DEGENERATE PINCH IN SEMICONDUCTORS

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The stationary state of a degenerate pinch is investigated, and the conditions under which the stationary state collapses and the pinch parameters (pinch channel radius, mean density over the channel) are determined. The population-inversion criterion (for transitions from the bottom of the conduction band to the top of the valence band[4]) is expressed in terms of the current in the pinch. Numerical estimates are made with the pinch in InSb as an example.

1. It is known that large carrier densities, at which the electron-hole gas becomes degenerate, are attained in a pinch channel[1-3]. In this paper we obtain equations describing the stationary state of a strongly degenerate pinch. These equations are solved by numerical methods for the case of linear volume recombination and a low rate of surface recombination, when the number of the carriers in the pinch channel is conserved. We derive a criterion for the collapse of the stationary state and calculate the pinch parameters (the radius of the pinch channel and the average density over the channel).

An expression is obtained for the spatial dependence of the carrier distribution near the surface of the pinch; this may be of interest in the calculation of recombination radiation from the pinch channel (in view of the strong absorption of the radiation by the crystal). If the electron gas is strongly degenerate, inversion of the carrier population takes place. The population-inversion criterion for transitions from the bottom of the conduction band to the top of the valence band[4] is expressed in terms of the current in the pinch (the calculation is based on the average carrier density in the channel). All the numerical calculations were made with the pinch in InSb as an example; the numerical values are given in the cgs esu system.

2. The study concerns the case of a cylindrical sample with equal number of electrons and holes ($n = p$). If the surface of the sample is sufficiently clean, so that the rate of surface recombination is lower than the rate of ambipolar diffusion ($s \ll D_a/R_0$), then, at small currents, the spatial distribution of the electron-hole plasma is homogeneous over the cross section of the sample (with density n_0). When the current is increased, a "compression" of the spatial distribution of the carriers towards the sample axis takes place, and at sufficiently strong currents this stationary state cannot be maintained. A pinch effect arises, in which the boundaries of the spatial distribution of the plasma narrow down and the new stationary state is characterized by an increased average density over the cross section of the current channel. During the "compression" process, we have $n < n_0$ near the sample surface, and volume generation dominates in this region, whereas in the region where $n > n_0$ (central part of the channel) volume recombination predominates[5]. In the case of a pinch made up of the crystal's own gas of electrons and holes, when $n_0 = n_{eq}$, where n_{eq} is the equilibrium concentration, the occurrence of this effect is not subject to any doubt. In the case of injection, when $n_0 > n_{eq}$, this effect is due to the fact that the injecting contact, regardless of the magnitude of the current, generates non-equilibrium carriers homogeneously over the entire cross section of the contact (it will henceforth be assumed that the cross section of the injecting contact coincides with the cross section of the sample).

We begin the analysis of the stationary state of the plasma with the equations of motion and continuity, and with Maxwell's equations, assuming the radial fluxes of the electrons and holes to be ambipolar. These equations are of the following form:

$$\frac{\nabla p_e}{m_e^* n} = -\frac{e}{m_e^*}\mathbf{E} - [\mathbf{v}_e\boldsymbol{\omega}_{eH}] - \frac{\mathbf{v}_e}{\tau_e},$$
$$\frac{\nabla p_h}{m_h^* n} = \frac{e}{m_h^*}\mathbf{E} + [\mathbf{v}_h\boldsymbol{\omega}_{hH}] - \frac{\mathbf{v}_h}{\tau_h},$$
$$\operatorname{div} n\mathbf{v}_e = -\frac{n-n_0}{\tau_0}, \qquad \operatorname{div} n\mathbf{v}_h = -\frac{n-n_0}{\tau_0},$$
$$v_{er} = v_{hr} = v_r, \qquad \operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \tag{1}*$$

where p_e and p_h are the pressures of the electrons and holes, $\mathbf{v}_e$ and $\mathbf{v}_h$ are the velocities of the electrons and $\omega_{iH} = eH/m_i^* c$, where m_i^* are the effective masses of the carriers ($i = e, h$), $\mathbf{H}$ is the magnetic field (in the case considered by us the magnetic field has only an azimuthal component due to the current), τ_i are the relaxation times, and τ_0 the carrier lifetime (assumed to be the same for the electrons and holes).

We shall solve the equations in (1) under the assumption that the spatial distribution of the carriers depends only on the radius (the electric field is directed along the Oz axis). It is easy to obtain from (1) an initial equation describing the stationary state of the pinch:

$$\frac{d(p_e+p_h)}{dr} = -\frac{4\pi e^2 v_d^2}{c^2}\frac{n}{r}\int_0^r n(r')r'\,dr' + \frac{e}{\tau_0 b_h}\frac{1}{r}\int_0^r (n-n_0)r'\,dr', \tag{2}$$

where $v_d = b_e E_z$ is the drift velocity, and the expression for the ambipolar electric field is

$$E_r = -\frac{1}{en}\frac{dp_e}{dr} - \frac{H_\varphi}{c}v_z. \tag{3}$$

In the derivation of (2) and (3) we have assumed that

*$[\mathbf{v}_e\boldsymbol{\omega}_{eH}] \equiv \mathbf{v}_e \times \boldsymbol{\omega}_{eH}$.

$$b_e / b_h \gg 1, \quad \omega_{eH}\tau_e\omega_{hH}\tau_h \ll 1, \quad p_h \lessgtr p_e. \tag{4}$$

The boundary condition on the surface of the sample is

$$nv_r(r = R_0) = -\frac{1}{\tau_0 R_0}\int_0^{R_0}(n - n_0)r'dr' = s(n - n_{eq})|_{r=R_0}, \tag{5}$$

where R_0 is the radius of the sample and s the rate of surface recombination.

In the classical case $p_e = p_h = nkT$ (it is assumed that the temperatures of the electrons and holes are equal), and in the degenerate case, when the chemical potential μ of one of the components (in our case, of the electrons, inasmuch as $m_e^* \ll m_h^*$) is much larger than the thermal energy,

$$\mu_e = \frac{(3\pi^2)^{2/3}}{2}\frac{\hbar^2}{m_e^*}n^{2/3} \gg kT, \tag{6}$$

the pressure of the electron-hole gas is determined by the relation[6]

$$p \approx p_e = {}^2/_5 n\mu_e = \alpha n^{5/3}, \tag{7}$$

where $\alpha = (3\pi^2)^{2/3}\hbar^2/5m_e^*$. In InSb, the effective mass is $m_e^* = 0.013\ m_0$, where m_0 is the mass of the free electron, and $m_h^* \approx 40\ m_e^*$[7], and accordingly $\alpha \approx 2 \times 10^{-25}$ cgs esu.

Equation (2) can be rewritten in the following dimensionless form:

a) in the classical case

$$\frac{dq}{dx} = -\frac{\Gamma_c q}{x}\int_0^x q(x')x'dx' + \frac{\gamma_c}{x}\int_0^x (q-1)x'dx', \tag{8}$$

where

$$q = \frac{n}{n_0}, \quad x = \frac{r}{R_0}, \quad \Gamma_c = \frac{2\pi e^2 v_d^2 n_0 R_0^2}{c^2 kT}, \quad \gamma_c = \frac{R_0^2}{l_{d.c}^2} \tag{8'}$$

and the diffusion length of the carriers in the nondegenerate gas is

$$l_{d.c} = (2b_h\tau_0 kT/e)^{1/2};$$

b) in the degenerate case

$$\frac{dq}{dx} = -\frac{\Gamma_{deg}\, q^{1/3}}{x}\int_0^x q(x')x'\,dx' + \frac{\gamma_{deg}}{x}q^{-1/3}\int_0^x (q-1)x'\,dx', \tag{9}$$

where

$$\Gamma_{deg} = \frac{12\pi}{5}\frac{e^2}{c^2\alpha}v_d^2 n_0^{1/3}R_0^2, \quad \gamma_{deg} = \frac{R_0^2}{l_{d.d.}^2},$$

$$l_{d.d.} = \left(\frac{5}{3}\alpha n_0^{2/3}\frac{b_h\tau_0}{e}\right)^{1/2} \tag{9'}$$

is the diffusion length in the degenerate gas. The boundary condition (5) then takes the form

$$\int_0^1 q(x')x'dx' = \frac{1}{2} - \lambda\left(q - \frac{n_p}{n_0}\right)\Big|_{x=1}, \quad \lambda = \frac{s\tau_0}{R_0}. \tag{10}$$

As seen from (8) and (9), volume recombination predominates near the axis of the pinch channel, where $q > 1$, while volume generation predominates near the surface ($q < 1$). Under conditions when $l_d \gg R_0$, the influence of the volume recombination (generation) in (8) and (9) can be neglected, and the volume recombination influences the formation of the pinch via the boundary condition (10).

As seen from the boundary condition (10), in the case of a pinch made up of the crystal's own carrier gas ($n_{eq} = n_0$), surface generation processes predominate, inasmuch as $q(z = 1) \ll 1$ and the number of particles in the pinch will increase, thus favoring the development of the pinch and leading to the appearance of S-shaped current-voltage characteristics[8]. In the case of an injection pinch ($n_0 \gg n_{eq}$—high injection level), surface recombination predominates, thus hindering the formation of the pinch. These conclusions are valid when $\lambda \gtrsim 1$. If $\lambda \ll 1$, then the surface effects do not influence the character of the pinch and the number of particles participating in the "compression" process remains constant. Let us investigate these effects, using as an example a classical pinch, for which we can obtain an exact solution if we neglect volume recombination in (8). This solution was obtained in 1934 by Bennet[9] and is given by

$$q = Q_0(1 + x^2/x_0^2)^{-2}, \tag{11}$$

where Q_0 and $1/x_0^2$ are connected, in accordance with (8) and (10), by the relations

$$\frac{1}{x_0^2} = \frac{\Gamma_c Q_0}{8}, \quad \left(1 + \frac{2\lambda}{1 + 1/x_0^2}\right)\frac{Q_0}{1 + 1/x_0^2} = 1 + 2\lambda\frac{n_{eq}}{n_0}. \tag{12}$$

Let us consider the cases mentioned above.

1) $\lambda \ll 1$ (surface effects play no role). From (12) we can easily get

$$Q_0 = \frac{8}{8 - \Gamma_c}, \quad \frac{1}{x_0^2} = \frac{\Gamma_c}{8 - \Gamma_c}, \tag{13}$$

from which we see that when $\Gamma_c > 8$ the stationary state of the plasma collapses and the current channel begins to narrow down. In this case, however, the theory does not yield the value of the new radius of the pinch channel, since in the classical case Γ_c does not depend on R when the number of particles in the channel is conserved ($n_0R_0 = \bar{n}R^2$, where $\bar{n}$ is the average density in a channel of radius $R < R_0$). At the instant when the stationary state collapses, the density on the axis becomes infinitely large.

2) In the case of pinch of the crystal's own gas ($n_{eq} = n_0$) and $\lambda \gg 1$, a similar calculation shows that the collapse of the stationary state occurs at $\Gamma_c > 2$, i.e., the pinching conditions are eased. The conductivity increases with increasing parameter Γ_c, and the current-voltage characteristic becomes S-shaped in the controlled-current mode.

3) In the case of an injection pinch ($n_0 \gg n_{eq}$) and $\lambda \gg 1$, the collapse of the stationary state occurs at $\Gamma_c > 4\lambda$, i.e., the pinching is hindered. The resistance of the sample increases in this case in proportion to λ.

3. We shall investigate Eqs. (9) and (10) for a degenerate pinch in the case when the number of particles in the pinch is conserved. To this end it is necessary to have $\lambda \ll 1$. When $R_0 \approx 10^{-2}$ cm and $\tau_0 \approx 10^{-6}$ sec[10], this condition is satisfied if the rate of surface recombination of the sample is $s \ll 10^4$ cm/sec. The equations describing the degenerate pinch in this case are

$$\frac{dq}{dx} = -\frac{\Gamma_{deg}\, q^{1/3}}{x}\int_0^x q(x')x'dx' + \frac{\gamma_{deg}}{x}q^{-2/3}\int_0^x (q-1)x'dx', \tag{14}$$

$$\int_0^1 q(x')x'dx' = {}^1/_2. \tag{15}$$

Figures 1 and 2 show the result of a numerical integration of Eqs. (14) and (15) for several values of the parameter Γ_{deg} and $\gamma_{deg} = 0$ and 10. With increasing Γ_{deg} (current) the carriers "compress" towards the axis; the effect of "compression" is much more strongly pronounced when $\gamma_{deg} = 0$ than when $\gamma_{deg} = 10$ (at the same value of Γ_{deg}). Thus, the volume recombination prevents the formation of the pinch even under conditions when the number of particles is conserved.

For definite values of Γ_{deg}, namely

$$\Gamma_{deg} > 6.5\ (\gamma_{deg} = 0 \text{ and } \Gamma_{deg} > 21\quad (\gamma_B = 10), \tag{16}$$

negative solutions are obtained for q, and the stationary state is violated. The density distribution at $\Gamma_{deg} = 6.5$ ($\gamma_{deg} = 0$) and $\Gamma_{deg} = 21$ ($\gamma_{deg} = 10$) correspond to the state prior to the pinch. We present below the values of Γ_{deg} at the instant of collapse ($\Gamma_{d.c}$) for different values of the parameter γ_{deg}:

γ_{deg}	0	1	3	10
$\Gamma_{d.c}$	6.5	7.3	8.8	21

Figures 3 and 4 show plots of the quantity

$$Y = \frac{1}{x}\int_0^x q(x')x'dx' = \frac{cH_\varphi}{4\pi e v_d R_0 n_0} \tag{17}$$

for the same values of the parameters Γ_{deg} and γ_{deg}. It is seen from these figures that at large values of the parameter Γ_{deg}, the azimuthal magnetic field passes through a maximum inside the column, this being due to the strong compression.

In the pre-pinch state of the degenerate pinch we have

$$H_{\varphi max} \approx \frac{2{,}4I}{cR_0}\quad (\gamma_{deg} = 0,\quad \Gamma_{deg} = 6.5,\quad x_{max} \approx 0.7),$$

$$H_{\varphi max} \approx \frac{4{,}4I}{cR_0}\quad (\gamma_{deg} = 10,\quad \Gamma_{deg} = 21,\quad x_{max} \approx 0.3). \tag{18}$$

From the conditions (16) we can determine the initial currents corresponding to the collapse of the stationary state, if account is taken of the fact that when the number of particles in the pinch channel is conserved, the current is given by $I = ev_d n_0 S$, where S is the area of the injection channel.

Conditions (16) take the form

$$I \geqslant 0.87\cdot10^{-2}(Sn_0^{5/3}\Gamma_{deg})^{1/2}. \tag{19}$$

For $n_0 \approx 10^{15}$ and $S \approx 10^{-3}$ cm^2 we get

$$I_0 \approx 0.7\ a\ \text{ for }\ \gamma_{deg} = 0,$$
$$I_0 \approx 1.3\ a\ \text{ for }\ \gamma_{deg} = 10. \tag{20}$$

When the current exceeds the limiting value I_0, the radius of the pinch channel decreases, and the transition to the new stationary state occurs in accordance with the equations

$$\dot{R} = b_h E_r = -b_h\left(\frac{1}{en}\frac{dp_e}{dR} + v_d\frac{H_\varphi}{c}\right), \tag{21}$$

from which we can readily estimate the time constant of the pinch

$$\tau_p \approx c^2R_0^2/4b_n v_d I. \tag{22}$$

The density distribution in the new stationary state at $\gamma_{deg} = 0$ corresponds to the case when the new value of the parameter $\Gamma_{deg}(R)$, determined from the new boundary R, is precisely equal to 6.5. This is connected with the fact that under conditions when the plasma breaks away from the surface of the sample, the states with sharp boundary ($\Gamma_{deg} < 6.5$) will be unstable and smear out into a state in which the density gradient on the boundary vanishes.

The radius of the new channel is determined from the condition $\Gamma_{deg}(R) = 6.5$, from which we can readily find, taking into account the conservation of the number of particles in the pinch channel, that

$$R = \beta I^{-1/4},\qquad \beta = \frac{1.6\cdot10^{-13}}{v_d^{5/4}m_e^{*3/4}}. \tag{23}$$

This formula was obtained earlier, apart from a numerical coefficient, by Akramov[11] from the condition that the magnetic and gas-kinetic pressures on the surface of the pinch be equal and under the assumption that the plasma is uniformly distributed over the channel cross section. Relation (23) is unconditionally valid for currents satisfying the condition (19).

During the course of the pinch, the drift velocity in

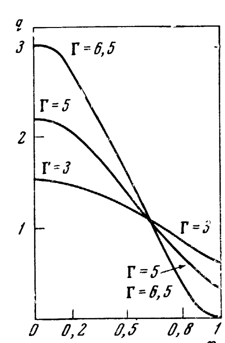

FIG. 1. Result of numerical integration of Eqs. (14) and (15), $\gamma_{deg} = 0$.

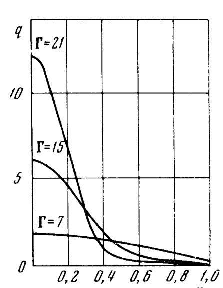

FIG. 2. Result of numerical integrations of Eqs. (14) and (15), $\gamma_{deg} = 10$.

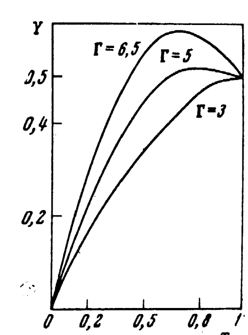

FIG. 3. Plot of the quantity Y, $\gamma_{deg} = 0$.

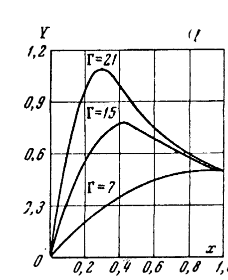

FIG. 4. Plot of the quantity Y, $\gamma_{deg} = 10$.

InSb reaches saturation: $v_d \approx 6 \times 10^6 - 2 \times 10^7$ cm/sec[12], apparently as a result of the decrease of the carrier mobility with increasing density in the current channel when $n > 10^{15}$ [7]. Electron and hole densities of this order are characteristic of the pinch in InSb[2,3]. Obviously, saturation of the drift velocity gives rise to the anomalous resistance observed in the InSb pinch by a number of authors[2,3,12]. If it is assumed that $v_d \approx 10^7$ cm/sec, then in formula (23) $\gamma \approx 1.5$ (for InSb). For example, $R \approx 4 \times 10^{-3}$ cm for a current I = 10 A. We shall henceforth put $\beta \approx 1.5$ in the various estimates concerning the pinch in InSb.

The average density in the pinch channel, taking (23) into account, varies in accordance with

$$\bar{n} = \eta I^{3/2}, \tag{24}$$

where in the case of InSb

$$\eta = 2.6 \cdot 10^{34} (v_d m_e^*)^{3/2} \approx 30. \tag{25}$$

With decreasing pinch radius, and accordingly with increasing average density, the parameter γ_{deg} tends to zero and the spatial distributions of the carrier in the pinch, corresponding to different initial values of γ_{deg}, tend in the case of large currents to the distribution of Fig. 1 ($\gamma_{\text{deg}} = 0$, $\Gamma_{\text{deg}} = 6.5$), while the radius of the channel and the average density are determined by the formulas (23) and (24).

From (14) and (15) it is easy to determine the asymptotic behavior of this distribution as $x \to 1$, by taking into account the fact that near the surface, in the case of strong "compression" we have

$$\int_0^\lambda q(t')t'dt' \approx 1/2.$$

In this case

$$\frac{dq}{dx} \approx -\frac{\Gamma_{\text{deg}} q^{1/3}}{2 \quad x} \approx -3\frac{q^{1/3}}{x}, \tag{26}$$

from which it follows that near the boundary

$$q \approx (-\ln x^2)^{3/2} \approx (1 - x^2)^{3/2}. \tag{27}$$

This expression may be of interest in the calculation of recombination radiation from the InSb pinch channel, since the relation $\kappa R \gg 1$ is satisfied as a rule in transitions with energy $\hbar\omega \geq E_g + \mu_e(x) + \mu_h(x)$ (κ—absorption coefficient).

The condition (6) for the strong degeneracy of the electron gas in the pinch channel assumes the form (for InSb), if the estimate is based on the average density in the channel,

$$I / kT \gg 0.4 a^{-1} \eta^{-2/3} \approx 2.4 \cdot 10^{23}. \tag{28}$$

Thus, in InSb at T = 20°K, strong degeneracy is attained in the pinch channel at currents $I \gg 0.2$ A.

It should be noted that in the state in which the plasma breaks away from the surface of the sample, the spatial distribution of the carriers in the channel assumes a dumbell shape (Fig. 5) (this is due to the homogeneity of the injection over the cross section of the contact) and depends on z near the contacts. The inhomogeneity near the contacts can be neglected if the height h of the dumbell head is smaller than the distance L between the contacts. This condition is satisfied if during the pinching time (τ_p) the particle injected from

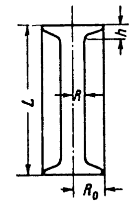

FIG. 5.

the contact drifts in the field over a distance much smaller than L:

$$b_{e,h} E_z \tau_p \ll L. \tag{29}$$

Taking (21) and (22) into account, this condition assumes the form (for electrons)

$$L \gg c^2 R_0^2 / 4 b_h I. \tag{30}$$

In InSb, at currents $I \approx 10$ A, $R_0 = 10^{-2}$ cm, and $b_h \approx 5 \times 10^6$, this condition is satisfied if $L > 10^{-1}$ cm.

4. Under conditions of strong degeneracy of the electron gas in the conduction band, transitions with energy[4]

$$\hbar\omega < E_g + \mu_e(x) + \mu_h(x) \tag{31}$$

occur from inverted-population levels. Quanta having this energy are no longer absorbed, inasmuch as the upper level in the conduction band is occupied, and the lower one, in the valence band, is free. For transitions between the bottom of the conduction band and the top of the valence band ($\hbar\omega \approx E_g$), the inversion condition is

$$\mu_e + \mu_h > 0. \tag{32}$$

This condition can be satisfied if the electron gas is degenerate while the hole gas is still described by Boltzmann's statistics (the case $m_h^* \gg m_e^*$). In such a situation, the condition (32) takes the form

$$\frac{a n^{2/3}}{kT} + 0.4 \ln \frac{4\pi^2 n}{\sqrt{\pi}(2m_h^*/\hbar^2)^{3/2}(kT)^{3/2}} > 0 \tag{33}$$

and for the case of InSb it is satisfied when

$$n^{2/3} / kT > 8.2 \cdot 10^{24}. \tag{34}$$

If the average density in the channel is given by (24), the condition (34) takes the form

$$I / kT > 10^{24} \cdot 0.8. \tag{35}$$

At T = 20°K this condition is satisfied at a pinch current $I \geq 0.8$ A. Thus, at large currents, recombination radiation from the channel of a degenerate pinch is determined by transitions from inverted-population levels (31). The question of recombination radiation of a degenerate pinch will be considered by us in the future.

In conclusion, I am deeply grateful to the participants of the seminar of M. A. Leontovich and A. S. Davydov for a useful discussion and valuable advice, and to B. B. Kadomtsev and A. P. Shotov for a number of important critical remarks.

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