INTERACTION OF OPPOSITELY DIRECTED WAVES IN A SOLID STATE RING LASER

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Interaction between oppositely directed waves during a transient process in a solid state ring laser is investigated. It is shown that when the initial values are unequal in the absence of scatter-induced coupling the intensities of oppositely directed waves can differ significantly in magnitude, although there is no rapid and complete suppression of the less intense wave. The suppression time is longer than or equal in the order of magnitude to the time it takes to establish stationary regime in the case of a single traveling wave. The interaction of oppositely directed waves changes significantly when scatter-induced coupling is taken into account. A periodic energy transfer occurs between the waves at a frequency proportional to the coupling coefficient. When the coupling coefficient is high enough the wave intensities begin rapid opposed-phase oscillations and the emission spikes have an oscillating structure.

INTRODUCTION

COUPLING between oppositely directed waves in a ring laser is caused by the nonlinearity of the active medium and by scattering of some energy from one wave into another. The scattering is due to diffraction on mirrors, inhomogeneities, reflection from crystal end faces, etc. The interaction of oppositely directed waves in a solid state laser with homogeneously broadened luminescence line was studied in[4,5]. It was shown that the single traveling wave regime is stable and the oppositely directed wave regime is unstable.

The instability of the latter regime in a solid state ring laser may be a factor favorable for the achievement of single-mode generation. In the single traveling wave regime there is a spatially homogeneous extinction of population inversion in the active medium and as shown in a number of studies (such as[6]) a laser with a homogeneously broadened luminescence line generates a single mode competition. This was verified experimentally in[7]. To obtain a traveling wave in a ring laser a non-reciprocal element was used that substantially spoiled the Q-factor for one of the oppositely directed waves. No narrowing of the generation spectrum was observed without the non-reciprocal element. This result indicates the absence of an effective competitive suppression of one of the oppositely directed waves while the laser is pumped with a pulsed lamp.

The present paper deals with the interaction of oppositely directed waves during a nonlinear transient process in which oscillations are established in a solid-state ring laser.

Because of the involved system of equations describing the field dynamics and the variation of population inversion in the transient process the problem was solved by numerical integration with an electronic computer. A qualitative analysis of the solutions was also performed. Two cases were considered: (a) the case of absence of back-scatter-induced coupling and (b) the case of equal coefficients of scatter-induced coupling. It is shown that just as in the case of gas ring lasers[2] back scatter-induced coupling can cause a periodic energy transfer from one wave to another at a frequency that depends on the magnitude of the coupling coefficient.

1. COMPUTATION OF THE POLARIZATION VECTOR

To compute the polarization vector $P_{1,2}$ we use the density matrix equations assuming that the active atoms are fixed:

$$\frac{\partial \rho_{ab}}{\partial t} = i\omega_0\rho_{ab} + \frac{id_{ab}}{\hbar}NE - \gamma_{ab}\,\rho_{ab},$$
$$\frac{\partial N}{\partial t} = -\frac{2i}{\hbar}(d_{ab}\,\rho_{ba} - \rho_{ab}\,d_{ba})E - \gamma(N - N_p), \tag{1.1}$$

Here $N = \rho_b - \rho_a$ is the population inversion, γ^{-1} is the luminescence time, γ_{ab} is the width of a homogeneous luminescence line, γN_p is the pumping rate, d_{ab} is the matrix element of the operator of the atomic dipole moment, and ω_0 is the natural frequency of atomic transition from level b to level a. In the case of single mode regime the electric field in a ring laser can be represented in the form

$$E = \frac{1}{2}\sum_{1,2}[E_{1,2}\,e^{i(\omega t\mp kx)} + \text{c.c.}] = \frac{1}{2}\sum_{1,2}(\tilde{E}_{1,2} + \text{c.c.}). \tag{1.2}$$

The solution of (1.1) is sought in the form

$$\rho_{ab} = \sum_{1,2}\rho_{ab}^{(1,2)}e^{i(\omega t\mp kx)},$$
$$N = N_0 + (N_1e^{-i2kx} + \text{c.c.}), \tag{1.3}$$

where $\rho_{ab}^{(1,2)}$, N_0 and N_1 are time functions that are slow in comparison to optical oscillations. This form of the solution takes into account the spatially inhomogeneous extinction of population inversion in the field of oppositely directed traveling waves. Terms with the time-dependent second harmonic in the population inversion are neglected since their contribution is small (of the order of γ/ω).

In a solid-state laser polarization follows the field in a quasi-static manner since it varies much faster than the field

$$\gamma_{ab} \gg \omega / Q, \qquad (1.4)$$

and the population inversion varies much slower than the field

$$\gamma \ll \omega / Q. \qquad (1.5)$$

The parameter $G = \omega/Q\gamma$ is of the order of $10^3 - 10^5$ in solid-state lasers. This condition imposes the spiking character on transient processes in solid-state lasers.

Taking (1.4) and (1.5) into account, Eq. (1.1) can be used to express polarization in terms of population inversion and the field, and population inversion can be expressed by differential equations (1.7) after separating the spatially homogeneous and inhomogeneous parts:

$$\rho_{ab}^{(1)} = \frac{d_{ab}}{2\hbar} \frac{N_0 E_1 + N_1 E_2}{\omega - \omega_0 - i\gamma_{ab}} \equiv \frac{\varkappa_1}{d_{ba}} E_1,$$

$$\rho_{ab}^{(2)} = \frac{d_{ab}}{2\hbar} \frac{N_0 E_2 + N_1^* E_1}{\omega - \omega_0 - i\gamma_{ab}} \equiv \frac{\varkappa_2}{d_{ba}} E_2,$$

$$\frac{\partial N_0}{\partial t} = i \frac{|d_{ab}|^2}{2\hbar^2} \left[\frac{N_0 |E_1|^2 + N_1 E_1^* E_2}{\omega - \omega_0 - i\gamma_{ab}} + \frac{N_0 |E_2|^2 + N_1^* E_1 E_2^*}{\omega - \omega_0 - i\gamma_{ab}} - \text{c.c.} \right] - \gamma (N_0 - N_p), \qquad (1.6)$$

$$\frac{\partial N_1}{\partial t} = i \frac{|d_{ab}|^2}{2\hbar^2} \left[\frac{N_0 E_1 E_2^* + N_1 |E_2|^2}{\omega - \omega_0 - i\gamma_{ab}} - \frac{N_0 E_1 E_2^* + N_1 |E_1|^2}{\omega - \omega_0 + i\gamma_{ab}} \right] - \gamma N_1. \qquad (1.7)$$

The polarization vector P is represented in the form

$$P = \sum_{1,2} P_{1,2}\, e^{i(\omega t \mp kx)} + \text{c.c.}, \qquad (1.8)$$

Then

$$P_{1,2} = d_{ba}\, \rho_{ab}^{(1,2)} n_0 \qquad (1.9)$$

(n_0 is the active atom density). Relations (1.6)—(1.9) express polarization in terms of the field and population inversion in the active medium.

2. SCATTER-INDUCED COUPLING BETWEEN OPPOSITELY DIRECTED WAVES

The coupling between oppositely directed waves due to scattering was considered in a number of papers[8,9] for the case of gaseous ring lasers. The scatter-induced coupling is typically assumed to be linear and is introduced into the wave equation in the form

$$\frac{\partial^2 \tilde{E}_{1,2}}{\partial t^2} + \frac{\omega}{Q_{1,2}} \frac{\partial \tilde{E}_{1,2}}{\partial t} - c^2 \frac{\partial^2 \tilde{E}_{1,2}}{\partial x^2} = -4\pi \frac{\partial^2 P_{1,2}}{\partial t^2} + \omega m_{1,2} \tilde{E}_{2,1}, \qquad (2.1)$$

where $Q_{1,2}$ are Q-factors of the ring resonator referred to the oppositely directed waves and $m_{1,2}$ are the complex coupling coefficients. The $m_{1,2}$ coefficients are expressed in terms of the reflection coefficients $\rho_{1,2}$ of backward waves in the following manner

$$m_{1,2} = i \frac{c}{L} \rho_{1,2}. \qquad (2.2)$$

Here L/c is the time of a single pass around the ring.

According to the reciprocity theorem the Q equality condition $Q_1 = Q_2$ should hold and the coupling coefficients should obey the complex conjugate relation $m_1 = m_2^*$. However deviations from these conditions do occur in real ring lasers[10].

Substituting (1.6) and (1.9) into the wave equation (2.1) we obtain the following abbreviation of the complex amplitude equation for the oppositely directed waves:

$$\dot{E}_{1,2} + \frac{\omega}{2Q_{1,2}} (E_{1,2} + i\tilde{m}_{1,2} E_{2,1}) = -4\pi n_0 \omega i \varkappa_{1,2} E_{1,2}. \qquad (2.3)$$

The following designations are used here:

$$\tilde{m}_{1,2} = \frac{Q_{1,2}}{\omega} m_{1,2} = i \frac{Q_{1,2}}{\omega} \frac{c}{L} \rho_{1,2}, \qquad \tilde{m}_{1,2} = m_{1,2}^{(r)} + i m_{1,2}^{(i)} \qquad (2.4)$$

The abbreviated equation system (2.3) for the complex amplitudes of the oppositely directed waves together with (1.7), expressing population inversion, represent the interaction of oppositely directed waves in a solid state ring laser. We note that the field is not assumed small and is not represented by a power-series expansion in the derivation of (1.7) and (2.3); in this sense these equations provide an exact expression for the coupling between oppositely directed waves due to nonlinearity of the active medium.

In the absence of scatter-induced coupling ($m = 0$) (1.7) and (2.3) are equivalent to the system of equations obtained by Zhelnov, Kazantsev, and Smirnov[4]. Dimensionless variables are convenient here:

$$x = a(|E_1|^2 - |E_2|^2), \qquad y = a(|E_1|^2 + |E_2|^2),$$
$$z = -a|E_1||E_2| \sin \Phi, \qquad u = a|E_1||E_2| \cos \Phi,$$
$$\eta = \frac{N_0 - N_{th}}{N_{th}}, \qquad n = \frac{\operatorname{Re} N_1}{N_{th}}, \qquad n_i = \frac{\operatorname{Im} N_1}{N_{th}} \qquad (2.5)$$

Dimensionless time is $\tau = \gamma t$. In (2.5) $a = d_{ab}^2/\hbar^2 \gamma_{ab} \gamma$, $|E_{1,2}|$ and $\Phi = \Phi_1 - \Phi_2$ are the amplitudes and phase difference respectively of the oppositely directed waves, and $N_{th} = \hbar \gamma_{ab} / 4\pi d_{ab}^2 Q n_0$ is the threshold value of population inversion N_0.

In terms of the (2.5) variables the system (2.3) and (1.7) assumes the following form

$$\dot{x} = G[\eta x + (m_1^r + m_2^r) z + (m_1^i + m_2^i) u], \qquad (2.6a)$$
$$\dot{y} = G[\eta y + 2(nu + n_i z) + (m_1^r - m_2^r) z + (m_1^i + m_2^i) u] \qquad (2.6b)$$
$$\dot{z} = G\left[\eta z + \frac{1}{2} n_i y - \frac{m_1^r + m_2^r}{4} x + \frac{m_1^r - m_2^r}{4} y \right], \qquad (2.6c)$$
$$\dot{u} = G\left[\eta u + \frac{1}{2} n y - \frac{m_1^i - m_2^i}{4} x + \frac{m_1^i + m_2^i}{4} y \right], \qquad (2.6d)$$
$$\dot{n} = -n - (1 + \eta) u - {}^1\!/_2 y n, \qquad (2.6e)$$
$$\dot{n}_i = -n_i - (1 + \eta) z - {}^1\!/_2 y n_i, \qquad (2.6f)$$
$$\dot{\eta} = \alpha - \eta - (1 + \eta) y - 2(nu + n_i z). \qquad (2.6g)$$

Here $G = \omega/Q\gamma$, and $\alpha = (N_p - N_{th})/N_{th}$ is the relative pump excess over threshold; a dot designates differentiation with respect to the dimensionless time γt.

Terms proportional to $(\omega - \omega_0)/\gamma_{ab}$ are dropped from (2.6) which formally corresponds to tuning the opposed wave frequency to the center of the amplification line. In reality however due to the large width of the luminescence line ($\gamma_{ab} \sim 10^{11}$ sec^{-1}) the dropped terms are small in solid state lasers when the mismatch is of the order of the distance between axial modes (10^8 sec^{-1}).

3. INVESTIGATION OF OPPOSITELY DIRECTED WAVE INTERACTION

In the absence of scatter-induced coupling ($\tilde{m}_{1,2} = 0$) the system of eqs. (2.6) has a stationary solution corresponding to the case of two oppositely directed waves with equal amplitude.

$$x_0 = 0, \quad y_0 = 2u_0 = {}^2/_3\alpha, \quad \eta_0 = -n_0 = \alpha/3, \quad n_{i_0} = z_0 = 0 \qquad (3.1)$$

It is typical that due to the spatially inhomogeneous extinction of population inversion in the oppositely directed wave regime the homogeneous part of N_0 has, according to (3.1), a value in the stationary state that is larger than the threshold N_{th} ($\eta_0 = {}^1/_3\alpha > 0$), while in the case of a completely homogeneous extinction of population inversion the stationary value of N_0 is equal to N_{th}.

In the case of small perturbations relative to the stationary state (3.1) we obtain from (2.6a)

$$\dot{x} = G\eta_0 x, \qquad (3.2)$$

i.e., the case of two oppositely directed waves in the absence of a scatter-induced coupling is unstable and the oppositely directed wave intensity difference increases with an increment

$$\lambda = \frac{\omega}{Q}\eta_0 = \frac{1}{3}\frac{\omega}{Q}\alpha = \frac{1}{3}G\alpha\gamma. \qquad (3.3)$$

This value of the instability increment was obtained in[4,5]. According to (3.3) competitive suppression of one of the oppositely directed waves should be rapid with the characteristic time of instability development being $t = 1/\lambda \ll 1/\gamma$.

As noted above no effective suppression of one of the oppositely directed waves was observed in the experiment[7] during a much longer period of time $t \gg 1/\lambda$. Therefore it is of interest to perform a theoretical investigation of the oppositely directed wave competition in a nonlinear transient process rather than in the case of small perturbations.

We first consider qualitatively the nature of oppositely directed wave competition during the transient process for the case of zero scatter-induced coupling. For $\tilde{m} = 0$ the system (2.6) can be simplified on the assumption that the initial value of phase difference $\Phi = 0$ and consequently $z = 0$ can be obtained by a proper choice of the origin of x. It then follows from equations for n_i and z that n_i and z are both zero at all instants of time. This is due to the fact that terms proportional to $(\omega - \omega_0)/\gamma_{ab}$ were neglected in (2.6); the polarization coefficient $\kappa_{1,2}$ of the system turns out to be purely imaginary and the phase difference of the oppositely directed waves is constant in time.

When $z \equiv 0$ and $n_i \equiv 0$ we obtain from (2.6)

$$\dot{x} = G\eta x, \qquad (3.4a)$$
$$\dot{y} = G[\eta y + 2nu], \qquad (3.4b)$$
$$\dot{u} = G[\eta u + {}^1/_2 ny], \qquad (3.4c)$$
$$\dot{n} = -n - (1+\eta)u - {}^1/_2 yn, \qquad (3.4d)$$
$$\dot{\eta} = \alpha - \eta - (1+\eta)y - 2nu. \qquad (3.4e)$$

Investigation of the nonlinear equations (3.4) is facilitated by the presence of the large parameter G. Similar equations were analyzed in[11,12]. When $G \gg 1$ a significantly nonlinear regime occurs with small deviations of η and n from zero:

$$|n| \ll 1, \qquad |\eta| \ll 1. \qquad (3.5)$$

Taking (3.5) into account we can in the first approximation drop small terms from (3.4d) and (3.4e); we then obtain the system

$$\dot{y} = G[\eta y + 2nu], \qquad (3.6a)$$
$$\dot{u} = G[\eta u + {}^1/_2 ny], \qquad (3.6b)$$
$$\dot{n} = -u, \qquad (3.6c)$$
$$\dot{\eta} = \alpha - y. \qquad (3.6d)$$

When $u = n \equiv 0$, i.e., in the case of a single traveling wave (3.6) describe an undamped oscillating process whose characteristics in the nonlinear regime were determined by Bespalov and Gaponov[11]. In the case of two oppositely directed waves these characteristics change significantly (the shape, repetition frequency, etc. of the pulses vary). It follows from (3.6c) that the condition $n \ll 1$ requires that the quantity $u = a|E_1||E_2|\cos\Phi$ change sign in free oscillation. When the sign of u changes one of the amplitudes of the oppositely directed waves turns to zero and the phase difference changes instantaneously by π. The intensity difference x of the oppositely directed waves is determined by (3.4a). Since in the nonlinear transient process η is a small quantity that changes sign in free oscillation, then according to (3.4a) the intensity difference increases in the region $\eta > 0$, while at $\eta < 0$ the oppositely directed wave intensities are equalized. This circumstance helps explain the significant deviation of the suppression rate of one of the oppositely directed waves in the transient process from the corresponding prediction of the linear theory (3.3).

The equations describing the interaction of oppositely directed waves were solved numerically since it was not possible to obtain their analytical solution even in the simplified form of (3.6).

In the absence of scatter-induced coupling the system (2.6) was solved with $\alpha = 0.1$, $G = 10^4$, and the initial conditions

$$\sqrt{a}|E_1| = 2\cdot 10^{-4}, \quad \sqrt{a}|E_2| = 10^{-4}, \quad \Phi = 0.$$

The dimensionless intensities of the oppositely directed waves $aE_{1,2}^2$ are shown as time functions in Fig. 1. The time interval used in the solution approxi-

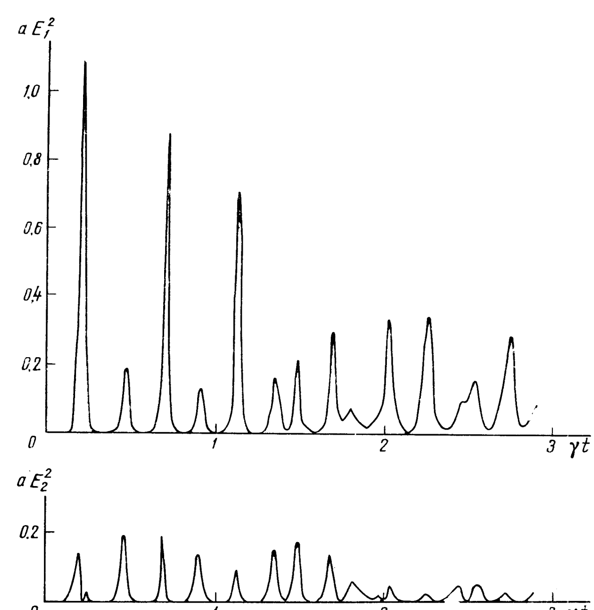

FIG. 1.

mately corresponds to the time it takes to set up stationary regime $t_1 \sim [\gamma(1+\alpha)]^{-1}$ in the case of a single wave $(u \equiv 0, n \equiv 0)$. According to Fig. 1 the laser emission spikes are irregular; wave intensities differ substantially in some spikes and are almost equal in others. The time of complete suppression of one of the oppositely directed waves is longer than t_1. We may conclude from the obtained solution that in the absence of scatter-induced coupling between the oppositely directed waves with unequal initial conditions wave intensities can significantly differ in magnitude although without a rapid complete suppression of the less intense wave. When the scatter-induced suppression is taken into account the character of the oppositely directed wave interaction can change significantly. We now consider the case of a sufficiently strong coupling between the oppositely directed waves in a solid state laser due to reflection from the crystal end faces.

Equations (2.6) were solved by electronic computer for equal real coupling coefficients $m = m_1^{(r)} = m_2^{(r)} = 0.05$ and $m_{1,2}^{(i)} = 0$. In accordance with (2.4) and for $Q = 10^7$ such a coupling value is obtained with a reflection coefficient ρ of the order of 1%. The chosen coupling value is real in the case when the crystal end faces are perpendicular to the direction of propagation of the waves. If the end faces are inclined the effective coupling coefficient is considerably smaller. The values of the α and G parameters and the initial conditions are then the same as in the case of $M = 0$.

According to the numerical solution the inclusion of coupling leads to a periodic transfer of energy from one wave into another: the energy flows from the more intense wave into one with lower intensity and amplifies it. As a result the second wave becomes more intense and the energy transfer then proceeds in the reverse direction. With the selected magnitude of the coupling coefficient of $m = 0.05$ the intensity oscillation period of the oppositely directed waves is much shorter than the duration of the emission spikes. The emission spikes thus have a complex structure, each consisting of a series of shorter pulses.

Figure 2 shows the envelopes of the emission spikes in the oppositely directed waves. According to these graphs the average intensities of the oppositely directed waves are approximately equal; on the other hand, in the absence of coupling under the same initial conditions they significantly differ from one another.

To illustrate the spike shape Fig. 3 shows the first two spikes in a different time scale. According to Fig. 3 the intensity oscillations are opposed in phase in the first spike and there are no oscillations and wave intensities are equal in the second spike. By comparing the spikes obtained in the absence of coupling shown in Fig. 1 with spikes observed when the coupling coefficient was $m = 0.05$ we can derive the following relationship between the two types of spikes: if in the absence of coupling the wave intensities in the spike are equal, they are also equal and there are no wave intensity oscillations when coupling is taken into account in the spike. On the other hand if the intensities differ significantly with $m = 0$ they are subject to strong oscillation in the presence of coupling in the spike.

Some characteristics of the oppositely directed wave competition process in the presence of scatter-induced coupling can be obtained analytically. If the coupling is sufficiently strong so that the following condition is satisfied:

$$m \gg \eta, \tag{3.7}$$

the Gm coefficient in (2.6) is a large parameter. Retaining only terms proportional to m in the first ap-

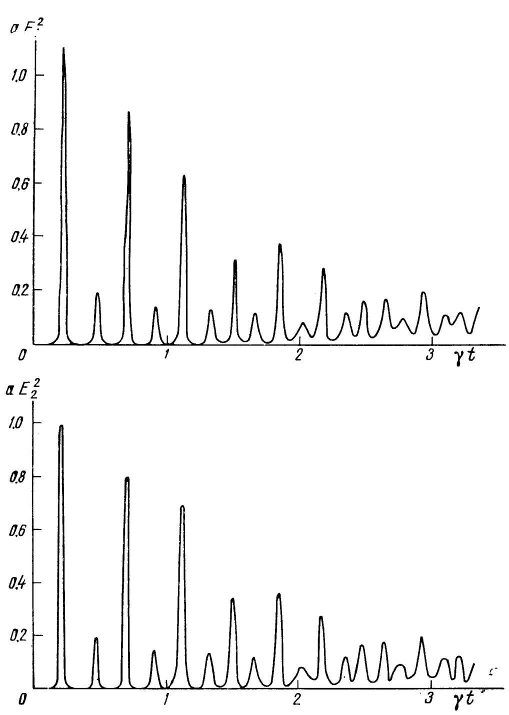

FIG. 2.

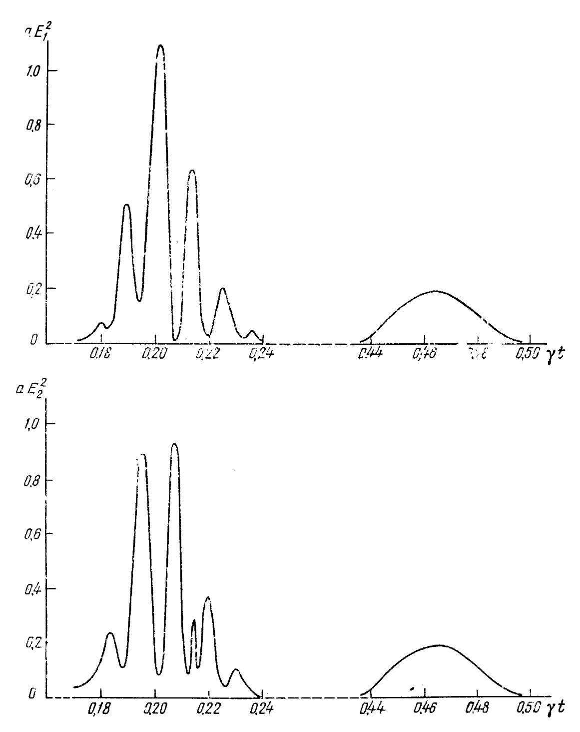

FIG. 3

proximation in (2.6) we obtain

$$\dot{x} = 2Gmz, \quad \dot{z} = -\tfrac{1}{2}Gmx, \tag{3.8}$$

hence

$$x = a(t)\cos(Gm\gamma t + \varphi(t)), \quad z = -\frac{a(t)}{2}\sin(Gm\gamma t + \varphi(t)), \tag{3.9}$$

i.e., the quantities x and z oscillate rapidly with a frequency Ω that is proportional to the coupling value $\Omega = Gm\gamma$. The amplitude a(t) and phase $\varphi(t)$ of the oscillations are slowly varying functions of time. Since according to (3.7) the sum of the wave intensities y is also a slowly varying function of time the oppositely directed wave intensity oscillations turn out to be opposed in phase. It follows from the equation for n_i (2.6) that n_i is a rapidly oscillating function of time

$$n_i \approx -\frac{a(t)}{2mG}\cos(Gm\gamma t + \varphi(t)). \tag{3.10}$$

Taking (3.9) and (3.10) into account we can neglect the rapidly oscillating term $n_i z$ in the equations for the slowly varying functions y, u, and η and thus we obtain a system of equations for y, u, n, and η that is identical with (3.4) and describes the oppositely directed wave competition in the absence of scatter-induced coupling. This result explains the above correspondence between spikes in the presence of coupling and when m = 0.

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