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THEORY OF PARTICLE FUSION BASED ON A FIVE-DIMENSIONAL SCHEME

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We derive equations that are covariant with respect to five-dimensional homogeneous proper Lorentz groups[1] (de-Sitter groups for 1 + 4 space), and with respect to the complete and incomplete five-dimensional homogeneous Lorentz groups. On the basis of the obtained equations with a periodic fifth coordinate, having the dimension of length and characterizing the rest mass of the particles, we construct a theory of particle fusion. A relativistic relation between the energy, momentum, and rest mass, which assumes unbounded quantized values, is satisfied. The latter circumstance, in our opinion, explains the very fact of occurrence of resonant particles, and also the discreteness of the energy levels of the virtual and bound states of nucleon systems. Stability of the product particle produced upon fusion is ensured when the fusing particles have identical spins but not when their spins are different. States are established which differ from the states determined by the general Pauli theorem concerning the connection between the spin and the sign of the energy and charge density.

THE known theoretical papers on particle fusion[2-5] lead to a finite number of the values of the rest masses of the product particle produced upon fusion. In the present paper we develop a theory of particle fusion with unbounded quantized mass spectrum for the product particle and for the particles resulting from the decay of the product particle. The theory leads to non-separating equations for the product particle upon fusion of an arbitrary number of particles with identical spins, equal to ½, 1, and 3/2, and to separating equations in the case of fusion of an arbitrary number of particles with different spins in the combinations 0–1 and ½–3/2.

The proposed fusion theory is based on equations that are covariant with respect to the incomplete $T'_\alpha(5)$ and complete $T''_\alpha(5)$ five-dimensional homogeneous Lorentz groups. The group $T'_\alpha(5)$ is made up of five-dimensional homogeneous proper Lorentz group $T_\alpha(5)$ by adding the reflections of the space coordinates, and the group $T''_\alpha(5)$ is formed by adding the reflections of the space coordinates and of the fifth coordinate.

1. LINEAR IRREDUCIBLE REPRESENTATIONS OF THE GROUPS $T'_\alpha(5)$ AND $T''_\alpha(5)$

The linear finite-dimensional irreducible representations of the group $T_\alpha(5)$ are specified by the pair of numbers $\sigma \sim (n_0, p_0)$ and are expressed by formulas (5), (6), (9), (12), and (14) of[1]. The irreducible representations of the groups $T'_\alpha(5)$ and $T''_\alpha(5)$ will be expressed by the same formulas with the corresponding addition of the operators S′ and S″, corresponding to the indicated reflections and having the following forms:

a) when $n \neq 0$

$$S'\xi_{l,m}^{p,n} = (-1)^l \xi_{l,m}^{p,-n}, \quad S''\xi_{l,m}^{p,n} = (-1)^{p+|n|+l}\xi_{l,m}^{p,-n}; \tag{1}$$

b) when $n = 0$

$$S'\xi_{l,m}^{p,0} = (-1)^l \xi_{l,m}^{p,0} \text{ and } S'\xi_{l,m}^{p,0} = (-1)^{l+1}\xi_{l,m}^{p,0}; \tag{2}$$

$$S''\xi_{l,m}^{p,0} = (-1)^{p+l}\xi_{l,m}^{p,0} \text{ and } S''\xi_{l,m}^{p,0} = (-1)^{p+l+1}\xi_{l,m}^{p,0}. \tag{3}$$

Here $\xi_{l,m}^{p,n}$—basis vectors in the space of the representations defined by the pair σ (canonical basis).

2. EQUATIONS COVARIANT WITH RESPECT TO THE GROUPS $T_\alpha(5)$, $T'_\alpha(5)$, AND $T''_\alpha(5)$

In analogy with the four-dimensional case[6] we shall write the five-dimensional equations in the form

$$\sum_{k=1}^{5} L_k \frac{\partial\psi}{\partial x_k} + i\varkappa\psi = 0, \tag{4}$$

$\psi(x_1, x_2, x_3, x_4, x_5) \equiv \psi(x, y, z, ct, x_5)$ and κ is a real constant. For the matrices L_k of Eqs. (4), which are covariant with respect to the group $T_\alpha(5)$, we can obtain

$$[L_i, I_{km}] = g_{ik}L_m - g_{im}L_k, \tag{5}$$

$$[I_{ik}, I_{jl}] = g_{il}I_{kj} + g_{kj}I_{il} - g_{ij}I_{kl} - g_{kl}I_{ij}, \tag{6}$$

$$g_{44} = -g_{11} = -g_{22} = -g_{33} = -g_{55} = 1, \quad g_{il} = 0 \quad (i \neq l),$$
$$i, k, j, l, m = 1, 2, 3, 4, 5.$$

Representing the matrix L_5 in the form

$$L_5\xi_{l,m}^{\sigma,p,n} = \sum_{\sigma', p', n', l', m'} C_{l,m,l',m'}^{\sigma,p,n,\sigma',p',n'} \xi_{l',m'}^{\sigma',p',n'}, \tag{7}$$

and taking into account its commutativity with H and F[1] and Schur's lemma, we obtain

$$C_{l,m,l',m'}^{\sigma,p,n,\sigma',p',n'} = C_{p,n}^{\sigma,\sigma'}\delta_{pp'}\delta_{nn'}\delta_{ll'}\delta_{mm'}. \tag{8}$$

Relations (7) and (8) in conjunction with

$$L_5 = [[L_5, I_{45}], I_{45}] \tag{9}$$

give a system of nine equations for $C_{p,n}^{\sigma,\sigma'}$, which has nonzero solutions under the following conditions:

$$1)\ n_0' = n_0,\ p_0' = p_0;\quad 2)\ n_0' = n_0,\ p_0' = p_0 \pm 1;\quad 3)\ n_0' = n_0 \pm 1,\ p_0' = p_0. \tag{10}$$

In this case $C_{p,n}^{\sigma,\sigma'}$ have the following form:

1) if $n_0' = n_0$, $p_0' = p_0$, then

$$C_{p,n}^{\sigma,\sigma} = C^{\sigma,\sigma}pn; \tag{11}$$

2) if $n_0' = n_0$, $p_0' = p_0 + 1$, then

$$C_{p,n}^{\sigma,\sigma'} = C^{\sigma,\sigma'}\sqrt{(p_0+n+2)(p_0-n+2)(p_0+p+2)(p_0-p+2)},$$

$$C_{p,n}^{\sigma',\sigma} = C^{\sigma',\sigma}\sqrt{(p_0+n+2)(p_0-n+2)(p_0+p+2)(p_0-p+2)}; \tag{12}$$

3) if $n_0' = n_0 + 1$, $p_0' = p_0$, then

$$C_{p,n}^{\sigma,\sigma'} = C^{\sigma,\sigma'}\sqrt{(n_0+n+1)(n_0-n+1)(p+n_0+1)(p-n_0-1)},$$

$$C_{p,n}^{\sigma',\sigma} = C^{\sigma',\sigma}\sqrt{(n_0+n+1)(n_0-n+1)(p+n_0+1)(p-n_0-1)}; \tag{13}$$

$C^{\sigma,\sigma}$, $C^{\sigma,\sigma'}$, and $C^{\sigma',\sigma}$ are arbitrary complex numbers.

We note that only (11) agrees with the results of Sokolik[7] in the case of classification of the five-dimensional equations of the form (4) in accordance with the representations of the orthogonal group O_5.

Using the formulas $L_5S' = S'L_5$ and $L_5S'' = -S''L_5$, we arrive at the following conclusion: the covariance of equations (4) relative to the group $T'_\alpha(5)$ requires $C^{\sigma,\sigma} = 0$ (no conditions are imposed on $C^{\sigma,\sigma'}$ and $C^{\sigma',\sigma}$); the covariance of Eqs. (4) with respect to the group $T''_\alpha(5)$ requires $C^{\sigma,\sigma'} = C^{\sigma',\sigma} = 0$ (no limitations are imposed on $C^{\sigma,\sigma}$).

3. THEORY OF FUSION

The pair of numbers $\sigma \sim (n_0, p_0)$ determines, by means of the relations[1]

$$-n_0 \leqslant n \leqslant n_0, \quad (n_0+1) \leqslant p \leqslant (p_0+1) \tag{14}$$

the set of those irreducible representations of the ordinary four-dimensional homogeneous proper Lorentz group $T_\alpha(4)$, which realize the irreducible representation of the group $T_\alpha(5)$. Each irreducible representation $T_\alpha(4)$ is specified by a pair of numbers $\tau \sim (n, p)$[6]. At $n_0 = 0$ and $p_0 = 1$, the irreducible representations $T_\alpha(4)$, which realize the irreducible representation $T_\alpha(5)$, are specified by two pairs $\tau \sim (n, p)$, namely $(0, 1)$ and $(0, 2)$. These pairs τ determine the relativistic equations for particles with zero spin. When $n_0 = p_0 = 1$, three pairs τ determine the relativistic equations for the particles with spin 1, and when $n_0 = p_0 = 1/2$, two pairs τ determine the Dirac equations, and when $n_0 = 1/2$ and $p_0 = 3/2$, four pairs τ determine the Pauli-Fierz equations for particles with spin $3/2$.

As is well known[6], the covariance of ordinary four-dimensional relativistic equations with respect to the four-dimensional complete Lorentz group $T'_\alpha(4)$ is ensured if the irreducible representations $T_\alpha(4)$, which realize the representation $T'_\alpha(4)$, are given by the following pairs τ: $(0, p)$ or (n, p) and $(-n, p)$. These conditions, as seen from (14), remain in force also when the representations $T_\alpha(5)$, $T'_\alpha(5)$ are constructed. The covariance of Eqs. (4) is ensured by the conditions (10). It is therefore clear that Eqs. (4), which are covariant with respect to $T_\alpha(5)$, $T'_\alpha(5)$ and $T''_\alpha(5)$ will always be realized by irreducible representations of $T'_\alpha(4)$. Eqs. (4), defined by the pairs $\sigma \sim (0, 1)$ and $\sigma' \sim (1, 1)$ are realized by the irreducible representations of $T'_\alpha(4)$ with pairs τ corresponding to spins 0 and 1; when $\sigma = \sigma' \sim (1/2, 1/2)$ we obtain a union of the representations $T'_\alpha(4)$ of spins $1/2$ and $1/2$, etc.

There is a possibility of joining by means of Eqs. (4), on the basis of the conditions (10), any number of identical or different irreducible representations of $T'_\alpha(4)$ corresponding to particles with spins 0 and 1 or $1/2$ and $3/2$, with the exception of the case of identical representations corresponding to zero spin, when all $L_i = 0$.

On the basis of the foregoing, it is natural to regard Eqs. (4) as equations describing the fusion of particles with identical or different spins. When $\kappa \neq 0$, Eqs. (4) do not break up, and this corresponds to a stable product particle. When $\kappa = 0$, Eqs. (4) break up into a number of equations equal to the number of the irreducible representations $T_\alpha(5)$ realizing (4). The equations obtained in the decomposition of (4) can be naturally regarded as describing particles produced in the decay of the product particle.

Comparing the five-dimensional equations (4) with the ordinary relativistic equations, we can interpret the operator $(-iL_5\partial/\partial x_5 + \kappa)$ as the mass operator of the particle, and the fifth coordinate, which has the dimension of length, can be interpreted as an internal parameter of the particle (internal degree of freedom), which characterizes its mass state. Let ψ be periodic in the fifth coordinate with period l_5 (l_5 is a certain elementary length characterizing the particle mass). Putting

$$\psi_\nu = C\exp\left[-\frac{i}{\hbar}(Et-\mathbf{pr}) + \frac{2\pi}{l_5}i\nu x_5\right], \tag{15}$$

where $\nu = 0, \pm 1, \pm 2, \ldots$ (C is independent of the coordinates), and substituting (15) in (4), we have in the rest system ($\mathbf{p} = 0$)

$$fL_4\psi_\nu - dL_5\psi_\nu - \varkappa\psi_\nu = 0,$$

$$f = m_0c/\hbar, \quad d = 2\pi\nu/l_5. \tag{16}$$

It is natural, in analogy with the four-dimensional case, to regard (16) as an equation on whose basis the values of the rest mass of the product particle, produced upon fusion, are determined. If we denote by λ_i the eigenvalues of the operator $D = fL_4 - dL_5$, then (16) yields

$$\varkappa = \lambda_i. \tag{17}$$

By determining the eigenvalues of the operator D, which turn out to be functions of m_0 and ν/l_5, we can obtain on the basis of (17) the rest-mass spectrum of the product particle. Owing to the contribution made by the fifth coordinate, the rest mass assumes quantized values, thus explaining the very existence of resonant particles, and also the discreteness of the energy levels of virtual and bound states of nucleon systems. This contribution can be determined also on the basis of the invariance of $|\mathbf{K}^2|$, the square of the modulus of the five-dimensional vector ($\mathbf{k} = \mathbf{p}/\hbar$, $k_4 = E/\hbar c$, $k_5 = 2\pi\nu/l_5$):

$$E^2 = c^2p^2 + m_0^2c^4, \quad m_0 = \pm\frac{\hbar}{c}\sqrt{|K^2| + \left(\frac{2\pi}{l_5}\nu\right)^2}. \tag{18}$$

The contribution of the fifth coordinate to the rest mass brings about the realization of states with $m_0 = 0$ ($\nu = 0$) and with $m_0 \neq 0$ ($\nu = \pm 1, \pm 2, \ldots$) for particles with spins $1/2$, 1, and $3/2$, which are produced in the decay of the product particle ($\kappa = 0$).

To separate the states corresponding to different values of the spin projection, we can use the method proposed by Fedorov[8]. It is easy to verify that $L_5 \sim I_{56}$, where I_{56} is the infinitesimal operator of the representation of the six-dimensional Lorentz group, taken in the canonical basis and satisfying relation (6) with i, k, j, l = 1, 2, 3, 4, 5, 6. On the basis of this fact and relation (5) we see that all the matrices L_k of Eqs. (4) commute with the infinitesimal operators corresponding to the spatial rotations. Therefore, for the specified momentum p and rest mass m_0, the states of the particle can also be characterized by different projections of the spin on the particle momentum. These states with different projections of the spin s_k are separated with the aid of the projection operator

$$\beta_k = \frac{Q_k(S)}{Q_k(s_k)}, \quad S = \frac{j}{|\mathbf{p}|}(p_1 I_{23} + p_2 I_{31} + p_3 I_{12}).$$

4. INVARIANT BILINEAR HERMITIAN FORM, LAGRANGE FUNCTION

We can obtain the following relations for the conditions of existence of an invariant nondegenerate bilinear hermitian form (ψ_1, ψ_2) in the space of the representations of the group T_α (5):

$$A^+ = A, \quad U_\alpha A = A U_\alpha, \quad F_3{}^+ A = A F_3, \tag{19}$$

$$I_{45}{}^+ A = -A I_{45}, \tag{20}$$

where A is the matrix of the bilinear form, U_α is the matrix of the representation of the group of spatial rotation, and the plus sign denotes the hermitian conjugate.

Relations (19), (as also for the group T'_α (4)) yield for the matrix elements of the matrix A at $\tau \sim (n, p)$ and $\tau \sim (-n, p)$ the conditions

$$a_l^{\tau\dot{\tau}} = (a_l^{\dot{\tau}\tau})^* = a^{\tau\dot{\tau}}(-1)^{l-|n|} = (a^{\tau\dot{\tau}})^*(-1)^{l-|n|}, \tag{21}$$

$a^{\tau\dot{\tau}}$ is any complex number, and the asterisk denotes the complex conjugate. Formula (20) gives for coupled[6] τ and τ', within the limits of each pair $\sigma \sim (n_0, p_0)$, the relation

$$a_l^{\tau\dot{\tau}}(\sigma) = a_l^{\tau\dot{\tau}} = -a_l^{\tau'\dot{\tau}'} = -a_l^{\tau'\dot{\tau}'}(\sigma'). \tag{22}$$

In the case of different pairs $\sigma \sim (n_0, p_0)$ and $\sigma'(n_0', p_0')$ formula (20) leads to two possibilities:

$$\begin{aligned} &1)\ p_0' = p_0,\ n_0' = -n_0 - 1; \\ &2)\ n_0' = n_0,\ p_0' = -p_0 - 3. \end{aligned} \tag{23}$$

Neither of these possibilities can be realized, since the numbers n_0, n_0', p_0, and p_0' are positive within the framework of the constructed representations[1].

The existence of an invariant bilinear hermitian form (ψ_1, ψ_2) in the space of representations of the groups T'_α (5) and T''_α (5) calls, on the basis of (1)–(3), for complex values of $a^{\tau\dot{\tau}}$ in the case of half-integer n_0 and p_0, and real values of $a^{\tau\dot{\tau}}$ for integer n_0 and p_0. The two possibilities in (2) and (3) then correspond, at n = 0, to two choices for $a^{\tau\dot{\tau}}$, namely $a^{\tau\dot{\tau}} = 0$ and $a^{\tau\dot{\tau}}$ arbitrary.

In analogy with the four-dimensional case[6], we can introduce the invariant Lagrange function

$$\mathcal{L} = \frac{1}{2i}\sum_{k=1}^{5}\left\{\left(\psi, \Lambda_k \frac{\partial\psi}{\partial x_k}\right) - \left(\Lambda_k \frac{\partial\psi}{\partial x_k}, \psi\right)\right\} + \varkappa(\psi, \psi) \tag{24}$$

and ascertain the conditions under which the vanishing of the variation of the action leads to Eq. (4). These conditions reduce to (19), (20), and the relation[1)]

$$L_5{}^+ A = A L_5, \tag{25}$$

from which it follows that when $p_0' = p_0$, $n_0' = n_0 + 1$ and $n_0' = n_0$, $p_0' = p_0 + 1$

$$(C^{\sigma,\sigma'})^* a_l^{\tau\dot{\tau}}(\sigma') = C^{\sigma',\sigma} a_l^{\tau\dot{\tau}}(\sigma), \tag{26}$$

$$C^{\sigma,\sigma'} C^{\sigma',\sigma} = (C^{\sigma,\sigma'})^* (C^{\sigma',\sigma})^*, \tag{27}$$

$$C^{\sigma,\sigma} = -(C^{\sigma,\sigma})^*. \tag{28}$$

Since (20) leads to the unrealizable possibilities (23), the invariant Lagrange function (24) can be constructed for $\kappa \neq 0$ only in the case $\sigma' = \sigma$, i.e., only in the case of fusion of any number of particles with identical spin.

If pairs $\sigma' \neq \sigma$, satisfying (10), exist besides the pair σ, then the invariant Lagrange function exists only for $\kappa = 0$, when the existence of the invariant function (ψ_1, ψ_2) is no longer necessary, and condition (25) alone suffices. But in this case Eqs. (4) break up and we cannot regard them as pertaining to a single particle. Thus, in fusion of particles with different spins, in the case $\kappa = 0$, stability of the product particle is not ensured within the framework of the developed theory. Certain interest may attach in this case only to states arising in the decay of the product particle. In fusion of particles with different spins, in the case when $\kappa \neq 0$, there is no invariant Lagrange function (24). However, Eqs. (4) continue to remain covariant with respect to the five-dimensional groups. We cannot exclude the possibility of reducing, for this variant of the theory, the five-dimensional scheme to a satisfactory ordinary four-dimensional Lagrangian formalism.

Connected with Eqs. (4) obtained from the Lagrange functions (24) are a five-dimensional current vector $j_k = \epsilon\,(L_k\psi, \psi)$ and a five-dimensional energy-momentum tensor

$$T_k{}^i \approx \frac{1}{2i}\left\{\left(\psi, L_k\frac{\partial\psi}{\partial x_i}\right) - \left(L_k\frac{\partial\psi}{\partial x_i}, \psi\right)\right\}.$$

We interpret j_4 as the ordinary charge density and j_5 as the charge density due to the fifth coordinate. The total charge equals

$$q = \int (j_4 + j_5)\, d^4x.$$

The quantity $-T_4^4 = W$ is interpreted by us as the ordinary energy density, and $-T_5^4 = W'$ as the additional energy density due to the fifth coordinate. The total field energy is

$$E = \int (W + W')\, d^4x.$$

The total charge density $(j_4 + j_5)$ is positive-definite if the eigenvectors $\psi^{(0)}$ of the operator D, correspond-

1)Satisfaction of relation (25) for the matrix L_5 ensures, just as in the four-dimensional case, satisfaction of a similar relation for the remaining four matrices L_k.

ing to nonzero eigenvalues, satisfy the inequality

$$((L_4 + L_5)\psi^{(0)}, \psi^{(0)}) \geqslant 0.$$

The total energy density $(W + W')$ is positive definite if the same eigenvectors $\psi^{(0)}$ satisfy the inequality

$$(\psi^{(0)}, \psi^{(0)}) \geqslant 0.$$

5. EXAMPLES OF FUSION OF TWO PARTICLES

1. Fusion of two particles with spin $\frac{1}{2}$ ($\sigma = \sigma' \sim (\frac{1}{2}, \frac{1}{2})$). Arranging the eight basis vectors of the column matrix ψ in a definite order and choosing on the basis of (21), (22) and (28) $C^{\sigma,\sigma} = 4i/3$ and $a^{\tau\dot{\tau}} = i$, we obtain eight-component equations (4) which are covariant with respect to the group T''_α (5) with matrices L_i and A in the form

$$L_i = \pm \begin{Vmatrix} 0 & L_i' \\ L_i' & 0 \end{Vmatrix}, \quad L_5 = \begin{Vmatrix} 0 & L_5' \\ L_5' & 0 \end{Vmatrix}, \quad A = \begin{Vmatrix} A' & 0 \\ 0 & A' \end{Vmatrix} \quad (i = 1, 2, 3, 4),$$

$$L_1' = \begin{Vmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{Vmatrix}, \quad L_2' = \begin{Vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{Vmatrix}, \quad L_3' = \begin{Vmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{Vmatrix},$$

$$L_4' = \begin{Vmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{Vmatrix}, \quad L_5' = \begin{Vmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{Vmatrix}, \quad A' = L_4'.$$

The matrices L_i and L_i' satisfy the Dirac algebra

$$L_k L_n + L_n L_k = 2g_{kn}.$$

The infinitesimal operators of the spatial rotations have a minimal polynomial

$$I_{ik}^2 + \tfrac{1}{4} = 0 \quad (i, k = 1, 2, 3).$$

All the eigenvalues of the matrix D are different from zero and equal $\lambda_i = \pm\sqrt{f^2 - d^2}$, each of these eigenvalues is fourfold, and (17) yields

$$m_0 = \pm \frac{\hbar}{c} \sqrt{\varkappa^2 + \left(\frac{2\pi}{l_5}\nu\right)^2}. \tag{29}$$

For $\lambda > 0$ the density of the total energy is positive, and for $\lambda < 0$ the density of the total energy is negative. The total charge density in both cases is positive[2)] ($j_5 = 0$). Thus, for four states (two with positive spin projection and two with negative) the general Pauli theorem holds at $\lambda < 0$; the four remaining states with $\lambda > 0$ are characterized by a charge density and energy density that are both simultaneously positive-definite[3)].

[2)]If, by virtue of the arbitrariness of $a^{\tau\dot{\tau}}$, we put $a^{\tau\dot{\tau}} = -i$, then for $\lambda > 0$ the density of the total energy is negative, for $\lambda < 0$ it is positive, and the total charge density is negative in both cases. The result is similar in the remaining cases of fusion.

[3)]Such states were established in [6] on the basis of infinitely-dimensional relativistic equations.

When $\kappa = 0$, the equations break up into two identical four-component equations with matrices L_i'.

2. Fusion of two particles with spins 1 ($\sigma = \sigma' \sim (1, 1)$). If we choose on the basis of (21), (22), and (28) $C^{\sigma\sigma} = i/2$ and $a^{\tau\dot{\tau}} = -1$ ($\tau \sim (0, 1)$, $l = m = 1$), we obtain 20-component equations (4), which are covariant with respect to the group T''_α (5) with matrices L_i satisfying the Duffin-Kemmer algebra. The matrix D has eight zero-value eigenvalues and 12 nonzero paired values $\lambda_i = \pm\sqrt{f^2 - d^2}$, and m_0 satisfies (29). The density of the total energy is positive at $\lambda_i > 0$ and $\lambda_i < 0$, the total charge density is positive when $\lambda_i > 0$ and negative when $\lambda_i < 0$ ($j_5 = 0$). When $\kappa = 0$, the equations break up into two identical ten-component equations with Duffin-Kemmer matrices.

3. Fusion of two particles with spins $\frac{3}{2}$ ($\sigma = \sigma' \sim (\frac{1}{2}, \frac{3}{2})$). $C^{\sigma,\sigma} = 4i/5$, and $a^{\tau\dot{\tau}} = i$, $(L_4^2 - 1)(L_4^2 - \frac{9}{25}) = 0$, $\lambda_{1,2} = \pm\sqrt{f^2 - d^2}$, $\lambda_{3,4} = \pm(\frac{3}{5})\sqrt{f^2 - d^2}$, and each of the four λ_i is eightfold. The density of the total energy is positive when $\lambda_i > 0$ and negative when $\lambda_i < 0$, and the total charge density is positive in all cases. When $\kappa = 0$, the equations break up into two identical 16-component equations.

4. Fusion of two particles with spins $\frac{1}{2}$—$\frac{3}{2}$. Eqs. (4), which are covariant with respect to T'_α (5), have 20 components, of which only 12 are independent. This agrees with the results of[3,4]. The minimal polynomial $L_4(L_4^2 + 1) = 0$ differs, but investigations reported in[9] indicate that the results of[3,4] do not exhaust all the possibilities.

5. Fusion of two particles with spins 0—1. Eqs. (4), covariant with respect to T'_α (5), have 15 components, and the matrices L_i satisfy the Duffin-Kemmer algebra.

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