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ADSORPTION OF ATOMIC IMPURITIES ON QUANTIZED VORTICES IN LIQUID HELIUM II

L. S. RENT and I. Z. FISHER

Odessa State University

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It is shown that impurity quasiparticles of a dilute solution of He^3 in superfluid He^4 interact effectively with the quantized vortices, becoming attracted to the latter. An analysis is presented of the interaction of one impurity quasi-particle with a linear vortex within the framework of the Landau superfluidity theory, and a general thermodynamic analysis is presented of the behavior of the solution as a whole in the presence of a superfluid linear vortex in the solution. The adsorption of the impurities on the vortex is appreciable at low temperatures, leading to the formation of a linear Fermi liquid of the subsystem along the vortex axis. The limits of applicability of the thermodynamic approach to the problem of adsorption of impurities on a vortex are discussed.

1. INTRODUCTION

A dilute solution of He^3 in He^4 is superfluid below the λ curve, and quantized vortex lines can be produced in it. It will be shown below that the quasiparticles belonging to the normal component, which correspond to impurity atoms, interact effectively with the vortex lines, and the sign of this interaction corresponds to attraction forces. Therefore, under equilibrium conditions, the impurity atoms will be adsorbed on the vortex lines. The adsorption efficiency depends on the temperature of the solution. If the temperature is sufficiently low, then appreciable saturation of the regions near the vortex lines by the impurity atoms may occur.

The drawing of the atomic impurities into the region adjacent to the vortex axis is not an exclusive phenomenon specifically connected with the properties of the given impurity. It is known that ions interact with vortex lines in helium II, that the vortices capture ions[1,2], and that analogous phenomena are known for colloidal particles[3]. The possibility of capture into a bound state of a roton by a linear vortex was noted in[4]. The increase of the phonon and roton densities near the vortex axis, which follows inevitably from the Landau superfluidity model, can also be interpreted in a certain sense as the result of interaction of the same type between helium excitations and the quantized vortex. Therefore the drawing of all the possible impurities and excitations into the superfluid vortex is apparently a universal phenomenon, although the mechanisms of attraction and capture (if the latter is possible) will be different in each case. It is easy to indicate the common cause of all these phenomena—inasmuch as the density of the kinetic energy of the superfluid vortex is maximal near the vortex axis and decreases with increasing distance from the axis, the crowding out or the extinction of the superfluidity in this region is energetically favored.

The purpose of the present paper is to estimate the interaction of the atomic impurities with the quantized linear vortex in helium II. Two different methods will be used to prove the existence of an interaction of the type

$$U(r) \sim -a(T)/r^2 \qquad (1)$$

For distances r that are not too close to the vortex axis. We also discuss the problem of adsorption of impurities from the solution by the vortex.

2. INTERACTION OF AN IMPURITY QUASIPARTICLE WITH A QUANTIZED VORTEX

Let us consider one He^3 atom in superfluid He^4. Let

$$\varepsilon(\mathbf{p}) = \varepsilon_0(\rho) + \mathbf{p}^2/2m_3^* \qquad (2)$$

be the energy of the impurity quasiparticle in the coordinate system in which $\mathbf{v}_s = 0$. Then in a coordinate system in which $\mathbf{v}_s \neq 0$ we have

$$E(\mathbf{p}) = \varepsilon(\mathbf{p}) + \mathbf{p}\mathbf{v}_s. \qquad (3)$$

The term $\mathbf{p}\cdot\mathbf{v}_s$ can be regarded here as the "kinematic part" of the interaction of the impurity quasiparticle with the field $\mathbf{v}_s$. However, a term of the same type would arise also for a perfectly free particle. Therefore, the real interaction of the impurity atom with the field $\mathbf{v}_s$ will be obtained by subtracting from $\mathbf{p}\cdot\mathbf{v}_s$ a similar expression for the free particle, which would move with the same velocity as the quasiparticle:

$$(\Delta E)_{\text{kin}} = \mathbf{p}\mathbf{v}_s - \frac{m_3}{m_3^*}\mathbf{p}\mathbf{v}_s = \frac{m^*_3 - m_3}{m_3^*}\mathbf{p}\mathbf{v}_s. \qquad (4)$$

It is necessary to add to this the "dynamic part" of the interaction due to the change of the binding energy $\epsilon_0(\rho)$ in the field $\mathbf{v}_s$. Let ρ_0 and $\rho(\mathbf{v}_s)$ be the density of helium II in the absence and in the presence of the superfluid motion, and let $\delta\rho = \rho(\mathbf{v}_s) - \rho_0$. Then

$$(\Delta E)_{\text{dyn}} = \varepsilon_0(\rho(\mathbf{v}_s)) - \varepsilon_0(\rho_0) = \frac{\partial\varepsilon_0}{\partial\rho}\delta\rho + \frac{1}{2}\frac{\partial^2\varepsilon_0}{\partial\rho^2}(\delta\rho)^2 + \ldots \qquad (5)$$

Not too close to the vortex axis we have $\delta\rho \sim \mathbf{v}_s^2$, and we put in lieu of (5)

$$(\Delta E)_{\text{dyn}} = -\beta\frac{m_4^2}{2m_3^*}\mathbf{v}_s^2, \qquad (6)$$

where β is a dimensionless coefficient, the magnitude and sign of which are to be determined. If we put

$$(m_3^* - m_3)/m_4 = \alpha, \qquad (7)$$

then we obtain, after changing over to the operators from (4) and (6),

$$\hat{U}^{\text{int}} = \frac{m_4}{2m_3^*}\{\alpha(\hat{\mathbf{p}}\mathbf{v}_s + \mathbf{v}_s\hat{\mathbf{p}}) - \beta m_4 \mathbf{v}_s^2\}. \tag{8}$$

Here $\mathbf{v}_S$ is regarded as the classical external field. In connection with the employed series expansions, the interaction in the form (8) should be valid starting with distances exceeding several atomic dimensions in helium II. For simplicity we assume $\mathbf{v}_n = 0$ throughout.

For one quantized vortex located along the z axis and having positive circulation, we have

$$\mathbf{v}_s = \frac{\hbar}{m_4 r^2}(-y, x, 0), \quad \mathbf{v}_s^2 = \frac{\hbar^2}{m_4^2 r^2}, \quad r = \sqrt{x^2 + y^2}. \tag{9}$$

Expression (8) reduces to

$$\hat{U}^{\text{int}} = \frac{\alpha\hbar}{m_3^* r^2}\hat{M}_z - \frac{\beta\hbar^2}{2m_3^* r^2} = -\frac{\hbar^2}{2m_3^* r^2}\left[2i\alpha\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \beta\right], \tag{10}$$

where $\hat{M}_Z$ is the operator of the z-projection of the angular momentum of the quasi-particle. If we denote by $l\hbar$ the eigenvalues of the operator $\hat{M}_Z$ ($l = 0, \pm 1, \pm 2, \ldots$), then we obtain for the states with specified l:

$$\langle l|\hat{U}^{\text{int}}|l\rangle = \frac{\hbar^2}{2m_3^* r^2}(2\alpha l - \beta). \tag{11}$$

We see therefore that when $l = 0$ the sign of the interaction is determined by the sign of β. When $l \neq 0$ and $\alpha \gg |\beta|$ (see below) we obtain an interaction of the attraction type if $l < 0$ and of the repulsion type if $l > 0$. This property of the interaction of the quasiparticles with the quantized vortex, as applied to rotons, was first noted by E. Lifshitz and Pitaevskiĭ[4].

Let us proceed to estimate and discuss the coefficients α and β. At very low temperatures we have $m_3^* = 2.4m_3$[5] and from (7) we get $\alpha = 1.05$. With increasing temperature, α increases slowly and at T ~ 1°K we have approximately $\alpha = 1.3$. In a solution, m_3^* and α increase slightly with increasing impurity concentration[5].

It is impossible to estimate β in (8) accurately, owing to the lack of reliable data on the derivative $\partial\epsilon_0/\partial\rho$. Ptukha[6] obtained an indirect estimate of this quantity, starting from the experimental data on the thermal conductivity of weak solutions of He^3 in He^4. A value $\partial\epsilon_0/\partial\rho \approx 4.5 \times 10^{-15}$ cm^5/sec^2 was obtained. However, the strong difference between this quantity and the corresponding estimate for rotons[7,8], obtained from more direct neutron-diffraction investigations, namely $\partial\Delta/\partial\rho \approx -5.2 \times 10^{-13}$ cm^5/sec^2, is strange. Taking the estimate from[6] and expressing the $\rho = \rho(\mathbf{v}_S^2)$ in terms of its value in thermodynamic equilibrium[9], we get from (5) and (6)

$$\beta = -\frac{m_3^*\rho^2}{m_4^2}\frac{\partial\varepsilon_0}{\partial\rho}\frac{\partial}{\partial P}\left(\frac{\rho_n}{\rho}\right) \approx -3.5\cdot 10^7\frac{\partial}{\partial P}\left(\frac{\rho_n}{\rho}\right), \tag{12}$$

where P is the pressure. The last factor in (12) can be calculated by representing it in the form of a sum of phonon and roton contributions and by using the experimental values of the dependence of the speed of sound and the roton-spectrum parameter on the pressure[7,8,10]. The final formulas and the entire calculations are very cumbersome and lead to the following picture: when T = 0 we also have $\partial(\rho_n/\rho)/\partial\rho = 0$. With increasing temperature, this quantity is negative and at T = 0.5°K a shallow minimum is reached, amounting to -1.7×10^{-13} cgs units. The roton contributions then begin to predominate and $\partial(\rho_n/\rho)/\partial P$ increases rapidly. At T = 0.064°K a zero value is reached, and with further increase of the temperature this quantity is positive. For example, a value 2.4×10^{-11} cgs units is reached at T = 1°K.

We therefore obtain from (12) $\beta = 0$ at T = 0 and then $\beta = 0$ at T = 0.4°K. In the interval between these temperatures, an extremal value is reached, namely

$$\beta \approx 6\cdot 10^{-6} \quad \text{at} \quad T = 0{,}5°\,\text{K}. \tag{13}$$

In the region T > 0.64°K, β reverses sign and increases rapidly in absolute magnitude, but still remains very small. For example, we get $\beta \approx -0.8 \times 10^{-3}$ at T = 1°K and $\beta \approx -3.6 \times 10^{-3}$ at T = 1.2°K.

The obtained estimate for β corresponds, in accordance with (11), to a very weak attraction of the impurity towards the axis of the vortex at low temperatures and to a weak repulsion from the axis of the vortex at higher temperatures in the state with $l = 0$. At $l \neq 0$ we always have $|2\alpha l| \gg |\beta|$, and the "dynamic" interaction serves as a small correction to the "kinematic" interaction. The sign of interaction of the impurity with the vortex is determined by the sign of l.

3. EXISTENCE OF BOUND STATES AND ABSORPTION ON THE LOWER ENERGY LEVELS

We consider the Schrödinger equation for a quasiparticle in the field of the vortex

$$-\frac{\hbar^2}{2m_3^*}\Delta\psi + (\varepsilon_0 + \hat{U}^{\text{int}})\psi = E\psi, \tag{14}$$

and are interested in the possible bound states. We therefore put

$$\psi(r, \varphi, z) = R_l(r)\, e^{ikz + il\varphi}, \tag{15}$$

$$E_l = \varepsilon_0 + \frac{\hbar^2 k^2}{2m_3^*} + E_l', \quad E_l' = -\frac{\hbar^2}{2m_3^*}\lambda_l^2. \tag{16}$$

Using (11), we obtain for $R_l(r)$ an equation of the Bessel type

$$R_l''(r) + \frac{1}{r}R_l'(r) + \left(-\lambda_l^2 + \frac{\beta - 2\alpha l - l^2}{r^2}\right)R_l(r) = 0. \tag{17}$$

It is obvious that the bound states are possible if $\beta - 2\alpha l - l^2 > 0$. We have seen above that at T = 0 we get $\alpha = 1.05$ and $\beta = 0$. Therefore the ground state corresponds to $l = -1$, and since $\sqrt{2\alpha - 1} = 1.05$, the solution of (17) that vanishes at infinity will be

$$R(r) = AK_{1.05\,i}(\lambda r), \quad r > r_0, \tag{18}$$

where $K_{i\tau}$ is a modified Bessel function of imaginary index, A is a constant, the symbol $l = -1$ has been left out, and r_0 denotes approximate distance, starting with which the interaction in the form (11) is already valid. For real λ (if they exist), which are still to be determined, expression (18) yields a series of eigenfunctions of the bound states with $l = -1$.

Another series of possible eigenfunctions of the bound states with $l = -2$ is given by

$$R(r) = BK_{0.45\,i}(\lambda r), \quad r > r_0. \tag{19}$$

Account is taken here of the fact that $\sqrt{4\alpha - 4} = 0.45$. At low temperatures, there are no solutions of (17) that vanish at infinity and have real λ and $|l| > 2$. At high temperatures, the indices of the functions on the right sides of (18) and (19) increase, and bound states with $l = 0$ can arise if $\beta > 0$.

To determine the possible values of λ, it is necessary to join together the functions (18) or (19) with the corresponding $R_l(r)$ in the region $r < r_0$, in which the explicit form of the interaction of the impurity atom with the vortex is unknown. For an approximate estimate of λ, we put $\hat{U}^{\text{int}} = \text{const}$ for $r < r_0$, and choose the constant equal to $\langle l|\hat{U}_{\text{int}}|l\rangle$ in accordance with (11) at the point $r = r_0$. For $l = -1$ this leads to the equation

$$R''(r)+\frac{1}{r}R'(r)+\left[\left(\frac{2.10}{r_0^2}-\lambda^2\right)-\frac{1}{r^2}\right]R(r)=0, \quad r<r_0, \tag{20}$$

with a solution that is regular at $r = 0$

$$R(r)=CJ_1\left(\frac{r}{r_0}\sqrt{2.10-\lambda^2r_0^2}\right), \tag{21}$$

where J_1—Bessel function of first order. The joining of the solutions (18) and (21) leads to a condition for the determination of λ in the form

$$\frac{\lambda r_0 K_{1.05i}(\lambda r_0)}{K_{1.05i}(\lambda r_0)}=\frac{\sqrt{2.10-\lambda^2r_0^2}\,J_1'(\sqrt{2.10-\lambda^2r_0^2})}{J_1(\sqrt{2.10-\lambda^2r_0^2})}. \tag{22}$$

This equation has infinitely many positive solutions with a condensation point at $\lambda = 0$. The largest solution is $\lambda r_0 = 0.18$, and after recalculation in accordance with (16) we obtain for the energy of the ground state at $T = 0$

$$E' = -1.5\cdot 10^{-17}/(r_0)_{A}^2\ [\text{erg}], \tag{23}$$

where r_0 is measured in Angstroms. The function $|R(r)|^2$ in accordance with (18) and (21) vanishes when $r = 0$ and has a maximum at $r \approx 2r_0$. Thus, the wave function of the ground state is localized in a narrow region encompassing the axis of the vortex if r_0 is sufficiently small.

It is possible to investigate analogously the remaining states of this series, and also the states of the series $l = -1$, for which there likewise exists an infinite set of negative values of the energy with a condensation point at $E' = 0$. It can also be noted that all the indicated properties of the bound states and their energy spectrum are valid qualitatively almost regardless of the manner in which the interaction of the impurity with the vortex is approximated at very small distances. Thus, for example, Eq. (22) has solutions of the type written out above for any right-hand side that is continuous at the point $\lambda r_0 = 0$.

The investigated properties suffice for a qualitative description of the absorption of the impurities on the vortex from a solution with very low concentration at low temperatures. However, to construct a quantitative theory of adsorption, more detailed information are necessary concerning the interaction of the impurities with the field of the vortex, especially at small distances.

4. THERMODYNAMIC ESTIMATE OF THE INTERACTION AND ADSORPTION

Instead of studying the behavior of the individual impurity quasi-particle, we proceed now to a study of the properties of a superfluid solution as a whole, in the presence of a quantized vortex in it. We assume that the entire system is in thermodynamic equilibrium. The region of applicability of such an approach will be discussed in detail below.

Assume that in helium II there is a mixture of He^3 atoms, and a stationary isothermal superfluid motion takes place in it. Then the following thermodynamic identity is satisfied[9]:

$$\rho d\mu = dP + Zdc - {}^1\!/_2\rho_n d\mathrm{v}_s^2, \tag{24}$$

where ρ, ρ_n—total density of the solution and density of the normal component, P—pressure, c—weight concentration of the impurities, and we put $v_n = 0$ and $dT = 0$; Z and μ are two chemical potentials, whose definition and connection with the ordinary chemical potentials of the particles (per unit mass) are given by

$$\mu=(\partial F/\partial\rho)_{T,c,v_s^2}=c\mu_3+(1-c)\mu_4, \tag{25}$$

$$Z=(\partial F/\partial c)_{T,\rho,v_s^2}=\rho(\mu_3-\mu_4), \tag{26}$$

where F is the free energy of the solution. We get therefore

$$\mu_3=\mu+(1-c)Z/\rho, \quad \mu_4=\mu-cZ/\rho. \tag{27}$$

The conditions for the stationary of the superfluid motion are obtained from general hydrodynamic equations for solutions in helium II in the form[9]

$$\operatorname{div}(\rho_s\mathrm{v}_s)=0, \quad \nabla P+{}^1\!/_2\rho_s\nabla\mathrm{v}_s^2=0, \tag{28}$$

$$d(\mu-cZ/\rho+{}^1\!/_2\mathrm{v}_s^2)=0. \tag{29}$$

From (27)—(29) follow new equilibrium conditions:

$$d(\mu_3+{}^1\!/_2\mathrm{v}_s^2)=d(\mu_4+{}^1\!/_2\mathrm{v}_s^2)=0. \tag{30}$$

When $v_s = 0$ they go over into the ordinary conditions $d\mu_3 = d\mu_4 = 0$.

We are further interested in the dependence of μ_3 on v_s^2 for relatively small motions. We then get from (24)

$$\mu(P,c,\mathrm{v}_s^2)=\tilde{\mu}(P,c)-\frac{\rho_n}{2\rho}\mathrm{v}_s^2, \tag{31}$$

and, in addition,

$$\frac{\partial^2\mu}{\partial c\,\partial\mathrm{v}_s^2}=\frac{\partial}{\partial\mathrm{v}_s^2}\left(\frac{Z}{\rho}\right)=-\frac{1}{2}\frac{\partial}{\partial c}\left(\frac{\rho_n}{\rho}\right). \tag{32}$$

From this we get, with the same accuracy

$$Z(P,c,\mathrm{v}_s^2)=\tilde{Z}(P,c)-\frac{\rho}{2}\frac{\partial}{\partial c}\left(\frac{\rho_n}{\rho}\right)\mathrm{v}_s^2. \tag{33}$$

From (27), (21), and (33) we get

$$\mu_3(P,c,\mathrm{v}_s^2)=\tilde{\mu}_3(P,c)-\frac{1}{2}\left[\frac{\rho_n}{\rho}+(1-c)\frac{\partial}{\partial c}\left(\frac{\rho_n}{\rho}\right)\right]\mathrm{v}_s^2. \tag{34}$$

In (31)—(34) we did not take into account terms of order $(v_s^2)^2$ and for brevity we have omitted the argument T of all the functions.

The equilibrium condition for the impurities, in accordance with (30) and (34), will be

$$\tilde{\mu}_3(P,c)-\frac{1}{2}\Bigl(\frac{\rho_n}{\rho}+(1-c)\frac{\partial}{\partial c}\Bigl(\frac{\rho_n}{\rho}\Bigr)\Bigr]\mathrm{v}_s^2+\frac{1}{2}\mathrm{v}_s^2=\text{const}. \tag{35}$$

In order to determine the dependence $c = c(v_s^2)$ under stationary conditions, we need one more equation. It is convenient to choose the second equation of (28), which can be written for slow motions in the form

$$P = P_0 - \rho_s v_s^2 / 2. \tag{36}$$

In addition, we have from (24)

$$\frac{\partial\mu}{\partial P} = \frac{1}{\rho}, \quad \frac{\partial}{\partial P}\left(\frac{Z}{\rho}\right) = \frac{\partial^2\mu}{\partial P\,\partial c} = \frac{\partial}{\partial c}\left(\frac{1}{\rho}\right), \tag{37}$$

so that it follows from (27) that

$$\frac{\partial\tilde{\mu}_3}{\partial P} = \frac{1}{\rho} + (1-c)\frac{\partial}{\partial c}\left(\frac{1}{\rho}\right). \tag{38}$$

Substituting (36) in $\tilde{\mu}_3(P, c)$ and expanding in powers of v_s^2, we get with the aid of (38), with the same accuracy as before,

$$\tilde{\mu}_3(P, c) = \tilde{\mu}_3(P_0, c) - \frac{1}{2}\left[\frac{1}{\rho} + (1-c)\frac{\partial}{\partial c}\left(\frac{1}{\rho}\right)\right]\rho_s v_s^2. \tag{39}$$

Finally, from (35) and (39) we obtain an equation for the determination of the equilibrium concentration

$$\tilde{\mu}_3(P_0, c) - \frac{1-c}{2}\left[\rho_s\frac{\partial}{\partial c}\left(\frac{1}{\rho}\right) + \frac{\partial}{\partial c}\left(\frac{\rho_n}{\rho}\right)\right] v_s^2 = \tilde{\mu}_3(P_0, c_0). \tag{40}$$

We now use the well known relation[9]

$$\frac{\rho_n}{\rho} = \left(\frac{\rho_n}{\rho}\right)_{c=0} + \frac{m_3^*}{m_4}c, \tag{41}$$

and go over to the ordinary chemical potential for the impurity, per unit particle: $\mu' = m_3\tilde{\mu}_3$. We then get in lieu of (40)

$$\mu'(c) - \frac{1-c}{2}\left[\frac{\partial}{\partial c}\left(\frac{m_3^*}{m_4}c\right) + \rho_s\frac{\partial}{\partial c}\left(\frac{1}{\rho}\right)\right] m_3 v_s^2 = \mu'(c_0), \tag{42}$$

where the argument of P_0 has been left out. Thus, in the equation for the equilibrium concentration, the chemical potential enters with an additive term proportional to v_s^2, which in classical thermodynamics is equivalent to the result of action of a second effective external field, equal in our case to

$$U^{\text{eff}} = -\frac{1-c}{2}\left[\frac{\partial}{\partial c}\left(\frac{m_3^*}{m_4}c\right) + \rho_s\frac{\partial}{\partial c}\left(\frac{1}{\rho}\right)\right] m_3 v_s^2. \tag{43}$$

For a linear vortex we get from (9)

$$U^{\text{eff}}(r) = -(1-c)\left[\frac{\partial}{\partial c}\left(\frac{m_3^*}{m_4}c\right) + \rho_s\frac{\partial}{\partial c}\left(\frac{1}{\rho}\right)\right]\frac{m_3\hbar^2}{2m_4^2 r^2}. \tag{44}$$

The expression in the square bracket is positive in this case, so that the effective potential (43) or (44) corresponds to attraction forces—the impurity particles will become drawn into the vortex.

The effective potential (44) depends on the temperature and concentration. For small concentrations we have

$$U^{\text{eff}}(r) = -\left(\frac{m_3^*}{m_4} + o_s\frac{\partial}{\partial c}\left(\frac{1}{\rho}\right)\right)_{c=0}\frac{m_3\hbar^2}{2m_4^2 r^2}. \tag{45}$$

In the temperature region ~1—1.5°K we have $m_3^* \sim 3m_3$ and $\partial\rho/\partial c \approx -0.4$[11], and after substituting the constants we obtain the approximate estimate

$$U^{\text{eff}}(r) \approx -\frac{1.5\cdot 10^{-31}}{r^2}\left(1 + 0.18\frac{\rho_s}{\rho}\right). \tag{46}$$

Using the effective potential, the equation for the equilibrium concentration in the field of the vortex takes the form

$$\mu'(c) + U^{\text{eff}}(r) = \mu'(c_0). \tag{47}$$

In the classical region $\mu' \approx kT \ln C$ and for small concentrations c_0, we obtain the Boltzmann distribution for c:

$$c(r) = c_0\exp\{-U^{\text{eff}}(r)/kT\}. \tag{48}$$

The concentration increases on approaching the vortex axis. In the more general case, information is necessary on the behavior of $\mu' = \mu'(c)$; in particular, the interaction between the impurities may become important[12]. However, inasmuch as $\partial\mu'/\partial c > 0$ in all cases, the positiveness of the adsorption of the impurities by the vortex always follows from (47). Assuming the adsorption to be small, we get from (47)

$$c(r) - c_0 = -U^{\text{eff}}(r) / (\partial\mu'/\partial c)_0. \tag{49}$$

We note in conclusion that the expression obtained for $U^{\text{eff}}(r)$, in the limits of its applicability, should be the result of the corresponding static averaging of the microscopic operator $\hat{U}^{\text{int}}$. In the approximation (10)—(11) we should have

$$U^{\text{eff}}(r) = \overline{\langle l|\hat{U}^{\text{int}}|l\rangle} = \frac{\hbar^2}{2m_3^* r^2}\overline{(2\alpha l - \beta)}. \tag{50}$$

We shall show below that (44) is an asymptotically exact expression at r → ∞. Comparing (50) and (45), we get

$$\overline{2\alpha l - \beta} = -\frac{m_3 m_3^*}{m_4^2}\left[\frac{m_3^*}{m_4} + \rho_s\frac{\partial}{\partial c}\left(\frac{1}{\rho}\right)\right], \tag{51}$$

from which it follows that $\bar{l} \approx -1.5$ at low temperatures and $\bar{l} \approx -1.3$ at high temperatures.

5. REGION OF APPLICABILITY OF THERMODYNAMIC ESTIMATES

We now investigate the region of applicability of the results obtained in the preceding section. At comparatively high temperatures and not too large impurity concentrations, the latter can be regarded as an excitation gas. It is clear from (30) that we have actually investigated the diffusion equilibrium in the impurity subsystem. Therefore, in order for the thermodynamic description of the excitation gas in the field of the superfluid vortex to be valid, it is necessary that the distances of interest to us exceed the diffusion length or the mean free path of the impurity quasiparticle:

$$r \gg l_i \approx \sqrt{Dt_i}, \tag{52}$$

where l_i and t_i are the length and free-path time. The quantities D, l_i, and t_i have been investigated in[13]. Using estimates from that paper, we obtain at T > 0.5°K the following inequality:

$$r \gg l_i \approx 5\cdot 10^{-10}(\rho_{n0}/\rho_n)e^{\Delta/kT}/\sqrt{T}\ [\text{cm}], \tag{53}$$

where ρ_{n0} and ρ_n are the normal density in the absence and in the presence of impurities:

$$\rho_n = \rho_{n0}(T) + (m_3^*/m_4)c\rho. \tag{54}$$

Two cases are further possible. If the concentration of the impurities is so small that $c\rho \ll \rho_n^0$, only collisions of impurities with rotons are significant, and from (54) we get

$$r \gg l_i \approx 5\cdot 10^{-10}T^{-1/2}e^{\Delta/kT}\ [\text{см}]. \tag{55}$$

This leads to $l_i \approx 1.2 \times 10^{-7}$ cm at T = 1.5°K, but at T = 1°K we already have $l_i = 2.7 \times 10^{-6}$. With further decrease of temperature, the free-path length increases very rapidly. Therefore, in the case of exceedingly small concentrations, the thermodynamic estimate of the adsorption is of interest only at temperatures close to 1.5°K and above, where the adsorption itself is small.

In the opposite case, $c\rho \gg \rho_{n0}$, only collisions of impurity quasi-particles with one another are significant, and from (54) we get

$$r \gg l_i \approx 0.7\cdot 10^{-8}/cT \ [\mathrm{cm}]. \tag{56}$$

The estimate had no sharp temperature dependence, and becomes more and more favorable with increasing concentration. For example, at T ~ 1°K and at a concentration of several per cent we get $l_i \approx 2 \times 10^{-7}$ cm.

Thus, at relatively high temperatures, there exist regions of variation of the quantities c and T, in which the thermodynamic results are valid when $r \gg 10^{-7}$ cm, and are in any case valid at asymptotic estimates as $r \to \infty$. In the most interesting region $r \sim 10^{-7}$ cm, the thermodynamic estimates are approximately correct only for practically large impurity concentrations.

In the region of very low temperatures and not too small impurity concentrations, the picture changes radically. The impurities can no longer be regarded as an almost classical gas of excitations, and it is necessary to take into account the Fermi-liquid properties of the impurity subsystem[12]. The only essential requirement which must be imposed in this region on the statistical or thermodynamic solution of the problem is the condition that the discussed distances exceed the average distance between particles:

$$r \gg (n_3)^{-1/3} = \left(\frac{c\rho}{m_3}\right)^{-1/3} \approx \frac{1.5\cdot 10^{-7}}{c^{1/3}} \ [\mathrm{cm}] \tag{57}$$

(where the concentration is measured in wt.%). Inasmuch as (57) should contain the local concentration at a given distance r, and at low temperatures the absorption of the impurities by the vortex, in accordance with (47), it is relatively large at any initial concentration of the solution c_0, it turns out that near T = 0 the theory based on analysis of the chemical potential of the impurities is valid at all distances $r \gtrsim 10^{-7}$ cm. The foregoing can be readily verified by using simple formulas for the degenerate Fermi gas. At still shorter distances, the terms of order $(v_s^2)^2$ and above, which represent the structure of the superfluid vortex at distances on the order of several atomic dimensions, and which are discarded in Sec. 4, become important.

6. CONCLUSION

It follows from the foregoing investigation that both an individual quasi-particle and an aggregate of all the impurity quasiparticles in a superfluid solution experience attraction to the quantized vortex lines. The resultant adsorption of impurities by the vortex should be particularly noticeable at low temperatures. The most interesting consequence of this picture is the possibility of formation of a linear Fermi-liquid system located along the vortex axis, with transverse dimensions on the order of 10^{-7} cm or higher, depending on the initial concentration of the solution. The existing attraction between the impurity quasiparticle[12] leads to an intensification of the adsorption effect and to an increase of the stability of the produced Fermi-liquid filament. A quantitative description of this phenomenon calls for a special investigation.

The adsorption of impurities by quantized vortices changes our notions concerning the structure of the vortices themselves in solutions of He^3 in He^4, especially at very low temperatures. Since the impurity quasiparticles belong to the normal component of the superfluid solution, the effect of impurity adsorption causes the relation between the normal and superfluid densities (ρ_n and ρ_s) to be different near the axis of the vortex in the case of the solution and for pure He^4. Actually this is the case always encountered in practice, since "pure" H^4 is actually a mixture of isotopes.

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