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LIMITING PARAMETERS OF ULTRA-SHORT PULSES EMITTED BY A LASER WITH RESONANCE LOSS MODULATION

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The paper presents the results of analyzing the parameters of pulses generated by a laser in a stationary mode-locking regime achieved by resonance modulation of the transparency of one of the mirrors (at a frequency equal to the frequency difference of the neighboring axial resonator modes). It is shown that in the absence of dispersion (infinite gain bandwidth and infinite number of equidistant resonator modes) the pulse has a finite length. The pulse shape is close to a triangle with a vertical leading edge.

AMONG the most promising methods of producing short light pulses of record intensity is the so-called method of capture (mode-locking) of the axial modes of the laser resonator. Mode locking is accomplished either by resonance variation of the laser parameters or by self-synchronization due to various nonlinear effects. Theoretical and experimental investigation of this phenomenon show that a suitable choice of laser parameters can make the length T_p of the emitted pulses significantly shorter than the travel time τ_p of light through the laser resonator so that the field in the resonator takes up at any time only a small portion ($T_p/\tau_p \ll 1$) of the cavity. Under these conditions the processes occurring in the laser can be more adequately described by the space-time[1-3] approach than by the spectral[4-18] approach, since in the latter case a large number of modes must be considered to describe the behavior of a small "bunch" of the field in the resonator. As it was noted in[3] the space-time approach represents the generation of a periodic pulse sequence as the result of a periodic emission and the subsequent amplification of one or several pulses that "travel" in the resonator. Consequently some results obtained in the study of short-pulse amplification in active media can be applied to the investigation of the generation process.

In the present paper this method is used to find the parameters of pulses generated by a laser in a stationary regime with resonance loss modulation. In the analysis it is assumed that the gain of the active medium is independent of frequency and there is no dispersive washout of the pulse in the medium filling the resonator. Under these conditions the pulse length is obviously minimal and the remaining parameters assume limiting values. The assumption of an unlimited gain bandwidth significantly simplifies the analysis since it permits us to use rate equations to describe the interaction between the field and matter.

1. BASIC RELATIONS IN A TRAVELING-WAVE LASER

The model of a traveling-wave laser given in Fig. 1 yields the sought answer in the simplest manner. Resonator losses are varied by changing the reflection coefficient of one or several mirrors. We assume that the reflection coefficient of all mirrors referred to a single mirror equals $\rho(t)$. The initial equations have the following form.

The equation for field intensity m in the resonator is

$$\frac{\partial m}{\partial t} + v\frac{\partial m}{\partial z} = Bmn, \tag{1}$$

where t and z are the time and coordinate, v is pulse velocity, and B is the Einstein coefficient. Attenuation in the medium is neglected and it is assumed that the basic losses are due to the departure of $\rho(t)$ from unity.

The equation describing the variation of population difference n when the pulse passes through point z under consideration is

$$\partial n / \partial t = -Bmn. \tag{2}$$

The effect of pumping and population relaxation during this time is neglected.

The equation for the population difference when there is no pulse (during the excitation of the material) is

$$\partial n / \partial t = -(n - n_0) / T_1, \tag{3}$$

where T_1 is the effective relaxation time of the material ($T_1 \gg T_p$) and n_0 is the stationary value of population difference at zero-field in the resonator.

A separate analysis of (2) and (3) is obviously possible only when the pulse length T_p is significantly shorter than the travel time of the pulse through the resonator. This condition is assumed, as noted above.

The solution of the system (1) and (2), provided that the quantity $n_1(z)$ (the value of n at point z at the instant before the pulse passes through this point) is given, can be readily obtained (see[19] for example):

$$n(z, t+\tau_1) = n_1(z) / \left(1 + K_1(z)\left\{\exp\left[B\int_{t_н}^{t_1} m_{l_1}(t'+\tau_1)\,dt'\right] - 1\right\}\right), \tag{4}$$

$$m(z, t+\tau_1) = m_{l_1}(t_1+\tau_1) / \left(1 - [1 - K_1^{-1}(z)] \times \exp\left[-B\int_{t_н}^{t_1} m_{l_1}(t'+\tau_1)\,dt'\right]\right), \tag{5}$$

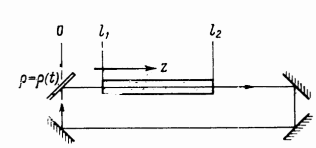

FIG. 1. Model of a traveling-wave laser

where t_i is the time of appearance of the pulse at the left mirror $\rho(t)$ (Fig. 1) at z = 0, τ_1 is the travel time of the pulse between points z = 0 and z = l_1, $t_1 = t - z/v$, $m_{l_1}(t)$ is the pulse envelope at the entry to the active material, and

$$K_1(z) = \exp\left[\frac{B}{v}\int_{l_1}^{z} n_1(z')dz'\right]$$

is the gain of the initial section of the pulse (weak-signal gain).

Equations (4) and (5) are used to relate the values of n(z) before ($n_1(z)$) and after ($n_2(z)$) the pulse:

$$n_2(z) = n_1(z)/\left(1 + K_1(z)\left\{\exp\left[B\int_{t_i}^{t_i+T_p} m_{l_1}(t'+\tau_1)dt\right] - 1\right\}\right), \quad (6)$$

and the values of light intensities at the input ($m = m_{l_1}(t)$) and output ($m = m_{l_2}(t)$) of the amplifying medium

$$m_{l_2}(t+\tau_1+\tau) = m_{l_1}(t+\tau_1)\Big/\Big(1-(1-G^{-1}) \times \exp\Big\{-B\int_{t_i}^{t} m_{l_1}(t'+\tau_1)dt'\Big\}\Big), \quad (7)$$

where $\tau = l/v$ is the travel time of the pulse through the active sector, and $G = K_1|_{z=l_2}$.

Considering that the pulse shape does not change in the remaining portion of the resonator (a medium without dispersion) but that only its time lag increases up to the value of the total travel time τ_p through the resonator and that pulse intensity drops off at the mirror with the effective reflection coefficient $\rho(t + \tau_p)$, we can write an integral equation for the envelope of a stationary pulse in the form

$$m_0(t+\tau_p) = \frac{m_0(t)\rho(t+\tau_p)}{1-(1-G^{-1})\beta(t)}, \quad (8)$$

where $\beta(t) = \exp\left[-B\int_{t_p}^{t} m_0(t')dt'\right]$ and $m_0(t)$ is the envelope of the pulse reflected by the left-hand mirror $\rho(t)$ (Fig. 1). Equation (8) is satisfied by solutions

$$m_0(t) = 0; \quad (9a)$$

$$m_0(t+\tau_p) = m_0(t), \quad 1 = \frac{\rho(t+\tau_p)}{1-(1-G^{-1})\beta(t)}. \quad (9b)$$

It is clear that (9b) are simultaneous equations provided that $\rho(t)$ is a periodic function with a period of $\tau_\rho = \tau_p/n$, where n is an integer, and that[1)]

$$m_0(t) = -\frac{1}{B}\frac{d}{dt}\left[\ln\frac{1-\rho(t+\tau_p)}{1-G^{-1}}\right] = \frac{\rho'(t+\tau_p)}{B[1-\rho(t+\tau_p)]}. \quad (10)$$

At first we consider that n = 1, i.e., $\tau_\rho = \tau_p$. From (9b) we see that $m_0(t) \neq 0$ ($\beta(t) < 1$) only when

$$F(t) = \frac{1-\rho(t)}{1-G^{-1}} < 1.$$

The pulse begins at time t_i determined from the condition

$$F(t_i) = \beta(t_i) = 1 \text{ or } \rho(t_i)G = 1. \quad (11)$$

The pulse ends at time t_f determined by the relation

$$\rho'(t)|_{t=t_f} = 0 \quad (12)$$

provided that $\rho(t)$ has a maximum[2)] at $t = t_f$.

Consequently the pulse envelope (see (10)) is completely determined by the form of the function $\rho(t)$ within a time interval $t_i < t \leq t_f$. To determine t_i and consequently the pulse shape we must find gain G expressed by an integral of $n_1(z)$. One relation between $n_1(z)$ and $n_2(z)$ is given by (6). Another is found from the solution of (3) for an initial distribution of population difference $n_2(z)$. The corresponding relation has the form

$$n_0 - n_1^{(1)}(z) = [n_0 - n_2(z)]e^{-\mu}, \quad (13)$$

where $n_1^{(1)}(z)$ is the population difference at the end of the excitation stage and $\mu = \tau_p/T_1$. Substituting $n_2(z)$ found from (13) into (6) and assuming that $n_1^{(1)}(z) = n_1(z)$ we obtain an integral equation for $n_1(z)$

$$n_1 e^{\mu} - n_0(e^{\mu}-1) = \frac{n_1}{1+(\bar\beta^{-1}-1)K_1(z)}, \quad \bar\beta = \beta(t_i + T_p), \quad (14)$$

which is reduced to the differential equation by the substitution $n_1 = vK_1'/BK_1$:

$$\frac{K_1'}{K_1}e^{\mu} - \frac{Bn_0}{v}(e^{\mu}-1) = \frac{K_1'}{K_1[1+(\bar\beta^{-1}-1)K_1]}. \quad (15)$$

Solution of (15) for $K_1|_{z=l_1} = 1$ has the form

$$(e^{\mu}-1)\left(\ln K_1 - \frac{Bn_0}{v}z\right) = -\ln\{\bar\beta[1+(\bar\beta^{-1}-1)K_1]\}. \quad (16)$$

It follows from (16) that

$$\left(\frac{G}{K_0}\right)^{e^{\mu}-1} = \{\bar\beta[1+(\bar\beta^{-1}-1)G]\}^{-1}, \quad K_0 = \exp\left(\frac{B}{v}n_0 l\right). \quad (17)$$

Another relation between G and $\bar\beta$ is found from (9b) at $t = t_f$

$$\bar\beta = \frac{1-\rho_0}{1-G^{-1}}, \quad \rho_0 = \rho(t_f). \quad (18)$$

We find from (17) and (18)

$$G\rho_0 = (K_0\rho_0)^{1-e^{-\mu}}. \quad (19)$$

The parameter μ is usually small and therefore (19) can be approximated in the form

$$G\rho_0 \approx 1 + \mu\ln(K_0\rho_0). \quad (20)$$

The obtained relations (10), (11), (12), and (19) permit us to determine both the length and the shape of the pulse in the limiting case under consideration where any dispersion is neglected. The pulse length found from (11) and (12) taking into account the small value of μ is equal to[3)]

$$T_p = \left\{-2\mu\frac{\rho_0}{\rho_0''}\ln(K_0\rho_0)\right\}^{1/2}, \quad (21)$$

where ρ_0'' is the second derivative of the function $\rho(t)$ at

1) It follows from (10) that when the maximum value of $\rho_{max}(+) = \rho_0$ tends to unity the field intensity in the resonator tends to infinity, while the field intensity in the output pulse $m_{out}(t) = (1-\rho(t))m_0(t)/\rho(t)$ remains finite.

2) If the function $\rho(t)$ has an inflexion point with $\rho'(t)|_{t=t_\kappa} = 0$ at $t = t_f$, then $m_0(t)$ turns to zero at this point beyond which the field again increases.

3) In (21) it is assumed that $\rho_0'' \neq 0$. When $\rho_0'' = 0$ and $\rho_0'''' \neq 0$ the expression for T_p is written as

$$T_p = \left\{-24\mu\frac{\rho_0}{\rho_0''''}\ln(K_0\rho_0)\right\}^{1/4}.$$

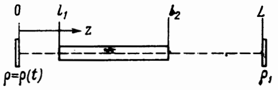

FIG. 2. Model of a two-mirror laser.

$t = t_f$. The energy in the emitted pulse is determined by integrating $m_0(t)$:

$$W = vh\nu \frac{1-\rho_0}{\rho_0} \int_{t_i}^{t_i+T_i} m_0(t)\,dt = \mu h\nu \frac{v}{B} \ln (K_0 \rho_0). \tag{22}$$

A more detailed discussion of results[4] is given below after an examination of the mode locking process in the "conventional" (two-mirror) laser.

2. BASIC RELATIONSHIPS IN A TWO-MIRROR LASER

The results obtained above are now generalized to include the case of a two-mirror laser (Fig. 2) in which the pulse passes through the active material in both directions. To make the case more definite we consider the reflection coefficient of the left mirror $\rho(t)$ variable and the reflection coefficient of the right mirror constant. The z coordinate is measured from the left mirror, and the active region of length l is located anywhere in the resonator whose total length is L. Let the coordinate of the left end face of the active element be l_1 and that of the right end face l_2; then $l_2 - l_1 = l$. The light intensity values at z = 0 and z = L in the forward ($m = m^{(+)}(t)$) and backward ($m = m^{(-)}(t)$) pulses are related as before by (7)[5]; the forward wave gain should be used in the expression for the forward wave

$$G_+ = \exp \left[\frac{B}{v} \int_{l_1}^{l_2} n_1^{(+)}(z')\,dz' \right],$$

and the backward wave gain in that of the backward wave

$$G_- = \exp \left[-\frac{B}{v} \int_{l_2}^{l_1} n_1^{(-)}(z')\,dz' \right].$$

When z = 0

$$m_0^{(+)}(t) = \rho(t) m_0^{(-)}(t), \tag{23a}$$

and when z = L

$$m_L^{(-)}(t) = \rho_1 m_L^{(+)}(t). \tag{23b}$$

For the sake of simplicity we introduce the notation

$$\beta_+(t) = \exp \left[-B \int_{t_i}^{t} m_0^{(+)}(t')\,dt' \right],$$

$$\beta_-(t) = \exp \left[-B \int_{t_i}^{t} m_L^{(-)}(t' + \tau_H/2)\,dt' \right] \tag{24}$$

Considering the above discussion we can write the following equations for the functions $\beta_+(t)$ and $\beta_-(t)$ provided that $\rho(t) = \rho(t + \tau_p)$:

$$\frac{\beta_+'}{\beta_+}[1 - (1 - G_-^{-1})\beta_-] = \rho(t) \frac{\beta_-'}{\beta_-},$$

$$\frac{\beta_-'}{\beta_-}[1 - (1 - G_+^{-1})\beta_+] = \rho_1 \frac{\beta_+'}{\beta_+}. \tag{25}$$

The solutions of (25) are

$$[1 - (1 - G_+^{-1})\beta_+][1 - (1 - G_-^{-1})\beta_-] = \rho_1 \rho(t),$$

$$\beta_- = \left[\frac{\beta_+}{G_+ - (G_+ - 1)\beta_+} \right]^{\rho_1} \tag{26}$$

According to (24) the beginning of the pulse ($t = t_i$) corresponds to the values $\beta_+(t_i) = 1$ and $\beta_-(t_i) = 1$; using (26) to determine the beginning of the pulse t_i we obtain

$$G_+^{-1} G_-^{-1} = \rho_1 \rho(t_i).$$

The pulse ends at time t_f corresponding to the minimum value of the functions $\beta_+(t)$ and $\beta_-(t)$ and, according to (26), to the maximum value of the function $\rho(t)$.

Thus (26) and (24) completely determine the shape of the pulse at z = 0 and z = L within the time interval $t_i < t \le t_f$ in terms of gains $G_+ = K_+(z)|_{z=l_2}$ and $G_- = K_-(z)|_{z=l_1}$.

The gain equations

$$K_+(z) = \exp \left[\frac{B}{v} \int_{l_1}^{z} n_1^{(+)}(z')\,dz' \right],$$

$$K_-(z) = \exp \left[-\frac{B}{v} \int_{l_2}^{z} n_1^{(-)}(z')\,dz' \right]$$

can be obtained in a manner similar to that employed in the case of a traveling-wave laser by using (6) and (3); the latter is now integrated within time limits from the passage of the pulse through point z in one direction to the passage through the same point in the opposite direction. As a result we write the following equations

$$\frac{K_\pm'}{K_\pm} = \pm \frac{B}{v} n_0 [1 + (\bar\beta_\pm^{-1} - 1) K_\pm] \tag{27}$$

$$\times \frac{e^\mu - 1 + (\bar\beta_\mp^{-1} - 1) K_\mp e^{\Phi_{2,1}} (e^{\Phi_{1,2}} - 1)}{[1 + (\bar\beta_+^{-1} - 1) K_+][1 + (\bar\beta_-^{-1} - 1) K_-] e^\mu - 1},$$

where

$$\bar\beta_\pm = \beta_\pm (t_i + T_p),$$

$$\Phi_1 = 2\mu \left(\frac{\tau_1}{\tau_p} + \frac{\tau}{\tau_p} \frac{z - l_1}{l_2 - l_1} \right), \quad \Phi_2 = 2\mu \left(\frac{\tau_2}{\tau_p} + \frac{\tau}{\tau_p} \frac{l_2 - z}{l_2 - l_1} \right),$$

and τ_1 is the travel time of the pulse between the points z = 0 and z = l_1, τ_2 is the travel time of the pulse between the points z = l_2 and z = L, τ is travel time of the pulse along the active region, $\mu = \tau_p / T_1$, and $\tau_p = 2(\tau + \tau_1 + \tau_2)$. Equations (27) are exact within the limitations stated above and can be used for the numerical solutions by computers. The analytic solution of (27) however is generally difficult. On the other hand it was noted above that the value of μ is usually small. We consider μ small and find an approximate solution of (27) retaining only terms of the order not exceeding μ. Assuming that

$$K_\pm = K_\pm^{(0)} + \mu K_\pm^{(1)}, \quad \beta_\pm(t) = 1 + \mu \beta_\pm^{(1)}(t), \tag{28}$$

$$G_\pm = G_\pm^{(0)} + \mu G_\pm^{(1)}, \quad \rho(t) = \rho_0 + \rho_0'' \frac{(t - t_f)^2}{2},$$

$$\rho_0'' = \rho''(t_f),$$

we find from (26) and (27) that

$$K_+^{(0)}(z) K_-^{(0)}(z) = G_+^{(0)} = G_-^{(0)} = (\rho_0 \rho_1)^{-1/2}, \tag{29}$$

$$\bar\beta_+^{(1)} + \bar\beta_-^{(1)} = -\frac{\ln \{K_0 (G_+^{(0)} G_-^{(0)})^{-1/2}]}{(G_+^{(0)} G_-^{(0)})^{1/2} - 1}.$$

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[4] Relations (10), (11), (12), and (20) and their preliminary analysis are given in a brief communication [3].

[5] The effects due to the superimposition of the forward and backward pulses in the active region are neglected. These effects vanish altogether when l_1/v and $(L - l_2)/v > T_p/2$.

The equations for the determination of the function $\beta_{\pm}^{(1)}(t)$ are found from (26) and written in the form

$$\mu[\beta_+^{(1)}(t)(G_+^{(0)}-1)+\beta_-^{(1)}(t)(G_-^{(0)}-1)+\rho_0\rho_1(G_+^{(0)}G_-^{(1)} + G_-^{(0)}G_+^{(1)})]+\rho_0''\rho_1 G_+^{(0)}G_-^{(0)}\frac{(t-t_f)^2}{2}=0,$$

$$\beta_-^{(1)}(t)=\rho_1 G_+^{(0)}\beta_+^{(1)}(t). \tag{30}$$

We find the pulse shape from (24) and (30)

$$m_0^{(+)}(t)=-\frac{1}{B}\mu\beta_+^{(1)\prime}(t)=\frac{1}{B}\frac{\rho_1^{1/2}\rho_0''(t-t_f)}{\rho_0^{1/2}(1-\rho_1)+\rho_1^{1/2}(1-\rho_0)}, \quad t_i<t\leqslant t_f,$$

$$m_L^{(-)}\left(t+\frac{\tau_p}{2}\right)=-\frac{1}{B}\mu\beta_-^{(1)\prime}(t)=\frac{1}{B}\frac{\rho_0^{-1/2}\rho_1\rho_0''(t-t_f)}{\rho_0^{1/2}(1-\rho_1)+\rho_1^{1/2}(1-\rho_0)},$$

$$m_0^{(+)}(t)=0, \quad m_L^{(-)}\left(t+\frac{\tau_p}{2}\right)=0 \quad \text{for} \quad t_f-\tau_p\leqslant t<t_i. \tag{31}$$

Using (30) and taking (29) into account we make the substitution $t = t_i$ and $t_f - t_i = T_p$ and obtain an expression for the pulse length:

$$T_p=\left\{-2\mu\frac{\rho_0}{''}\ln[K_0(\rho_0\rho_1)^{1/2}]\right\}^{1/2}. \tag{32}$$

Equations (31) determine a triangular pulse with a vertical leading edge at $t = t_i$. The pulse length is proportional to $\mu^{1/2}$ and is the same for pulses traveling in the $+z$ and $-z$ directions.

The energy emitted by the laser per cycle, i.e. the energy $W^{(-)}$ in a pulse emitted through mirror $\rho(t)$ plus energy $W^{(+)}$ in a pulse emitted through mirror ρ_1 can be determined by integrating $m_0^{(+)}(t)$ and $m_L^{(-)}(t)$. Retaining only terms not exceeding the order of μ we find

$$W^{(-)}+W^{(+)}=vh\nu\left[\frac{1-\rho_0}{\rho_0}\int_{t_i}^{t_i+T_p}m_0^{(+)}(t)\,dt + \frac{1-\rho_1}{\rho_1}\int_{t_i}^{t_i+T_p}m_L^{(-)}\left(t+\frac{\tau_p}{2}\right)dt\right]=\mu h\nu\frac{v}{B}\ln[K_0(\rho_0\rho_1)^{1/2}]. \tag{33}$$

3. DISCUSSION OF RESULTS AND NUMERICAL COMPUTATIONS

The above equations show that in a small μ approximation there is no significant difference in the mode-locking process between the ordinary laser and the traveling-wave laser. Indeed, (21) and (32) defining pulse length and (22) and (33) defining the total emitted energy differ only in the form of the argument under the logarithmic sign: $K_0\rho_0$ and $K_0(\rho_0\rho_1)^{1/2}$ respectively. However these quantities coincide in lasers with the same intensity of excitation of the active material (the same value of K_0) and the same pump threshold value ($(\rho_0)_{tw} = (\rho_0\rho_1)^{1/2}_{ord}$) and consequently the lengths and total energies of the pulses also coincide. The pulse shape is also the same in both cases, representing a triangle with a steep leading edge. We see from (10) that the pulse shape is different only when the expression

$$\rho(t_i)=\rho_0+\rho_0''(t_i-t_f)^2/2$$

is essentially invalid. Using (9b) and (19) we can find the required law of variation of $\rho(t)$ from a given pulse shape. We note however that when $T_p \ll \tau_\rho$ pulse shapes that differ from the triangular can be obtained only with nonsinusoidal loss modulation.

In the case of the traveling wave laser the above computations can be readily shown valid also for $\tau_p = n\tau_\rho$ if μ is interpreted as $\mu_{new} = \tau_\rho/T_1 = \mu/n$ in the corresponding equations. The pulse length is shortened here (in comparsion to the $\tau_p = \tau_\rho$ regime) $n^{1/2}$ times and the energy per pulse is increased n times. When $\tau_p = n\tau_\rho$ pulses of other periodicity (multiples of τ_ρ), such as pulses with the period τ_p, can also be generated. The problem of stability of any particular regime calls for a special analysis.

Incidentally we note that in the investigation of the corresponding system of equations we did not consider the known unstable solutions. These include the trivial solution $m_0(t) \equiv 0$ for example that is unstable when $K_0\rho_0 > 1$. The equations are also satisfied by a solution that coincides over a portion of the interval (for example in the region $t_1 < t \leq t_f$, where $t_i < t_1 \leq t_f$) with the function $m_0(t)$ determined by (10), turns to infinity at $t = t_1$ (as $\delta(t - t_1)$) with the energy

$$\int_{t_i}^{t_1} m_0(t)\,dt$$

and is equal to zero at all other times. One of the unstable solutions, $\delta(t - t_1)$ at $t_1 = t_f$ with the energy

$$\int_{t_i}^{t_f} m_0(t)\,dt,$$

is obtained in the case of exact phase-locking of the laser modes without dispersion. We readily find that in all these cases the effective gain $\rho(t)K(L, t)$ is larger than unity in the interval $t_i < t < t_1$ and consequently in real systems such a solution is transformed into a stable solution because of spontaneous emission or even a small dispersion.

A significant feature of all pulses (regardless of the form of $\rho(t)$) is the vertical leading edge. This is due to the fact that the quantity $\rho(t)K(L, t)$ is less than unity at $t < t_i$ and in the absence of dispersion for any initial distribution the field, beginning with some time instant, damps out at $t < t_i$ and tends to zero. When $t > t_i$ in the absence of a pulse the quantity $\rho(t)K(L, t) > 1$; consequently with weak initial pulses the field grows until the quantity $\rho(t)K(L, t)$ turns into unity due to the gain saturation effect. These two processes cause a jump in the field at $t = t_i$. The finite value of the jump is due to the fact that $\rho(t_i)K(L, t) = 1$.

If the stationary pulse began at time t_1 such that $\rho(t_1)K(L, t_1) > 1$ the field intensity at the leading edge would be infinite as was noted above.

Numerical computation for a traveling wave laser as performed in[3] for the function

$$\rho(t)=\rho_0\left(1-\alpha\cos\frac{2\pi}{\tau_p}t\right)\Big/(1+\alpha), \tag{34}$$

showed that for the solid state laser (with $K_0\rho_0 = 3$, $\tau_p = 10^{-8}$ sec, and $T_1 = 10^{-3}$ sec) the pulse length $T_p^{(0.5)}$ measured at 0.5 of the maximum intensity equals 0.5×10^{-11} sec for $\alpha = 1$ and 10^{-11} sec for $\alpha = 0.2$. For a gas laser (with $K_0\rho_0 = 1.05$, $\tau_p = 10^{-8}$ sec, and $T_1 = 10^{-7}$ sec) the pulse length measured at the same level equals 10^{-10} sec for $\alpha = 1$ and 2×10^{-10} sec for $\alpha = 0.2$.

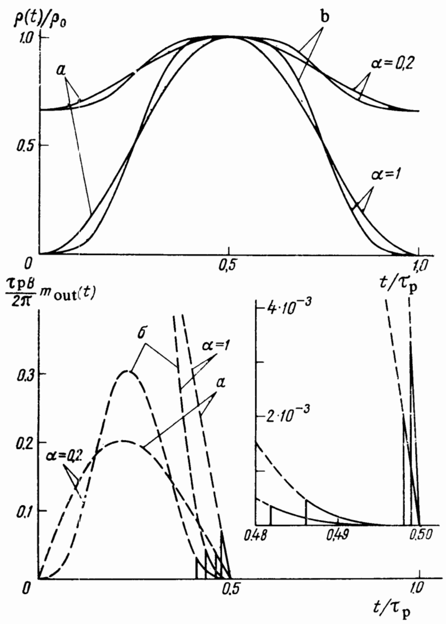

FIG. 3. Graphs of the function $\rho(t)$ and the corresponding shapes of output pulses from a traveling-wave laser for $K_0\rho_0 = 1.05$, $\tau_p = 10^{-8}$ sec, and $T_1 = 10^{-7}$ sec. Pulse lengths are determined by (21) or (21′). Dashed lines are envelopes plotted from the formula $\tau_p\rho'(t)/2\pi\rho(t)$. The output pulses of a traveling-wave laser for $K_0\rho_0 = 3$; $\tau_p = 10^{-8}$ sec, and $T_1 = 10^{-3}$ sec are drawn to a larger scale.

These computations remain valid for the ordinary laser if $(\rho_0\rho_1)^{1/2}_{\text{ord}} = \rho_{0\,\text{tw}}$.

To illustrate the effect of the form of the function $\rho(t)$ on pulse parameters we consider the function

$$\rho(t) = \rho_0\left[1 - \frac{\alpha}{8}\left(9\cos\frac{2\pi}{\tau_p}t - \cos\frac{6\pi}{\tau_p}t\right)\right] \Big/ (1+\alpha), \qquad (35)$$

for which $\rho_0'' = 0$. It is easily shown that with the same laser parameters the pulse length $T_p^{(0.5)}$ at the 0.5 level is in the above case five times longer at $\alpha = 1$ and four times longer at $\alpha = 0.2$ in a solid state laser. In a gas laser the pulse length remains approximately the same. This example shows that flattening of the curve $\rho(t)$ near the maximum can result in a considerable lengthening of the pulse and a corresponding decrease of power in the pulse.

Figure 3 shows the shapes of output pulses

$$m_{\text{out}} = \frac{1-\rho(t)}{\rho(t)} m_0(t),$$

obtained from expressions for the traveling-wave laser with reflection coefficients $\rho(t)$ determined by (34) (curves a) and (35) (curves b).

We note that the pulse shape can be significantly changed by a very slight change in the quantity that determines modulation, for example in the case of a small increase in voltage at the electro-optical shutter, producing a weak over-modulation condition.

In conclusion we again note that the above results are obtained without taking dispersion in the elements and material of the laser into account. However when the length of the pulse obtained in this manner is significantly larger than the reciprocal width of the corresponding band (the gain bandwidth of the active medium for example) the finite value of the latter will merely cause a slight deformation of the leading edge of the pulse and a smoothing of its trailing edge.

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