INFLUENCE OF IMPURITIES ON THE EXISTENCE OF AN INHOMOGENEOUS STATE IN A FERROMAGNETIC SUPERCONDUCTOR

L. G. ASLAMAZOV

Institute of Theoretical Physics, USSR Academy of Sciences

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When the impurity concentration is decreased, a ferromagnetic alloy becomes unstable against pairing of the electrons with non-zero total momentum, even if the electron scattering by the impurities is appreciable.

1. INTRODUCTION

THE transition of a ferromagnet into the superconducting state can be accompanied by pairing of electrons with different momenta[1,2]. This gives rise to pairs whose momentum differs from zero, so that the ordering parameter turns out to be a periodic function of the coordinates. The resultant structure has a cell dimension on the order of the pair dimension in the superconductor, and has a number of interesting properties.

It is natural to seek this effect in alloys with paramagnetic impurities in which, according to Abrikosov and Gor'kov[3], the ferromagnetic ordering of the impurity spins is due to the exchange interaction of the conduction electrons with the impurity atoms. At sufficiently low temperatures and at low impurity concentrations, such an alloy becomes superconducting. However, besides the exchange interaction between the electrons and the impurity spins, there is always present in such a system ordinary scattering which as a rule is several times larger than the exchange scattering.

Electron scattering interferes with production of pairs with non-zero momentum, since, by virtue of the uncertainty principle, the momentum ceases to be a quantum number in this case. We shall show that the appearance an inhomogeneous superconducting state is nonetheless possible in the alloys.

2. NON-EXCHANGE INTERACTION OF ELECTRONS WITH IMPURITIES

The condition for the instability of the ground state of the metal with respect to pairing of the electrons with total momentum q is obtained from the linearized Gor'kov equations[4] and is given by

$$1+|\lambda|T\sum_{\omega}\int\frac{1}{2}\operatorname{Sp}\{\hat{K}_{\omega}(\mathbf{p},\mathbf{q}-\mathbf{p})\}\frac{d^3\mathbf{p}}{(2\pi)^3}=0, \quad (1)$$

where $\hat{\mathbf{K}}_{\omega}(\mathbf{p}_1,\mathbf{p}_2)$ is the Fourier component of the expression

$$\overline{\hat{G}_{\omega}(\mathrm{s},\mathrm{r})\,\hat{\sigma}_y\,\hat{G}_{-\omega}(\mathrm{s},\mathrm{r}')\,\hat{\sigma}_y}. \quad (2)$$

The bar in (2) denotes averaging over the impurity positions, $\hat{G}_{\omega}(\mathbf{s},\mathbf{r})$ is the Green's function of the normal metal with the impurities, and λ is the electron-phonon interaction constant. The averaging is by means of a diagram technique[4]. This yields for $\hat{K}_{\omega}$ an integral equation that can be solved in the case of isotropic scattering. After substituting the expression for $\hat{K}_{\omega}$ in (1), we obtain

$$\frac{(2\pi)^2}{\lambda m p_0}+T\sum_{\omega}\frac{\int G^+(\omega,\xi)G^-(-\omega,\xi-vqx)\,d\xi\,dx}{1-\dfrac{1}{4\pi\tau}\int G^+(\omega,\xi)G^-(-\omega,\xi-vqx)\,d\xi\,dx}=0. \quad (3)$$

The diagonal elements of the averaged Green's functions are here

$$G^{\pm}(\omega,\xi)=(i\omega-\xi\pm I+i\operatorname{sign}\omega/2\tau)^{-1}. \quad (4)$$

The shift 2I of the Fermi surface is due to the exchange interaction between the electrons and the impurity atoms, which reduces in first approximation to a self-consistent exchange field I proportional to the concentration of the impurities and to their average spin. The contribution of ordinary scattering to the G-function is $i\ \mathrm{sign}\ \omega/2\tau$, where $1/\tau$ is the reciprocal of the time between collisions in the Born approximation.

At a specified temperature, we can obtain from (3) the value of the exchange field $I(q)$ at which the normal state becomes unstable against pairing of the electrons with total momentum q. If the maximum value of I corresponds to a non-zero momentum q, this means that an instability appears with respect to the production of pairs with non-zero momentum. The quantity $1/\tau$ in Eq. (3) can be regarded as a parameter.

In the general case, no such analysis can be made, owing to the complicated character of Eq. (3). However, for a contaminated metal the left side of the equation can be expanded in powers of τ. As will be shown later, it is necessary in this case to retain also the first-order terms in τ. As a result we get

$$\ln\frac{T}{T_{c0}}+\frac{1}{2}\left[\psi\left(\frac{1}{2}+iz\right)-\psi\left(\frac{1}{2}\right)\right]+\frac{\tau v^2q^2}{6\pi T}\psi'\left(\frac{1}{2}+iz\right)$$
$$-\frac{4}{3}\tau^2v^2q^2\left[\psi\left(\frac{1}{4\pi\tau T}\right)-\psi\left(\frac{1}{2}+iz\right)\right]$$
$$+\frac{\tau^2v^4q^4}{36\pi^2T^2}\psi''\left(\frac{1}{2}+iz\right)+\text{c.c.}=0, \quad (5)$$

where $\psi(\zeta)$ is the logarithmic derivative of the Gamma-function, $z = I/2\pi T$, and T_{c0} is the critical temperature of the clean superconductor.

The foregoing equation corresponds, accurate to terms of first order in τ to the equation derived for the critical temperature of a ferromagnetic alloy by Fulde and Maki[5]. However, the equation of[5] cannot be used with a higher degree of accuracy.

At low temperatures $T \ll I$, Eq. (5) takes the form

$$\ln\frac{2I}{\Delta_0}+\frac{2\tau v^2q^2}{3}\left(\frac{\pi e^{-I/T}}{T}-4\tau\ln\frac{1}{2\tau I}\right)+\frac{2\tau^2v^4q^4}{9I^2}=0, \qquad (6)$$

where Δ_0 is the gap in the spectrum of the clean superconductor. The temperature T^*, above which pair production with zero momentum is most convenient, is determined from the equation

$$\pi e^{-\Delta_0/2T^*}-4\tau T^*\ln(1/\tau\Delta_0)=0. \qquad (7)$$

We then obtain, with logarithmic accuracy,

$$T^*=-\Delta_0/2\ln\tau\Delta_0.$$

At temperatures lower than T^*, at the instability point

$$I=\frac{\Delta_0}{2}\left(1+\frac{a^2}{2}\right),\quad \alpha=2\tau\Delta_0\ln\frac{1}{\tau\Delta_0}-\frac{\pi\Delta_0}{2T}\exp\left(-\frac{\Delta_0}{2T}\right), \qquad (8)$$

pairs are produced with momentum $q = (1/v)(3\alpha\Delta_0/4\tau)^{1/2}$. Thus, at sufficiently low temperatures, the instability of the normal metal against pairing of electrons with non-zero total momentum always occurs earlier than for pairing with zero momentum.

The temperature T^* can be determined in the general case without assuming that the mean free path of the electrons in the metal is small. In fact, at this temperature the derivative d^2I/dq^2 should vanish at $q = 0$ (at $T < T^*$ we have $d^2I/dq^2 > 0$ and the maximum shifts towards $q \neq 0$). Using Eq. (3), which specifies in implicit form the connection between I and q, we write down the condition $d^2I/dq^2 = 0$ in the form of a system of equations determining the boundary values T^* and I^*:

$$\ln\frac{T^*}{T_{c0}}-\psi\left(\frac{1}{2}\right)+\frac{1}{2}\left[\psi\left(\frac{1}{2}+iz^*\right)+\text{к.с.}\right]=0,$$

$$4\pi\tau T^*\left[\psi\left(\frac{1}{2}+iz^*+\frac{1}{4\pi\tau T^*}\right)-\psi\left(\frac{1}{2}+iz^*\right)\right]$$

$$-\psi'\left(\frac{1}{2}+iz^*\right)+\text{c.c.}=0. \qquad (9)$$

The numerical solution of the system (9) at $1/\tau = 0$ gives values $T^* = 0.13\,\Delta_0$ and $I^* = 0.12\,\Delta_0$. For a contaminated metal we obtain Eq. (7).

Larkin and Ovchinnikov[2] found that at zero temperature a first-order transition into the homogeneous superconducting state takes place at $I = \Delta_0/\sqrt{2}$. This value turned out to be less than the shift $I = 0.755\Delta_0$ at which the clean ferromagnet becomes unstable at $T = 0$ against production of pairs with non-zero momentum.

Non-exchange scattering by impurities does not affect the point of the first-order transition into the homogeneous superconducting state, and shifts the instability point towards lower values of I. For a contaminated metal we obtain from formula (8) that the instability point is in the metastable region.

3. EXCHANGE SCATTERING OF ELECTRONS

Exchange-scattering effects influence the position of the point of the first order transition, for ferromagnetic ordering of the impurity spins in the homogeneous superconducting phase sets in at arbitrarily small exchange free path of the electrons[6]. This gives rise to an electron-spin density

$$\langle\sigma\rangle=T\sum_\omega\int \text{Sp}\left[\frac{1}{2}\sigma G(p,\omega)\right]\frac{d^3\mathbf{p}}{(2\pi)^3}. \qquad (10)$$

An expression for the Green's function of a superconducting ferromagnetic alloy was obtained by Fulde and Maki[5]. Using their results, we obtain in the approximation linear in I

$$\langle\sigma\rangle=-\rho I\left[1-\pi T\sum_\omega\left(\Delta(1+u^2)^{3/2}-\frac{1}{\tau_2}-\frac{3\langle S_x{}^2\rangle}{\tau_2\langle S^2\rangle}(1-u^2)\right)^{-1}\right], \qquad (11)$$

where $\rho = mp_0/\pi^2$ is the level density, $1/\tau_2$ is the reciprocal exchange free-path time, and the parameter u is given by

$$\frac{\omega}{\Delta}=u-\frac{1}{\tau_2\Delta}\frac{u}{(1+u^2)^{1/2}}. \qquad (12)$$

The average impurity spin entering in (11) should itself be calculated in the field of the magnetized electrons. It can be assumed that $\langle S_X^2\rangle/\langle S^2\rangle = 1/3$ on the line of transition from the paramagnetic into the ferromagnetic phase, and formula (11) gives the result of Gor'kov and Rusinov[6]. At zero temperature, however, regarding the spin as a classical vector, we have $\langle S_X^2\rangle = 0$. We then find that even though the impurity spins are fully ordered, $\langle\sigma\rangle = 0$.

If the impurity spin is regarded as a quantum mechanical operator, then at zero temperature we have $\langle S_X^2\rangle = S/2$, where S is the projection of the impurity spin on the z axis, and formula (11) is altered as follows: $\langle S_X^2\rangle$ is replaced by $\langle S_X^2\rangle \pm \langle S_Z\rangle/2$, and the average of the obtained expression is taken. Then $\langle\sigma\rangle$ is already different from zero.

Since we are interested also in the region of larger values of I, we obtain the value of $\langle\sigma\rangle$ without expansion in I

$$\langle\sigma\rangle=-\rho I\left[1-\frac{\pi T}{2iI}\sum_\omega\left(\frac{u_+}{\sqrt{1+u_+^2}}-\frac{u_-}{\sqrt{1+u_-^2}}\right)\right], \qquad (13)$$

where the parameters u_+ and u_- are obtained from the system

$$u_\pm=\frac{\omega}{\Delta}\pm\frac{iI}{\Delta}+\frac{u_\pm}{\sqrt{1+u_\pm^2}}\frac{\langle S_z{}^2\rangle}{\langle S^2\rangle}\frac{1}{\tau_2\Delta}+\frac{u_\pm+u_\mp}{\sqrt{1+u_\mp^2}}\frac{\langle S_x{}^2\rangle\pm\langle S_z\rangle/2}{\langle S^2\rangle}\frac{1}{\tau_2\Delta}. \qquad (14)$$

The exchange scattering is usually small, so that we can assume that $1/\tau_2\Delta \ll 1$. Replacing at $T = 0$ the sum over ω by an integral, we obtain from (13) and (14)

$$\langle\sigma\rangle=2\rho I\frac{1}{\tau_2\Delta}\frac{1}{S+1}\int_0^\infty\frac{x^2dx}{[x^2+(1+I/\Delta)^2]^{3/2}[x^2+(1-I/\Delta)^2]^{3/2}}$$
$$=\frac{\rho}{2\tau_2(S+1)}\frac{K(z')-E(z')}{z'}, \qquad (15)$$

where $K(z')$ and $E(z')$ are complete elliptic integrals, and the parameter $z' = I/\Delta$ at $I < \Delta$ and z'

$= \Delta/I$ at $I > \Delta$. When $z' = 1$, the integral in (15) diverges logarithmically and should be replaced by $\ln \tau_2 \Delta$.

The ordering parameter Δ itself depends on the exchange field I. An equation for this dependence can be readily obtained when $\tau_2 \Delta \gg 1$, and has the following form when $I < \Delta$:

$$\frac{2}{|\lambda|\rho} = \ln\frac{2\omega_D}{\Delta} - \frac{\pi}{4\tau_2\Delta} - \frac{1}{\tau_2\Delta}\frac{1}{2(S+1)}\left[K\left(\frac{I}{\Delta}\right) - \frac{\pi}{2}\right]. \quad (16)$$

In the region $I > \Delta$ the equation for Δ has a similar form, and its solution can be disregarded, since it corresponds to a larger energy.

Neglecting scattering effects $(1/\tau_2\Delta = 0)$, Eq. (16) has a solution $\Delta = \Delta_0$ at $I < \Delta_0$, and no solution at larger I. The energy of the corresponding superconducting state $-(\frac{1}{4})\rho\Delta_0^2$ does not depend on I and becomes smaller than the energy $-\rho I^2/2$ of the normal metal at the point $I = 1/\sqrt{2}\Delta_0$[2].

In the approximation linear in $1/\tau_2\Delta$, the solution of (15) at values of I not too close to Δ_0 is given by

$$\Delta = \Delta_0\left(1 - \frac{\beta}{\tau_2\Delta_0}\right), \quad \beta = \frac{\pi}{4} + \frac{1}{2(S+1)}\left[K\left(\frac{I}{\Delta_0}\right) - \frac{\pi}{2}\right], \quad (17)$$

and the energy

$$\Omega(I) = \Omega(0) - \int_0^I \langle\sigma\rangle\, dI$$

$$= -\frac{\rho\Delta_0^2}{4}\left[1 - \frac{\pi}{\tau_2\Delta_0} + \frac{2}{\tau_2\Delta_0(S+1)}\left(\frac{\pi}{2} - E\left(\frac{I}{\Delta_0}\right)\right)\right] \quad (18)$$

differs little from $-\rho\Delta_0^2/4$ already in the entire region $0 < I < \Delta_0$. Equating (I) to the energy of the normal metal, we obtain the first-order transition point

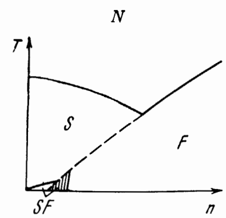
$$I = \frac{1}{\sqrt{2}}\Delta_0\left(1 - \frac{\pi}{2\tau_2\Delta_0} + 0.21\frac{1}{\tau_2\Delta_0(S+1)}\right), \quad (19)$$

which lies in the region where formulas (15) and (17) are still applicable.

The second term in (19), which results from the influence of the paramagnetic impurities on the energy of the superconductor, is always larger than the last term. Therefore allowance for electron scattering by the impurities shifts the first-order transition points towards smaller values of I. However, the instability point shift in this case more strongly, since usual scattering by impurities exceeds the exchange scattering.

4. CONCLUSION

A ferromagnetic alloy becomes an inhomogeneous superconductor with decreasing impurity concentration if its instability against pairing of the electrons occurs before the first-order transition to the homogeneous superconducting state. To satisfy this condition it is necessary that the reciprocal free path time of the electron, $1/\tau$, be small at impurity concentration $n \sim \Delta_0/u_2S$, where u_2 is the exchange part of the scattering of the electrons by the impurities. It is easy to see that such a requirement can be satisfied even when the non-exchange part of the amplitude u_1 is several times larger than u_2S. Indeed, the free-path effects are of second order in the scattering amplitude, and the Born parameter is usually small.



The phase diagram has in this case the form shown in the figure. The region of the inhomogeneous superconducting state is shaded in the diagram. The dashed line is the line of the first-order phase transition into the ordinary superconducting state.

Besides paramagnetic impurities, the metal can contain an appreciable amount of nonmagnetic impurities, on which spin-orbit scattering of the electrons takes place. The spin-orbit scattering of the electrons can make the transition into the superconducting state a second-order transition, even when the scattering by the impurities is large. This, however, hinders the pairing of electrons with non-zero total momentum. At those concentrations of the nonmagnetic impurities at which the transition into the superconducting state becomes a second-order transition, pairing of electrons with a zero total momentum becomes already more convenient.

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