EFFECT OF BOUNDARY CONDITIONS ON THE ELECTRON TEMPERATURE IN A PLASMA HEATED BY A HIGH-FREQUENCY FIELD

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Submitted April 15, 1968

Zh. Eksp. Teor. Fiz. 55, 1096–1104 (September, 1968)

The spatial dependence of the electron temperature in a boundary layer is determined for various boundary conditions at the vacuum-semiconductor interface. The effect of these boundary conditions on the nature of electromagnetic wave propagation in the medium is considered.

IN a paper that has appeared earlier[1] the present authors have formulated a theory for the heating of plasma electrons by an alternating electric field. In this work it was assumed that the means for removal of heat is the crystal lattice in the case of a plasma in a semiconductor, and the molecules and ions in the case of a gaseous plasma. Among other things, if the plasma occupies a finite volume the heat from the electrons can also be lost through the boundary. This situation leads to certain features in the dependence of electron temperature on coordinate and also has an effect on the damping of the electromagnetic field inside the plasma.

In solving the problem we shall make use of the system of equations formulated in[1], in which we limit ourselves to the case of normal incidence of the electromagnetic wave from the vacuum on the half-space occupied by the plasma. In this formulation the problem can be treated as a one-dimensional problem. We take the $z = 0$ plane to be the plane that separates the vacuum and the plasma (the plasma occupies the region $z > 0$). In this case all the quantities that appear in the problem depend only on the coordinate z. The complete system of equations consists of the thermal-conductivity equation and Maxwell's equations.

The thermal-conductivity equation is written in the form[1]

$$T\frac{d}{dz}\varkappa(v)\frac{dv}{dz}+\bar{B}_{ik}(v)E_iE_k^{*}=NT\tilde{\nu}(v)(v-1). \tag{1}$$

Maxwell's equations are written in the usual way:

$$\frac{d^2E_x}{dz^2}+\frac{\omega^2}{c^2}[A(v)E_x-iB(v)E_y]=0, \tag{2}$$

$$\frac{d^2E_y}{dz^2}+\frac{\omega^2}{c^2}[iB(v)E_x+C(v)E_y]=0,$$

$$E_z=\frac{\sin\varphi}{\varepsilon_{11}(v)\sin^2\varphi+\varepsilon_{33}(v)\cos^2\varphi}[\varepsilon_{12}(v)E_x+(\varepsilon_{33}(v)-\varepsilon_{11}(v))\cos\varphi E_y].$$

Here, $\mathbf{E}$ is the amplitude of the electric field (it is assumed that the wave is monochromatic, that is to say, the time dependence is given by $e^{-i\omega t}$); $v = \Theta/T$ is the dimensionless electron temperature (T is the lattice temperature, Θ is the temperature of the electron gas); $\kappa(v)$ is the electronic thermal conductivity; $\tilde{\nu}(v)$ is the frequency of collisions between electrons and heavy particles in which energy transfer occurs; $\epsilon_{ik}(v)$ is the complex conductivity tensor, which depends on v. Expressions for $\kappa(v)$, $\bar{B}_{ik}(v)$, $\tilde{\nu}(v)$, $A(v)$, $B(v)$, $C(v)$ and $\epsilon_{ik}(v)$ are given in[1]. In the derivation of Eq. (1) it is assumed that the electron temperature v is time independent. This is the case when $\omega \gg \tilde{\nu}'$, where ω is the frequency of the electromagnetic wave[2], or for any frequency when the wave is circularly polarized.[3]

The boundary conditions for the thermal-conductivity equation at $z = 0$ are taken to be boundary conditions of the third kind[4]

$$\left.\frac{dv}{dz}\right|_{z=0}=\eta(v)(v-1)|_{z=0}. \tag{3}$$

The quantity η characterizes the transfer of heat from the plasma to the vacuum. When $\eta = 0$ the gradient of the electron temperature vanishes at the boundary as does the heat flux, so that no heat transfer occurs outward from the plasma. When $\eta = \infty$ the quantity v is unity at the boundary and the heat transfer in the outward direction is a maximum. In the formulation of (3) it is assumed that the temperature outside the plasma is the same as the temperature of the ions and the molecules for the case of a gas plasma, or the temperature of the crystal lattice for the case of a semiconductor plasma.

The boundary conditions for Maxwell's equations are written in the usual way:

$$E_{x,y}(-0)=E_{x,y}(+0),\qquad \frac{\partial E_{x,y}(-0)}{\partial z}=\frac{\partial E_{x,y}(+0)}{\partial z}. \tag{4}$$

$$E(\infty)=0.$$

We first consider the case of weak plasma heating, that is to say, we assume

$$v=1+v' \tag{5}$$

where $v' \ll 1$. The problem will be solved by successive approximations.

In the first approximation we write $v = 1$ in (2). Solving this problem we find

$$\mathbf{E}=\mathbf{E}_0\exp\left(i\frac{\omega}{c}nz-\xi_0z\right), \tag{6}$$

where $\mathbf{E}_0$, n and ξ_0 are the amplitude, refractive index and damping as determined from the linear theory. Substituting (6) in (1) and linearizing (1) and the boundary conditions (3) we can write the following equations and boundary conditions for v':

$$\frac{d^2v'}{dz^2} - \delta^2 v' = -De^{-2\xi_0 z}, \tag{7}$$

$$\left.\frac{dv'}{dz}\right|_{z=0} = \eta_0 v'|_{z=0}, \quad v'|_{z=\infty} = 0. \tag{8}$$

Here

$$\delta^2 = \frac{N\tilde{\nu}(1)}{\varkappa(1)}, \quad D = \frac{B(1)|E_0|^2}{T\varkappa(1)}, \quad \eta_0 = \eta(1).$$

For reasons of simplicity we shall assume that the magnetic field is along the z axis. In this case $E_z = 0$, $E_x = \pm iE_y$ and by $|E_0|$ we are to understand either $|E_x|$ or $|E_y|$ at $z = 0$. We note that $\delta \sim \tilde{l}^{-1}$ where $\tilde{l}$ is the mean free path associated with the energy transfer.[1]

Solving Eq. (7) with the boundary conditions in (8) we find the following relation for

$$v' = \frac{D}{4\xi_0^2 - \delta^2}\left[\frac{\eta_0 + 2\xi_0}{\eta_0 + \delta} e^{-\delta z} - e^{-2\xi_0 z}\right]. \tag{9}$$

It follows from (9) that v′ exhibits a maximum when

$$z = z_0 = \frac{1}{\delta - 2\xi_0} \ln \frac{\delta}{2\xi_0} \frac{\eta_0 + 2\xi_0}{\eta_0 + \delta},$$

this maximum value of v′ being given by the expression

$$v'(z_0) = \frac{D}{4\xi_0^2 - \delta^2}\left[\frac{\eta_0 + 2\xi_0}{\eta_0 + \delta}\left(\frac{\delta}{2\xi_0}\frac{\eta_0 + 2\xi_0}{\eta_0 + \delta}\right)^{\delta/(2\xi_0 - \delta)} - \left(\frac{\delta}{2\xi_0}\frac{\eta_0 + 2\xi_0}{\eta_0 + \delta}\right)^{2\xi_0/(2\xi_0 - \delta)}\right]. \tag{10}$$

We note that when $\eta_0 = 0$, then $z_0 = 0$ but $v'(0) = D/(Z\xi_0 + \delta)\delta$. When $\eta_0 = \infty$

$$z_0 = \frac{1}{\delta - 2\xi_0}\ln\frac{\delta}{2\xi_0}, \quad v'(0) = 0,$$

$$v'(z_0) = \frac{D}{4\xi_0^2 - \delta^2}\left[\left(\frac{\delta}{2\xi_0}\right)^{\delta/(2\xi_0 - \delta)} - \left(\frac{\delta}{2\xi_0}\right)^{2\xi_0/(2\xi_0 - \delta)}\right].$$

Using the requirement $v'(z_0) \ll 1$ we can obtain a condition on the amplitude of the incident field. This condition, in the form of an inequality, is written the following way for the two limiting cases:

$$|E_0|^2 \ll \frac{T\varkappa(1)\delta^2}{B(1)}\left(\frac{\delta}{2\xi_0}\frac{\eta_0 + 2\xi_0}{\eta_0 + \delta}\right)^{2\xi_0/\delta}, \quad \delta \gg 2\xi_0,$$

$$|E_0|^2 \ll \frac{4T\varkappa(1)\xi_0^2}{B(1)}\frac{\eta_0 + \delta}{\eta_0 + 2\xi_0}\left(\frac{\delta}{2\xi_0}\frac{\eta_0 + 2\xi_0}{\eta_0 + \delta}\right)^{-\delta/2\xi_0}, \quad \delta \ll 2\xi_0. \tag{11}$$

We can take account of the effect of electron heating on the coefficients of reflection and refraction. For this purpose it is necessary to solve the Maxwell equations (2) to a second approximation in v′ and to match the vacuum field by means of the boundary conditions in (4). The correction to the reflection and refraction coefficients is given by

$$\gamma = \frac{i\omega c^{-1} D d\varepsilon(1)/dv}{(4\xi_0^2 - \delta^2)(1 + n + ic\omega^{-1}\xi_0)} \times \left[\frac{\delta(\eta_0 + 2\xi_0)}{(\eta_0 + \delta)[(i\omega c^{-1}n - \xi_0 - \delta)^2 + \omega^2 c^{-2}\varepsilon(1)]} - \frac{2\xi_0}{(i\omega c^{-1}n - 3\xi_0)^2 + \omega^2 c^{-2}\varepsilon(1)}\right]; \tag{12}$$

this quantity is analogous to the nonlinearity coefficient in the static theory.[5]

The problem on the nonlinear propagation of electromagnetic waves in a semiconductor or plasma contains two parameters with the dimensions of length that characterize the electron gas: the mean free path associated with momentum transfer l and the mean free path associated with energy transfer $\tilde{l}$. If the energy scattering is elastic, and this is the only case considered here, then $l \ll \tilde{l}$.[1]

The normal skin effect is obtained if the depth of penetration of the field L is significantly greater than both of the mean free paths mentioned above. This condition can be written $L \gg l, \tilde{l}$. The anomalous skin effect pertains to propagation of electromagnetic waves in which the inequality $l \ll L \lesssim \tilde{l}$ is satisfied (this point is discussed in greater detail in[1]).

We now investigate the normal skin effect, in which $\delta \gg \xi$; that is to say, in this case the energy mean free path is much smaller than the field penetration. When this condition is satisfied, as in indicated in[1], the equation for thermal conductivity can be written without the derivative terms, in which case the relation between the field and the temperature is a local one and the scale on which the temperature exhibits a significant change coincides with the depth of penetration of the field.

This is the case everywhere with the exception of the boundary layer at the interface z = 0, which is of width δ^{-1}. In a distance δ^{-1} there occurs a significant adjustment of the temperature from its value as determined by the field to the value which is imposed by the boundary conditions. We note at the outset that the behavior of the temperature at this narrow boundary has essentially no effect on the form of the field and the coefficients of reflection and refraction. This is due to the fact that the assumptions that have been made imply that the field within the boundary layer can be regarded as constant and equal to its value at the boundary. Outside the boundary layer the field is given by the formulas derived in[1].

In what follows it will be convenient to introduce a new variable w, which is related to v as follows:[1]

$$w = \int_0^v \varkappa(v)\,dv \Big/ \int_0^1 \varkappa(v)\,dv. \tag{13}$$

Written in terms of w, the thermal-conductivity equation (1) assumes the form

$$\frac{d^2w}{dz^2} - \delta^2 Q(w) = -\delta^2 P_{ik}(w) E_i E_k^* \tag{14}$$

and the boundary condition (3) becomes

$$\left.\frac{dw}{dz}\right|_{z=0} = \beta(w)[v(w) - 1].$$

Here

$$\delta^2 = N\tilde{\nu}_0 \Big/ \int_0^1 \varkappa(v)\,dv, \quad Q(w) = \frac{\tilde{\nu}(w)}{\tilde{\nu}_0}[v(w) - 1], \quad P_{ik}(w) = \frac{\overline{B}_{ik}(w)}{NT\nu_0},$$

$$\beta(w) = \eta(v(w))\varkappa(v(w)) \Big/ \int_0^1 \varkappa(v)\,dv, \quad \tilde{\nu}_0 = \tilde{\nu}(1).$$

We shall first consider the behavior of the temperature in the boundary layer. It will be necessary to consider two limiting cases: $\eta \ll \delta$ and $\eta \gtrsim \delta$. In the first case the heat in the boundary layer is transferred primarily by the lattice and in the second case primarily through the surface of the surrounding medium. As we have indicated above, outside the boundary layer the heat is transferred by the lattice when the normal skin effect obtains. When $\eta \ll \delta$ the solution of Eq. (14) can be written in the form

$$w = w' + w'', \tag{16}$$

where $w' \gg w''$. The function w' is given by Eq. (14), in which we neglect the first term, which describes the heat transfer. The solution of this equation has been investigated in detail in[1]. In order to obtain an equation for w'' we must substitute (16) in (14) and expand all quantities that appear in this equation in w'', limiting ourselves to the linear terms. As will be evident from the solution, the characteristic distance in which there is a significant change in w'' is of order δ^{-1}, so that all quantities that depend on w' and E can be regarded as constant because w' and E vary in distances of the order of $L \gg \delta^{-1}$ and can be replaced by w_0' and E_0; by w_0' and E_0 we mean the value of these quantities at the boundary $z = 0$.

In view of the above considerations the equation for w'' can be written in the form

$$\frac{d^2 w''}{dz^2} - \tilde{\delta}^2(w_0')\, w'' = 0, \tag{17}$$

where $\delta^2(w_0')$ denotes

$$\tilde{\delta}^2(w_0') = \delta^2 Q(w_0') \frac{d}{dw_0'} \ln \frac{Q(w_0')}{P(w_0')}, \quad P(w_0') = P_{ik}(w_0') \frac{E_{i0}E_{k0}^*}{|E_0|^2}$$

In the derivation of the last equation we have used the equation for w_0'. The solution of Eq. (17) that remains finite at infinity is of the form

$$w'' = w_0'' e^{-\tilde{\delta}(w_0')z}, \tag{18}$$

and w_0'' is determined from the boundary condition (15)

$$w_0'' = \frac{dw_0'/dz - \beta(w_0')[v(w_0') - 1]}{\tilde{\delta}(w_0')}. \tag{19}$$

It will be evident that the dimensionless temperature can also be written in a form similar to (16):

$$v = v' + v'', \tag{20}$$

where

$$w' = \int_0^{v'} \varkappa(v)\,dv \Big/ \int_0^1 \varkappa(v)\,dv, \quad w'' = \varkappa(v')\,v'' \Big/ \int_0^1 \varkappa(v)\,dv. \tag{21}$$

Finally we have

$$v(z) = v'(z) + v_0'' e^{-\tilde{\delta}(v_0')z}, \tag{22}$$

where

$$v_0'' = \frac{dv_0'/dz - \beta(v_0')[v_0' - 1]}{\tilde{\delta}(v_0')},$$

$$\beta(v_0') \equiv \beta(w'(v_0')), \quad \tilde{\delta}(v_0') \equiv \tilde{\delta}(w'(v_0')).$$

In[1] we have given expressions for $w'(z)$ and $v'(z)$ for various particular cases. Using these expressions it is possible to write v_0'' and $\tilde{\delta}$ as functions of the incident electric field and a fixed magnetic field.

It will be evident from (22) that the initial assumption that w'' is small compared with w' (v'' compared with v') is satisfied. Physically this means that at small values of η the electrons primarily transfer energy to the lattice as a consequence of which the existence of boundaries has a weak effect on the temperature, leading only to a small correction $(\beta + \xi)/\delta$. However, this small correction must have a derivative of the main term in order to provide matching of the boundary conditions. Using the expression for $v'(z)$ given in[1] we find that when $z = z_0$ where

$$z_0 = \frac{1}{\tilde{\delta}(v_0')} \ln \left[\left(- \frac{\tilde{\delta}(v_0')\, v_0''}{2\xi(v_0')} \frac{d}{dv'} \ln D(v_0') \right) \right], \tag{23}$$

the quantity $v(z)$ exhibits a maximum, the maximum value being given by v_0'; $D(v') = Q(v')/P(v')$.

As is evident from Eq. (9) and elementary physical considerations, when $\eta \gtrsim \delta$, in a boundary layer of width δ^{-1} the quantity $\mathbf{E}$ in Eq. (14) can be replaced by $\mathbf{E}_0$, which case this equation can be solved by quadratures. The answer is of the form

$$\sqrt{2}\,\delta z = \int_{w_0}^{w} dw \left[\int_{w}^{w_\infty} F(w)\,dw \right]^{-1/2}, \tag{24}$$

where

$$F(w) = P(w)\,|E_0|^2 - Q(w). \tag{25}$$

Converting from w to v we rewrite Eq. (25) in the form

$$\sqrt{2}\,\delta z = \int_{v_0}^{v} dv\,\varkappa(v) \left[\int_{v}^{v_\infty} dv\,\varkappa(v)\,\Phi(v) \right]^{-1/2} \Big/ \left[\int_0^1 dv\,\varkappa(v) \right]^{1/2}. \tag{26}$$

Here $\Phi(v) = F[w(v)]$.

In order to obtain (24) and (26) we have chosen a solution of (14) that corresponds to $dw/dz > 0$ ($dv/dz > 0$). This choice is dictated by the fact that $dw/dz > 0$ at $z = 0$ by virtue of the boundary condition in (15). The solution in (26) contains two arbitrary constants v_0 and v_∞ which implies that two equations are necessary. However, one of these can be obtained by substituting the solution (26) in the boundary condition (15) and is of the form

$$\frac{\sqrt{2}\,\delta}{\varkappa(v_0)} \left[\int_0^1 \varkappa(v)\,dv \right]^{1/2} \left[\int_{v_0}^{v_\infty} \Phi(v)\,\varkappa(v)\,dv \right]^{1/2} = \beta(v_0)\,(v_0 - 1). \tag{27}$$

A second equation for determining the constant of integration can be obtained from the requirement that the solution (26), which applies in the boundary layer, be smoothly matched to the solution that applies in the remainder of the volume. If we denote by $v'(z)$ the temperature outside the boundary layer (this temperature has been investigated in detail in[1]) then when $z > \delta^{-1}$ the quantity $v_\infty = v'(0)$. Actually, the expression for the temperature obtained in[1] holds when $z \gg \delta^{-1}$ while the solution in (26) which has been obtained here holds when $z \ll \xi^{-1}$; consequently there exists an interval of z ($\delta^{-1} \ll z \ll \xi^{-1}$) where these two solutions must coincide and in this interval $v'(z) \approx v'(0)$. In[1] this quantity has been denoted by v_0.)

In order that the value of v be close to v_∞ when $z \gg \delta^{-1}$ this quantity must satisfy the equation

$$\Phi(v_\infty) = 0. \tag{28}$$

Equation (28) obviously coincides with the equation for $v'(0)$ that has been obtained in[1]; the expression for v_∞ as a function of the amplitude of the variable electric field and the fixed magnetic field is also given there.

It is an easy matter to obtain an asymptotic expression for $v(z)$ when $z \gg \delta^{-1}$. Using the method developed in[1] we have

$$v(z)=v_\infty-S_v\exp\left\{-\delta\left[\frac{d^2\psi(v_\infty)}{dv^2}\int_0^1\varkappa(v)\,dv\right]^{1/2}z\right\}. \tag{29}$$

Here

$$\psi(v)=\varkappa^{-2}(v)\int_v^{v_\infty}\Phi(v)\varkappa(v)\,dv,$$

$$S_v=(v_\infty-v_0)\exp\left\{\left(\frac{1}{2}\frac{d^2\psi(v_\infty)}{dv^2}\right)^{1/2}\int_{v_0}^{v_\infty}\left[\psi^{-1/2}(v)+\left(\frac{1}{2}\frac{d^2\psi(v_\infty)}{dv^2}\right)^{-1/2}\frac{1}{v-v_\infty}\right]dv\right\}.$$

Using the solution in (26) we can obtain an interpolation formula for the temperature that applies over the whole range of z. In this case we must replace v_∞ by $v'(z)$ in Eq. (26). When $z \ll \xi^{-1}$ we have $v'(z) \approx v_\infty$ and we recover (26). When $z \gg \delta^{-1}$ as is evident from the asymptotic behavior we find $v(z) \approx v'(z)$. Clearly the maximum value of $v(z)$ is v_∞.

We now wish to investigate the anomalous skin effect $\xi \gg \delta$. The anomalous skin effect has been studied for the case $\eta = 0$ in[1]. The method of analysis and all the results obtained there can be extended without change to finite values of η if the inequality $\eta \ll \xi$ is satisfied. The only difference lies in the equation for determining the boundary value of the temperature v_0. For finite values of η this equation becomes

$$\frac{\delta^2 P(v_0)\,|E_0|^2}{2\xi(v_0)}=\sqrt{2}\,\delta\left[\int_1^{v_0}\tilde{v}(v)\varkappa(v)\,dv\Big/\tilde{v}_0\int_0^1\varkappa(v)\,dv\right]^{1/2}+\beta(v_0)(v_0-1). \tag{30}$$

When $\eta \ll \delta$ we recover the equation for v_0 obtained in[1]. For other values of η we must have the actual expression that shows the dependence of η on v_0. When $\xi \gg \eta \gg \delta$ we can neglect the first term on the right in (30). When $\eta \ll \xi$, as follows from[1], the expression for $v(z)$ exhibits a maximum at the point

$$z=\frac{1}{2\xi(v_0')}\ln\left[-\frac{\sqrt{2}\,\tilde{v}_0^{1/2}\delta(v_0')\varkappa(v_0')v_0'}{\delta\left[\int_0^{v_0'}\varkappa(v)\nu(v)(v-1)\,dv\right]^{1/2}\left[\int_0^1\varkappa(v)\,dv\right]^{1/2}}\right], \tag{31}$$

where the maximum value of $v(z)$ is v_0'.

Now let us assume that $\eta \gtrsim \xi$. We limit our analysis to the case of weak damping of the electromagnetic waves and assume that the propagation occurs along the fixed magnetic field. As follows from[1], in this case the Maxwell equations and the energy balance equation can be written

$$d^2w/dz^2+\delta^2P(w)u^2=0,\qquad du/dz+\xi(w)u=0. \tag{32}$$

Here, u is the modulus of the normal wave E ($E = ue^{i\omega nz/c}$, n is the refractive index). This way of writing the first equation in (32) holds only when $z \ll \delta^{-1}$. Actually, this equation can be obtained from (14) by neglecting the term $\delta^2 Q(w)$ which describes the transfer of heat to the lattice.

For the assumptions that have been made here in the layer $z \ll \delta^{-1}$ the basic heat transfer occurs by virtue of the transfer of heat through the surface because the condition $\eta \gtrsim \xi$ indicates that the value of η is much larger than δ.

Outside the layer $z \lesssim \xi$ we can neglect the right side of Eq. (14) (this is discussed in more detail in[1]) in which case this equation can be solved by quadratures. The quantity $P(w)$ can always be written in the form $P(w) = P_0\xi(w)$. This result follows immediately from (14) and is intuitively obvious because the second term describes the Joule heating, which is proportional to the damping of the energy of the electromagnetic field. It is evident that the system in (32) has the first integral

$$\frac{dw}{dz}-\frac{1}{2}\delta^2P_0u^2=\text{const.} \tag{33}$$

The constant on the right side of Eq. (33) is equal to zero because if this is not the case when $z \gg \xi^{-1}$ the quantity w increases or decreases with z, since u^2 is negligibly small when $z \gg \xi^{-1}$. This is not possible physically because of the absence of sources and sinks of energy in the region $\xi^{-1} \ll z \ll \delta^{-1}$; the field in this region does not generate energy and the lattice does not absorb energy. Substituting u^2 from (33) in the first equation in (32) and carrying out the integration we find

$$-2z=\int_{w_0}^{w}dw\left[\int_{w_\infty}^{w}dw\,\xi(w)\right]^{-1},\qquad u^2=\frac{4}{\delta^2P_0}\int_w^{w_\infty}\xi(w)\,dw. \tag{34}$$

The constants w_0 and w_∞ are determined from the boundary conditions (4) and (15). Converting from w to v we can write (34) in the form

$$-2z=\int_{v_0}^{v}dv\,\varkappa(v)\left[\int_{v_\infty}^{v}\xi(v)\varkappa(v)\,dv\right]^{-1},$$

$$u^2=4\left[\delta^2P_0\int_0^1\varkappa(v)\,dv\right]^{-1}\int_v^{v_\infty}dv\,\varkappa(v)\xi(v). \tag{35}$$

It is evident from (35) that $v(z) = v_0$ when $z = 0$ and $v(z) \approx v_\infty$ when $z \gg \xi^{-1}$.

As we have indicated above, the equation that describes the temperature outside the boundary layer can be solved by quadratures and can be written as an implicit function in the following form:

$$-\sqrt{2}\,\delta z=\tilde{v}_0^{1/2}\int_{v_\infty}^{v}dv\,\varkappa(v)\left[\int_1^{v}\varkappa(v)\tilde{v}(v)(v-1)\,dv\right]^{-1/2}\Big/\left[\int_0^1\varkappa(v)\,dv\right]^{1/2}. \tag{36}$$

The constants of integration are chosen in such a way that when $z \ll \delta^{-2}$ we have $v \approx v_\infty$ and when $z \gg \delta^{-1}$, we have $v \approx 1$. With this choice of the constants in the region of small z (36) matches to (35). Here we have the same situation as applies in the case of the normal skin effect for large values of η.

We now obtain asymptotic expressions for the temperature and field. Without exhibiting the calculations here (these are conventional[1]) we simply write down the result

$$v=v_\infty-S_ve^{-2\xi(v_\infty)z},$$

$$u^2=4\xi(v_\infty)\varkappa(v_\infty)S_ve^{-2\xi(v_\infty)z}\Big/\delta^2P_0\int_0^1\varkappa(v)\,dv. \tag{37}$$

Here,

$$S_v=(v_\infty-v_0)\exp\left\{-\int_{v_0}^{v_\infty}\left[\varkappa(v)\Big/\int_{v_\infty}^{v}dv\,\xi(v)\varkappa(v)-\frac{\xi^{-1}(v_\infty)}{v-v_\infty}\right]dv\right\}.$$

As in the case of the normal skin effect for large values of η, here we can also write an interpolation formula for $v(z)$ which holds over the entire space. This can be done by replacing v_∞ in the first equation in (35) with $v(z)$ as determined from (36). As in the case above, the maximum value of $v(z)$, which describes the electron temperature over the entire half space, coincides with v_∞. In order to find v_0 and v_∞ from the boundary conditions we must specify the explicit dependence of κ and ξ on v and the dependence of η on v_0. We shall take $\eta \equiv \infty$. Then, as is evident from the boundary condition (15) on v_0, we have $v_0 = 1$.

We write $\xi(v) = \xi_0 v^{-q}$ and $\kappa(v) = \kappa_0 v^{1+q}$. This corresponds to the case of relatively high frequencies. Now, using the boundary conditions for the field (4) we can obtain a formula for v_∞:

$$v_\infty = \left[1 + \frac{2\delta^2 P_0 \tilde{u}^2}{(2+q)(1+n)^2}\right]^{1+q/2}. \tag{38}$$

Here, $\tilde{u}$ is the amplitude of the field incident from the vacuum side. An investigation of these results shows [(cf. (23) and (31)] that the qualitative behavior of the temperature as a function of z is the same as for weak heating.

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Translated by H. Lashinsky
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