuations on the coordinate** x_0 **of the center of the interval between the probes.** In the case of slow drift and large distance between the probes, if $\omega \gg 2\nu(d/L_D)^2$, then the noise at $x_0 = \pm(L - d)$ is half again as large as noise deep in the sample. When the probes are close together and $\omega \ll \omega_3$, the noise at the contact is white. When $\omega \gg D_a/d^2$, the noise at the contact is twice as large than inside the sample. When $L_d \gg L_D$ and $d \gg D_a/v_a$, the noise at the contact differs from the noise in the interior only if $\omega \gg \omega_0$, when it is twice as large at the contact.

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