

DIELECTRIC CONSTANT OF ALKALI-METAL VAPOR OR HYDROGEN AT

RESONANCE FREQUENCIES

Yu. A. VDOVIN and N. A. DOBRODEEV

Moscow Engineering Physics Institute

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The dielectric constant of alkali-metal vapor or of hydrogen is found for frequencies corresponding to transitions between the ground state and doublet levels under conditions when pair collisions with resonant energy transfer make the largest contributions. The effect of Doppler broadening is taken into account. The variation of the dielectric constant on going from heavy alkali metals to lithium and hydrogen is considered. The line wings are investigated. Allowance is made for the effect of population of the excited levels. The case when transitions occur between one of the doublet levels and an excited level is also considered.

1. IN this paper we determine the dielectric constant of alkali-metal vapors and of hydrogen at frequencies corresponding to the resonant transition from the ground-state level  $S_{1/2}$  to the doublet levels  $P_{1/2}$  and  $P_{3/2}$ . We consider also transitions between doublet levels and some excited level. The study is made in the pressure region in which the dielectric constant is determined by pair collisions of the excited and unexcited atoms with resonant energy transfer. Since the excited states  $P_j$  ( $j = 1/2, 3/2$ ) are connected with the ground state by optically allowed dipole transitions, a resonant dipole-dipole interaction is realized between the excited and unexcited atoms and leads to a large effective collision cross section. These collisions play an important role provided  $n\lambda^3 \gg 1$ , where  $n$  is the density of the atoms in the ground state and  $\lambda$  is the wavelength corresponding to the considered resonant transition,  $\lambda = c/c\omega_j$ . The influence of collisions of this type on the shape of the spectral line was first considered by Vlasov and Fursov.<sup>[1]</sup>

As is well known,<sup>[1, 2]</sup> an important role is played in collisions with resonant excitation transfer by the impact parameters  $\rho \sim \rho_0$ , where  $\rho_0$  is the Weisskopf radius

$$\rho_0 = g_j / \gamma v. \tag{1}$$

By  $v$  we denote the relative velocity of the colliding particles, and  $g_j^2 = d_j^2 / (2j + 1)$ , where  $d_j$  is the reduced matrix element of the dipole transition between the states  $S_{1/2}$  and  $P_j$  ( $j = 1/2$  or  $3/2$ ). Here and throughout we use the atomic system of units:  $\hbar = m = e = 1$ . The condition for the pairing of the collisions, which is assumed satisfied, is given by

$$n\rho_0^3 \ll 1. \tag{2}$$

To determine the dielectric constant  $\epsilon$  we shall make use of a procedure developed for transitions of the  $0 \rightleftharpoons 1$  type in a paper by one of the authors and Galitskiĭ.<sup>[3]</sup> A somewhat different calculation method was proposed by Kazantsev.<sup>[4]</sup> The method of temperature Green's functions<sup>[5]</sup> is used for the calculations. The dielectric constant  $\epsilon(\omega)$  is expressed in terms of

the polarization operator  $\Pi_{00}$  or, which is the same, in terms of the two-particle Green's function  $K_{mm_0}^{\mu\mu_0}$ :

$$\epsilon(\omega) = 1 - \frac{4\pi g_j^2}{k^2} \sum_{\substack{\mu\mu_0 \\ mm_0 \\ v v_0}} (1j_0 v \mu | jm) (1j_0 v_0 \mu_0 | j m_0) (-1)^{v k - v k_0} \int d\mathbf{p} K_{mm_0}^{\mu\mu_0}(\mathbf{p}, \mathbf{k}, \omega), \tag{3}$$

where the terms of the type  $(1j_0 v \mu | jm)$  denote the Clebsch-Gordan coefficients,  $k = 1/\lambda = \omega/c$ , and  $j_0, \mu, \mu_0$ , and  $j, m, m_0$  denote the total angular momentum of the atom and its projections in the ground and in the excited states. The equation for the function  $K_{mm_0}^{\mu\mu_0}$  is of the form<sup>[3]</sup>

$$K_{mm_0}^{\mu\mu_0}(\mathbf{p}, \mathbf{k}, \omega) = \sum_{m'\mu'} K_{mm'}^{0\mu\mu'}(\mathbf{p}, \mathbf{k}, \omega) \left\{ \delta_{\mu'\mu_0} \delta_{m'm_0} - \frac{1}{M} \sum_{m''\mu''} \int d\mathbf{p}' \text{Re} \tilde{f}_{m''m''}^{\mu''\mu''} \left( \mathbf{q} - \frac{\mathbf{k}}{2}, \mathbf{q} + \frac{\mathbf{k}}{2} \right) K_{m''m_0}^{\mu''\mu_0}(\mathbf{p}', \mathbf{k}, \omega) \right\}, \tag{4}$$

where

$$K_{mm'}^{0\mu\mu'}(|\mathbf{p}|, \mathbf{k}, \omega) = -\frac{i}{2j_0 + 1} \frac{n\varphi(|\mathbf{p}|) \delta_{\mu\mu'} \delta_{mm'}}{\omega - \omega_j - \Sigma_m(|\mathbf{p}|) - p k_z / M}. \tag{5}$$

We have written formula (5) in a coordinate system in which the  $z$  axis is directed along the vector  $\mathbf{p}$ . An expression for the functions  $K^0$  in an arbitrary system can be obtained with the aid of the usual rotation matrices. The expression for  $K^0$  was obtained under the assumption that the population of the excited levels can be neglected. A generalization of the results to the case when the densities of the excited and unexcited atoms are of the same order will be presented below. The self-energy part  $\Sigma_m$  is given by

$$\Sigma_m(|\mathbf{p}|) \delta_{mm'} = \frac{i}{2j_0 + 1} \frac{n}{M} \int d\mathbf{p}' \varphi(\mathbf{p}') \sum_{\mu} \text{Im} f_{mm'}^{\mu\mu}(\mathbf{q}, \mathbf{q}), \tag{6}$$

$$\int d\mathbf{p} \varphi(\mathbf{p}) = 1,$$

where  $\varphi(\mathbf{p})$  is the Maxwell distribution function,  $M$  is the mass of the atom, and  $\mathbf{q} = 1/2(\mathbf{p} - \mathbf{p}')$ .

By  $f_{mm'}^{\mu\mu'}(\mathbf{q}, \kappa)$  we denote the scattering amplitude (this definition differs from the usual one by a factor  $-1/4\pi$ ):

$$f_{mm_0}^{\mu\mu_0}(\mathbf{q}, \boldsymbol{\kappa}) = M \sum_{m'\mu'} \int d\mathbf{R} e^{-i\mathbf{q}\mathbf{R}} V_{mm'}^{\mu\mu'}(\mathbf{R}) \Psi_{m'\mu_0}^{\mu\mu_0}(\boldsymbol{\kappa}, \mathbf{R}), \quad (7)$$

where  $V_{mm'}^{\mu\mu'}(\mathbf{R})$  is the matrix element of the dipole-dipole interaction operator, taken over the atomic wave functions. The wave function  $\Psi_{m'\mu_0}^{\mu\mu_0}(\boldsymbol{\kappa}, \mathbf{R})$  is a solution of the Schrödinger equation for relative motion with potential  $V_{mm'}^{\mu\mu'}(\mathbf{R})$ , corresponding to a momentum  $\boldsymbol{\kappa}$  and polarizations  $m_0$  and  $\mu_0$  prior to scattering. The amplitude  $\tilde{f}_{m'\mu}^{\mu\mu'}$  differs from the amplitude  $f_{mm'}^{\mu\mu'}$  in that the self-consistent field is subtracted:

$$\tilde{f}_{m'\mu}^{\mu\mu'}\left(\mathbf{q} - \frac{\mathbf{k}}{2}, \mathbf{q} + \frac{\mathbf{k}}{2}\right) = f_{m'\mu}^{\mu\mu'}\left(\mathbf{q} - \frac{\mathbf{k}}{2}, \mathbf{q} + \frac{\mathbf{k}}{2}\right) - \int d\mathbf{R} V_{m'\mu}^{\mu\mu'}(\mathbf{R}) e^{i\mathbf{k}\mathbf{R}}. \quad (8)$$

This subtraction corresponds to allowance for only the irreducible diagrams in the calculation of the polarization operator.

Replacement of the effective four-poles by the real and imaginary parts of the scattering amplitude in the integral equation (4) is valid, as shown in [3], in the frequency region  $|\omega - \omega_j| \leq g_j^2/\rho_0^3$ , where  $g_j^2/\rho_0^3$  has the meaning of the reciprocal effective collision time. The wings of the line, i.e., the region  $|\omega - \omega_j| \gg g_j^2/\rho_0^3$ , will be considered later.

2. We assume first that the Doppler line width can be neglected compared with the impact width. Then we can omit the term  $\mathbf{p}\mathbf{k}_Z/M$  from the denominator of formula (5). In this case an approximate solution of (4), consisting of replacing the scattering amplitude by its value averaged over the direction of the vector  $\mathbf{q}$ , leads to the following expression for  $\varepsilon(\omega)$ :

$$\begin{aligned} \varepsilon_j(\omega) = 1 - \frac{2j+1}{2j_0+1} \frac{4\pi}{3} n g_j^2 \left\{ \omega - \omega_j + \frac{n}{M(2j_0+1)(2j+1)} \right. \\ \times \left[ \frac{(2j+1)^2}{9} 4\pi g_j^2 M - \sum_{\substack{m'\mu\nu \\ m'\mu'}} (1j_0\nu\mu' | jm) (1j_0\nu\mu | jm') \operatorname{Re} f_{m'\mu}^{\mu\mu'}(\boldsymbol{\kappa}, \boldsymbol{\kappa}) \right. \\ \left. \left. - i \sum_{\mu m} \operatorname{Im} f_{m\mu}^{\mu\mu}(\boldsymbol{\kappa}, \boldsymbol{\kappa}) \right] \right\}^{-1}. \quad (9) \end{aligned}$$

As shown in [3, 4], this approximation is accurate to 1-2%. Thus, the dielectric constant  $\varepsilon$  is determined by the zero-angle scattering amplitude. However, the real part of the scattering amplitude is not an analytic function of the scattering angle. [3] Therefore, on going

over to the limiting value of  $\operatorname{Re} f_{mm'}^{\mu\mu'}(\boldsymbol{\kappa}, \boldsymbol{\kappa})$  from the value  $\operatorname{Re} f_{mm'}^{\mu\mu'}(\boldsymbol{\kappa} + \mathbf{k}, \boldsymbol{\kappa})$  (the phonon momentum  $\mathbf{k}$  is much smaller than the thermal momentum  $\boldsymbol{\kappa}$  of the atom) an additional term  $4\pi g_j^2 n / (2j_0+1)(2j+1)$  appears in the denominator of (9). This term takes into account exactly the frequency shift obtained in accordance with the classical Lorentz-Lorenz formula (see in this connection also [6, 7], where the classical field is averaged in a system of stationary dipole centers).

To find the amplitudes, we use the method proposed by Vainshtein and Galitskii. [8] The wave function

$\Psi_{mm_0}^{\mu\mu_0}(\boldsymbol{\kappa}, \mathbf{R})$  (see (7)) is sought in the form

$$\Psi_{mm_0}^{\mu\mu_0}(\boldsymbol{\kappa}, \mathbf{R}) = e^{i\boldsymbol{\kappa}\mathbf{R}} S_{mm_0}^{\mu\mu_0}(\boldsymbol{\kappa}, \mathbf{R}). \quad (10)$$

Substituting (10) in the Schrödinger equation and neglecting the second derivatives of the function  $S$  (see, for example, [9]), we get

$$2i \frac{\boldsymbol{\kappa}}{M} \nabla S_{mm_0}^{\mu\mu_0} = \sum_{m'\mu'} V_{m'\mu'}^{\mu\mu_0} S_{m'\mu_0}^{\mu\mu_0}. \quad (11)$$

As noted in [3], the developed approximation is valid if  $1 \gg v \gg 1/M^2$ , which in practice is always satisfied.

We note that for all the alkali metals, except the lightest Li, the condition  $\Delta E \gg g^2/\rho_0^3$  is usually satisfied, where  $\Delta E = E_{3/2} - E_{1/2}$  is the difference of the energy levels of the doublet. Under these conditions, when finding the dielectric constant, it is possible to neglect collisions with a transition between the doublet components, i.e., to consider each level of the fine structure independently. This is precisely the case to which Eqs. (11) and (20) pertain. The imaginary part of the scattering amplitude  $\operatorname{Im} f_{mm_0}(\mathbf{q}, \boldsymbol{\kappa})$  is the amplitude of the non-exchange scattering, i.e., scattering without excitation transfer, while the real part is the amplitude of the exchange scattering, i.e., scattering with excitation transfer. [3] It is easy to show, using the general unitarity properties of the scattering, that the following relation is satisfied

$$-\frac{1}{Mv} \operatorname{Im} f_{mm}^{\mu\mu} = \frac{1}{2} (\sigma_m^{\mu} + \sigma_m'^{\mu}), \quad (12)$$

where  $\sigma_m^{\mu}$  and  $\sigma_m'^{\mu}$  denote the total collision cross sections without and with excitation transfer. The dices  $m$  and  $\mu$  in the cross sections denote the initial states of the colliding atoms, and summation is carried out over the final states. The cross sections for exchange and non-exchange scattering can be expressed directly in terms of the integrals of the function  $F$ , taken at  $z \rightarrow \infty$  ( $R^2 = \rho^2 + z^2$ ,  $\rho$ —impact distance):

$$\begin{aligned} \sigma_m^{\mu} &= \frac{\pi g^2}{v} \sum_{\mu'm'0} \int_0^{\infty} du (\operatorname{Im} S_{m'm'}^{\mu\mu})^2, \\ \sigma_m'^{\mu} &= \frac{\pi g^2}{v} \sum_{\mu'm'0} \int_0^{\infty} du (\operatorname{Re} S_{m'm'}^{\mu\mu} - \delta_{m'm'} \delta_{\mu\mu'})^2, \end{aligned} \quad (13)$$

where  $u = v\rho^2/g^2$ .

Averaging relation (12) over the initial state, we get

$$-(Mv)^{-1} (2j_0+1)^{-1} (2j+1)^{-1} \sum_{\mu m} \operatorname{Im} f_{mm}^{\mu\mu}(\boldsymbol{\kappa}, \boldsymbol{\kappa}) = \frac{1}{2} (\sigma + \sigma'), \quad (14)$$

where  $\sigma$  and  $\sigma'$  are the average values of the corresponding cross section.

The system (11) was solved with a computer. The following results were obtained:

a) For the transitions  $S_{1/2} \rightleftharpoons P_{1/2}$

$$\sigma' = 1.20 \frac{\pi g^2}{v}, \quad \sigma = 1.13 \frac{\pi g^2}{v}, \quad (15)$$

$$\Delta_{1/2} = -0.148 \pi g^2 M, \quad (16)$$

where

$$\Delta_j = \sum_{m\mu\nu} (1j_0\nu\mu' | jm) (1j_0\nu\mu | jm') \operatorname{Re} f_{m'\mu}^{\mu\mu'}(\boldsymbol{\kappa}, \boldsymbol{\kappa}). \quad (17)$$

We note that the system of equations (11) for the transitions  $S_{1/2} \rightleftharpoons P_{1/2}$  reduces in fact to the system of equations considered in [3] for the transitions  $S_0 \rightleftharpoons P_1$ . The excitation-transfer cross section for such transitions was determined in [8, 10-12].

b) For the transitions  $S_{1/2} \rightleftharpoons P_{3/2}$

$$\sigma' = 2.06 \frac{\pi g_{1/2}^2}{\nu}, \quad \sigma = 1.66 \frac{\pi g_{1/2}^2}{\nu}, \quad (18)$$

$$\Delta\nu_{1/2} = 0.768\pi g_{1/2}^2 M. \quad (19)$$

Using the results (3) and (11)–(15) we obtain expressions for the dielectric constant at the resonant frequencies of the transitions  $1/2 \rightleftharpoons 1/2$  ( $\epsilon_{1/2}$ ) and  $1/2 \rightleftharpoons 3/2$  ( $\epsilon_{3/2}$ ):

$$\epsilon_{1/2}(\omega) = 1 - \frac{4}{3}\pi n g_{1/2}^2 [\omega - \omega_{1/2} + (0.48 + 1.16i)\pi n g_{1/2}^2]^{-1}, \quad (20)$$

$$\epsilon_{3/2}(\omega) = 1 - \frac{8}{3}\pi n g_{3/2}^2 [\omega - \omega_{3/2} + (0.79 + 1.86i)\pi n g_{3/2}^2]^{-1}.$$

The matrix elements  $g_j^2$  are connected with the oscillator strengths  $f_j$  of the transition from the ground state to the given excited state by the relation

$$g_j^2 = \frac{3}{2j+1} \frac{f_j}{\omega_j}. \quad (21)$$

The numerical values of  $g_{1/2}$  and  $g_{3/2}$  are very close to each other.

We now consider the case when  $ng^2 \ll \Delta E \ll g^2/\rho_0^3$ . This means that the components of the doublet are optically resolved, but the distance between the levels is small compared with the reciprocal effective collision time. This condition, at definite densities ( $n \lesssim 10^{18} \text{ cm}^{-3}$ ), is valid for lithium and hydrogen. In this case the multiple structure of the terms plays no role in the analysis of the collision problem. The expression for  $\epsilon$  takes the form

$$\epsilon_j(\omega) = 1 - \frac{2j+1}{2j_0+1} \frac{4\pi}{3} n g_j^2 \left\{ \omega - \omega_j - \frac{n}{M} \left[ \frac{2j+1}{2j_0+1} \frac{1}{9} (-4\pi g_j^2 M + \sum_m \text{Re} f_{mm}(\boldsymbol{\kappa}, \boldsymbol{\kappa})) + \frac{i}{3} \sum_m \text{Im} f_{mm}(\boldsymbol{\kappa}, \boldsymbol{\kappa}) \right] \right\}^{-1}. \quad (22)$$

The index  $m$  (projection of orbital angular momentum of the excited state) runs here through the values  $\pm 1$  and  $0$ , while  $f_{mm}$  denotes the amplitudes that reduce to those introduced in [3] when the vector index  $\alpha$  is replaced by  $m$ . Substituting the corresponding numerical values, we get

$$\epsilon_{1/2}(\omega) = 1 - \frac{4\pi}{3} n g_{1/2}^2 [\omega - \omega_{1/2} + (0.37 + 2.33i)\pi n g_{1/2}^2]^{-1}, \quad (23)$$

$$\epsilon_{3/2}(\omega) = 1 - \frac{8\pi}{3} n g_{3/2}^2 [\omega - \omega_{3/2} + (0.74 + 2.33i)\pi n g_{3/2}^2]^{-1}.$$

We note that in this case the widths of the doublet components coincide, and the shifts differ by a factor of 2.

3. We now consider the more general case, when the Doppler line width cannot be neglected. Replacement of the true amplitudes by their average is possible in this case, too, but the accuracy of the calculation decreases. The most unfavorable is the case when the Doppler width is of the same order as the impact width. As shown by estimates, the error in this case is of the order of 10%. On the other hand, if the Doppler width is small or large compared with the impact width, then the accuracy of such an approximation increases, reaching, as already noted, 1–2%. The expression for  $\epsilon$ , as follows from (3)–(5) and (17), is

$$\epsilon_j(\omega) = 1 - \frac{2j+1}{2j_0+1} \frac{4\pi}{3} n g_j^2 I$$

$$\times \left\{ 1 + \frac{n}{M(2j+1)(2j_0+1)} \left[ \frac{(2j+1)^2}{9} 4\pi g_j^2 M - \Delta_j \right] I \right\}^{-1}, \quad (24)$$

where

$$I = \int d\mathbf{p} \varphi(\mathbf{p}) \left[ \omega - \omega_j - \frac{\mathbf{p}\mathbf{k}}{M} - i \frac{n}{M(2j+1)(2j_0+1)} \sum_{mm'} \text{Im} f_{mm}^{\mu\mu}(\boldsymbol{\kappa}, \boldsymbol{\kappa}) \right]^{-1}. \quad (25)$$

Using the results (15)–(19) and (23), and expressing the integral (25) in terms of the tabulated functions [13]  $Z(\xi, \eta)$ :

$$Z(\xi, \eta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-x^2} dx}{x - \xi - i\eta} = 2i \int_{-\infty}^{\xi} dt e^{-(t^2 + \zeta^2)}, \quad \zeta = \xi + i\eta, \quad (26)$$

we get

$$\epsilon_j(\omega) = 1 + \frac{2\pi}{3} (2j+1) \frac{n g_j^2}{\delta} \times Z(\xi_j, \eta_j) \left\{ 1 - \frac{\pi n g_j^2 \beta_j Z(\xi_j, \eta_j)}{\delta} \right\}^{-1}, \quad (27)$$

where

$$\xi_j = \frac{\omega - \omega_j}{\delta}, \quad \eta_j = \frac{\pi n g_j^2 \alpha_j}{\delta}, \quad \delta = k \sqrt{\frac{2T}{M}};$$

$\delta$  is the Doppler width,  $T$  the gas temperature, and  $\alpha_j$  and  $\beta_j$  are numerical constants with the following values:

a) for Na, K, Rb, and Cs

$$\alpha_{1/2} = 1.16, \quad \beta_{1/2} = 0.48, \\ \alpha_{3/2} = 1.86, \quad \beta_{3/2} = 0.79;$$

b) for Li and H

$$\alpha_{1/2} = 2.33, \quad \beta_{1/2} = 0.37, \\ \alpha_{3/2} = 2.33, \quad \beta_{3/2} = 0.74.$$

We note that the absorption line contour, which is determined by the imaginary part of the dielectric constant (27), differs from the customarily employed Voigt profile (see in this connection the review article by Rautian and Sobel'man [14]).

The obtained expression for the dielectric constant can be readily generalized also to the case when the population of the excited levels cannot be neglected. The collision widths of the line is then determined not only by the width of the excited level, but also by the width of the ground state. A contribution to the broadening of the ground state is made by collisions with excited atoms, which are at any one of two states ( $P_{1/2}$  or  $P_{3/2}$ ). The expression for  $\epsilon$  has the form (27), as before, provided that

$$\eta_{1/2} = \frac{\pi}{\delta} [1.16(n + n_{1/2})g_{1/2}^2 + 1.86n_{3/2}g_{3/2}^2], \quad (28)$$

$$\eta_{3/2} = \frac{\pi}{\delta} [1.86(n + n_{3/2})g_{3/2}^2 + 1.16n_{1/2}g_{1/2}^2],$$

and the density  $n$ , which enters not in the parameter  $\eta$ , is replaced by  $n - 2n_j/(2j+1)$ . By  $n_j$  we denote the density of the atoms in the state  $P_j$ . Our generalization of formula (27) is not valid for lithium or hydrogen.

4. We have considered so far  $\epsilon$  at frequencies corresponding to a resonant transition, i.e., a transition between the ground state  ${}^2S_{1/2}$  and excited states ( ${}^2P_{1/2}$ ,  ${}^2P_{3/2}$ ).

We now consider the dielectric constant at frequen-

cies corresponding to a transition between one of the levels of a doublet and some excited level (with angular momentum  $j'$ ). We assume that the population of the excited levels is small. In this case we can neglect the integral term in (4), since it is proportional to the density of the excited atoms. We assume that the upper excited level  $j'$  is not connected with the ground state by a dipole transition. Then the line broadening is determined by the resonant collisions of the atoms excited at one of the doublet levels with the atoms in the ground state, and is proportional to the density of the latter. Under these conditions we can obtain an analytic expression for  $\varepsilon(\omega)$  without averaging the amplitudes over the angles. The expression for the dielectric constant is

$$\varepsilon_{3/2}(\omega) = 1 + \frac{4\pi}{3} (2j' + 1) g_{1/2}^2 \left( \frac{n_{j'}}{2} - \frac{n'}{2j' + 1} \right) \frac{1}{\delta} Z \left( \frac{\omega - \omega'_{1/2}}{\delta}, \eta_{3/2} \right), \quad (29)$$

where  $\omega'_{1/2} = E_{j'} - E_{1/2}$ ,  $n'$  is the density of the atoms at the considered excited level  $j'$ , and  $g'_{1/2}$  is the corresponding matrix element of the transition

$$\varepsilon_{3/2}(\omega) = 1 - \frac{1}{3\pi} g_{1/2}^2 [(2j' + 1)n_{j'} - 4n'] (I_1 + I_2), \quad (30)$$

where

$$I_1 = \int d\mathbf{p} \varphi(\mathbf{p}) \left[ \omega - \omega'_{1/2} + \frac{\mathbf{p}\mathbf{k}}{M} + i\pi n g_{1/2}^2 (1.74 + 0.36\chi(x)) \right]^{-1}, \quad (31)$$

$$I_2 = \int d\mathbf{p} \varphi(\mathbf{p}) \left[ \omega - \omega'_{1/2} + \frac{\mathbf{p}\mathbf{k}}{M} + i\pi n g_{1/2}^2 (1.98 - 0.36\chi(x)) \right]^{-1} \quad (32)$$

The function  $\chi(x)$  is given by

$$\chi(x) = 1 - \frac{1}{x^2} + \frac{1}{x^3} e^{-x^2} \int_0^x e^{y^2} dy, \quad x = \sqrt{\frac{p^2}{2MT}}.$$

For light elements (Li and H) the number 1.16 in (28) is replaced by 2.33, and the expression in the brackets in the denominators of (31) and (32) is replaced respectively by  $2.18 + 0.44\chi(x)$  and  $2.48 - 0.44\chi(x)$ .

An approximate expression for  $\varepsilon$  is obtained, as already noted, by averaging the amplitude  $f(\mathbf{q}, \mathbf{q})$  over the directions of the vector  $\mathbf{q}$ . This averaging is equivalent to replacing the function  $\chi(x)$  in (31) and (32) by its value at  $x = 0$ ,  $\chi(0) = 1/3$ . Then expression (30) for  $\varepsilon_{3/2}$  takes the form

$$\varepsilon_{3/2}(\omega) = 1 - \frac{2\pi}{3} g_{1/2}^2 [(2j' + 1)n_{j'} - 4n'] Z \left( \frac{\omega - \omega'_{1/2}}{\delta}, \eta_{3/2} \right). \quad (33)$$

5. All the foregoing expressions were valid in the frequency region  $|\omega - \omega_j| \ll g^2/\rho_0^3$ . However, the procedure developed in [3] makes it possible to consider also the wings of the lines, i.e., the frequency region  $\Delta E \gg |\omega - \omega_j| \gg g^2/\rho_0^3$ . The results obtained in this case coincide with those obtained from the statistical theory of the broadening of spectral lines (see, for example, [15]). Thus, to find the line width in this frequency region it is sufficient to determine the behavior of the electronic terms of the quasimolecule.

We present an expression for the imaginary part of the dielectric constant:

$$\text{Im } \varepsilon_{3/2} = \frac{8}{3} \pi n g_{1/2}^2 \frac{2.15\pi n g_{1/2}^2}{(\omega - \omega'_{1/2})^2} \quad \text{Im } \varepsilon_{3/2} = \frac{4\pi}{3} n g_{1/2}^2 \frac{1.39\pi n g_{1/2}^2}{(\omega - \omega'_{1/2})^2} \quad (34)$$

since these expressions are valid in the frequency region  $\Delta E \gg |\omega - \omega_j| \gg g^2/\rho_0^3$ , they pertain to those al-

kali metals in which  $\Delta E \gg g^2/\rho_0^3$ , i.e., Na, K, Rb, and Cs. Thus, the width changes on going over to the line wings, and the character of the dependence on the frequency remains the same as before. The contribution of the Doppler broadening on the line wings, as is well known, is negligibly small and is therefore disregarded.

6. The imaginary part of the dielectric constant, as is well known, yields under certain conditions the form of the absorption line. From the obtained results (see (20) and (34)) it follows in particular that in the approximation  $g_{1/2}^2 = g_{3/2}^2$  the widths of the doublet components are related respectively as 1.59 and 1.52. The approximate calculation by Foley [16] gives for this ratio the value  $\sqrt{2}$ . However, the accuracy of the available experimental data (see the review of [17]) is insufficient to reveal this difference. It is of interest to note that for light elements, hydrogen and lithium, the collision widths of the doublet components are equal to each other.

The values of the widths obtained in the later experimental papers [18, 20] do not agree with each other and contradict the results obtained on the basis of the "impact" and "statistical" approximation. A root dependence of the width on the density was observed in [19, 20], but the widths themselves exceeded the calculated values by approximately three orders of magnitude. It is possible that such large widths are due to radiation-diffusion processes. Attempts to explain the results of [18] theoretically, undertaken in [21], pertain only to the case of immobile atoms and can hardly serve as a basis for the interpretation of the indicated experiments.

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