# *ACOUSTIC PLASMA WAVES IN THIN FILMS*

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It is shown that weakly damped acoustic plasma waves can propagate along thin films when the transverse electron motion is quantized. In the case of a highly degenerate electron gas (to which the present analysis is limited) it is found that there are several such waves with phase velocities close to the Fermi velocity for electrons in the upper film levels.

1. Weakly damped acoustic waves can propagate in any equilibrium plasma consisting of two or more species of charged particles. These waves correspond to oscillations in the density of particles of the different species with phase shifts that adjust themselves to provide overall neutrality of the total oscillations.

If particle collisions are neglected, the basic contribution to the damping of the acoustic wave is Landau damping. In a uniform infinite plasma that is not located in a magnetic field this damping can be small if all of the plasma particles can be effectively divided into two groups which exhibit significantly different Fermi (thermal) velocities in the direction of propagation of the wave.[1-3] The presence of a quantizing magnetic field leads to the separation of even a single-component plasma into particle classes with different discrete velocities along the magnetic field. In this case, as has been recently pointed out by Konstantinov and Perel[4] there exist branches of the acoustic wave that propagate along the magnetic field with zero Landau damping.

A similar division into particle classes occurs in thin films. The transverse motion of the current carriers is quantized in the film. In this case the particle energy is determined by the quasi-momentum **k** which lies in the plane of the film and some discrete quantum number n, which characterizes the energy of the transverse motion. Consequently, the current carriers divide into groups characterized by different quantum numbers n and Fermi (thermal) velocities along the film. In a system of this kind, as we have indicated above, it is possible to support acoustic waves.

In the present work we consider plasma waves that propagate along a film in which film quantization occurs.[5] In the analysis, for reasons of simplicity we shall assume that the electron gas in the film is highly degenerate and that the electron collisions with all possible defects are so rare that they can be neglected. It is shown that in sufficiently thin films there exist several branches of weakly damped plasma waves that are characterized by an acoustic spectrum. The existence of these waves requires that the criteria given in (4), (5), (8), and (12) be satisfied. These criteria can be satisfied in semiconductors and in semimetal films.

2. We start with the dispersion equation given in[6,7] for an even E wave propagating along the plasma film. At phase velocities much smaller than the phase velocity of light this expression is

$$\frac{2}{L}\sum_{q_3}\frac{q}{(q^2+q_3^2)\,\varepsilon^l(q,q_3,\omega)}=-\frac{1}{\varepsilon_0} \tag{1}$$

where L is the film thickness; q is the wave vector for the plasmon which lies in the plane of the film; $q_3 = 2\pi\nu/L$, $\nu = 0, \pm1, \pm2\ldots$, $\omega$ is the plasmon frequency; $\epsilon^l$ is the longitudinal dielectric constant of the plasma and $\epsilon_0$ is the dielectric constant of the medium surrounding the plasma layer.

Equation (1) has been obtained for a uniform classical plasma under the assumption of specular reflection of the plasma particles at the boundaries. A rigorous derivation of the dispersion relation for a bounded quantum plasma involves considerable difficulties, these difficulties being associated primarily with the inhomogeneity near the edge. If this inhomogeneity is neglected then, as shown in[7], the dispersion relation in (1) still holds but the dielectric constant that appears in it $\epsilon^l(q, q_3, \omega)$ is now given by the expression

$$\varepsilon^l(q,q_3,\omega)=1+\frac{8\pi e^2}{(q^2+q_3^2)L}\sum_n\int\frac{d^2k}{(2\pi)^2}\frac{f(\varepsilon_{n+\nu,\,k+q})-f(\varepsilon_{n,\,k})}{\hbar\omega+\varepsilon_{n,k}-\varepsilon_{n+\nu,\,k+q}+i\delta} \tag{2}$$

where $\epsilon_{n,k} = \epsilon_n + \hbar^2k^2/2m_\parallel$ is the electron energy in the film, $\epsilon_n$ is the energy of the quantized transverse motion of the electrons [in order to make estimates we will take $\epsilon_n = (\hbar^2/2m_\perp)(\pi n/L)^2$], $n = \pm1, \pm2\ldots$, $m_\parallel$ ($m_\perp$) in the electron mass which corresponds to the longitudinal (transverse) motion in the film, f is the Fermi function, $\delta \to +0$, and particle collisions are neglected; this last procedure is valid if the wave frequency is much larger than the collision frequency.

Equation (2) differs from the expression for the longitudinal dielectric constant in an inhomogeneous plasma in that the integration over the transverse momentum of the electron is replaced by a summation over the quantization number n.

This form of the dispersion equation is to be expected if one starts from qualitative considerations regarding the spectrum of an electron in the film. For this reason it is to be hoped that the results that follow from this equation will, at least qualitatively, remain valid if one takes account of the inhomogeneity of the equilibrium plasma near the edge of the film. We now write Eq. (1) in the form

$$\frac{2}{qL\varepsilon^l(q_3=0)}+\frac{2}{L}\sum_{q_3\neq0}\frac{q}{(q^2+q_3^2)\,\varepsilon^l(q,q_3,\omega)}=-\frac{1}{\epsilon} \tag{3}$$

The first term in (3) describes transitions within a band, i.e., transitions between states with fixed quantization number n; the second term describes transitions between different bands.

In what follows we shall limit our analysis to longwave plasmons with energies comparable with the energy spacing between bands, that is to say, we shall assume that the following inequalities are satisfied:

$$qL \ll 1, \tag{4}$$

$$\hbar\omega \ll \varepsilon_{n+1} - \varepsilon_n. \tag{5}$$

It is easy to show that when (5) is satisfied then $\epsilon^l (q_3 \neq 0) > 1$ and consequently the second term in (3) is smaller than the quantity $qL/6$. Thus, we can neglect the effect of transitions between different bands on the formation of the longwave plasmons. Under these conditions the dispersion equation (3) assumes the simple form

$$\varepsilon^l(q_3 = 0) = -2\,\varepsilon_0 / qL. \tag{3$'$}$$

We now wish to calculate the dielectric constant $\epsilon^l (q_3 = 0)$. It will be assumed that the electron gas in the film is highly degenerate. Replacing the distribution function for $\epsilon_{n,k} \leq \epsilon_F$ ($\epsilon_F$ is the Fermi level for the current carriers in the film) by unity and integrating Eq. (2) with respect to the longitudinal component of the electron momentum, which is perpendicular to the vector $\mathbf{q}$, we have

$$\varepsilon^l(q_3 = 0) = 1 + \frac{4m_{\parallel}e^2}{\pi\hbar^2 L q^3} \sum_n k_n \int_{-1}^{1} dx \left( \frac{\sqrt{1-x^2}}{x - p_-} - \frac{\sqrt{1-x^2}}{x - p_+} \right), \tag{6}$$

where we have introduced the notation $p_{\mp} = s/v_n \mp q/2k_n$, $s = \omega/q$ is the phase velocity of the plasma wave and $v_n (K_n)$ is the Fermi velocity (Fermi wave vector) of an electron in the n-th band.

Using the relation

$$\int_{-1}^{1} dx \frac{\sqrt{1-x^2}}{x-p} = \pi(\sqrt{p^2-1} - p),$$

we can write the expression for the longitudinal dielectric constant in the form

$$\varepsilon^l(q_3 = 0) = 1 + \frac{4}{aLq^2} \left\{ N - \sum_{n=1}^{N} \frac{k_n}{q} \left[ \sqrt{\left( \frac{s}{v_n} + \frac{q}{2k_n} \right)^2 - 1} - \sqrt{\left( \frac{s}{v_n} - \frac{q}{2k_n} \right)^2 - 1} \right] \right\}. \tag{7}$$

Here, N is the number of filled bands in the film, $a = \hbar^2/m_{\parallel}e$ is the effective Bohr radius.

Analysis of Eq. (3′) with the dielectric constant in (7) shows that in sufficiently thin films, in addition to the usual surface waves characterized by frequency $\omega \approx \omega_{\text{film}} \sqrt{qL/2\epsilon_0}$ there can also be a new kind of plasma wave which is characterized by an acoustic dispersion relation. These waves are associated with the quantization of the transverse motion of the electron and do not appear in thick films in which the film quantization in unimportant.

In order to obtain the dispersion relation for the acoustic waves we shall assume that the following inequality is satisfied in addition to those in (4) and (5):

$$\left| \frac{s^2}{v_n^2} - 1 \right| \gg \frac{q}{k_n} \left( \frac{s}{v_n} + \frac{q}{4k_n} \right). \tag{8}$$

In this case the dispersion equation is simplified considerably and assumes the form

$$N - \sum_n \left[ 1 - \left( \frac{v_n}{s} \right)^2 \right]^{-1/2} = 0. \tag{9}$$

In the derivation of (9) we have made use of the condition $aq \ll 1$ which is always satisfied for longwave oscillations.

Equation (9) is actually the equation $\epsilon^l = 0$; consequently the effect of the surrounding dielectric medium on the acoustic wave spectrum is found to be unimportant. Physically this result is associated with the fact that the electromagnetic field produced by the acoustic wave is small (this is the only mechanism that can have an effect on the surrounding medium).

3. Assume that the phase velocity of the wave lies between the Fermi velocities of the $m - 1$ and $m + 1$ layers, that is to say

$$v_N < v_{N-1} < \ldots < v_{m+1} < s < v_{m-1} < \ldots < v_1.$$

We divide s into real and imaginary parts $s = s_1 - is_2$ and assume that $s_2 \ll s_1$. It will be shown below that the phase velocity $s_1$ is close to the Fermi velocity $v_m$ and for this reason in all of the terms aside from the m-th term we can write $s = v_m$. This procedure is valid provided

$$|s^2 - v_m^2| \ll |v_n^2 - v_m^2|, \quad n \neq m. \tag{8$'$}$$

We can now write Eq. (9) in the form

$$(a + ib) \left[ 1 - \frac{v_m^2}{s_1^2} - 2i\frac{s_2}{s_1} \right]^{1/2} = 1, \tag{10}$$

where

$$a = N - \sum_{n=m+1}^{N} \frac{v_m}{\sqrt{v_m^2 - v_n^2}}, \quad b = \sum_{n=1}^{m-1} \frac{v_m}{\sqrt{v_n^2 - v_m^2}}.$$

It then follows that

$$s_1 = v_m \left[ 1 - \frac{a^2 - b^2}{(a^2 + b^2)^2} \right]^{-1/2}, \quad s_2 = s_1 \frac{ab}{(a^2 + b^2)^2}. \tag{11}$$

It is evident from (11) that the damping of the acoustic wave is small and that $s_1 \approx v_m$ (as assumed above) provided

$$\frac{\max(a^2, b^2)}{(a^2 + b^2)^2} \ll 1. \tag{12}$$

When $a^2 > b^2$ we have $s_1 > v_m$ and when $a^2 < b^2$ we have $s_1 < v_m$.

Analysis of Eq. (11) shows that the most weakly damped wave is the one characterized by $m = N$. For this wave $a = N > b$ and $s_1 \sim v_N(1 + 1/2N^2)$, $s_2 \lesssim s_1/N^2$. As the number m diminishes the wave damping increases rapidly. However the phase velocity of the wave approaches the corresponding Fermi velocity ($v_m$) and then becomes somewhat smaller. The approximation used in the derivation of Eq. (11) is not satisfied for waves characterized by small values of m and the determination of the corresponding roots requires a more exact solution of Eq. (9).

We can estimate the magnitude of phase velocity and the damping of the acoustic wave in a film for a particular case. Assume that the electron density in the conductivity band is $\sim 10^{18}$ cm$^{-3}$, $L \sim 5 \times 10^{-6}$ cm, $m \sim 0.01\, m_e$. Under these conditions five film levels will be filled ($N = 5$) and the corresponding Fermi velocities are $(0.8, 2.1, 2.7, 3, 3.2) \times 10^8$ cm/sec. In this case there exist at least three weakly damped acoustic waves with velocities $s_1$ equal to 1.02 $v_5$; 1.01 $v_4$ and 0.99 $v_3$. The damping of these waves is respectively 0.006 $v_5$; 0.025 $v_4$ and 0.03 $v_3$. It follows from Eq. (8) that $q \ll 2 \times 10^4\ \text{cm}^{-1}$ (the other criteria are less stringent) and consequently $\omega_{max} \sim 10^{12}\ \text{sec}^{-1}$. A suitable material for observing these waves might be a semiconductor or a semimetal with a small effective carrier mass, for example InSb or Bi.

The analysis given above refers to the case of a single component plasma with an isotropic electron dispersion in the film plane. The generalization to the case of an anisotropic dispersion relation and the existence of several energy minima should not represent any particular difficulty.

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173