SOVIET PHYSICS JETP VOLUME 28, NUMBER 1 JANUARY, 1969

EFFECT OF CONCENTRATION NON-UNIFORMITY ON THE PROPERTIES OF SUPERCONDUCTORS WITH PARAMAGNETIC IMPURITIES

I. O. KULIK and O. Yu. ITSKOVICH

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

Submitted June 17, 1967; resubmitted March 18, 1968

Zh. Eksp. Teor. Fiz. **55**, 193–207 (July, 1968)

The possibility of calculating the critical temperature of a superconducting alloy with a nonuniform distribution $c(\mathbf{r})$ of paramagnetic impurities is investigated by means of a generalization of the Abrikosov-Gor'kov theory (the characteristic inhomogeneity dimension δ_0 and coherence length ξ_0 are assumed to be large compared to the mean distance between the impurities). By analogy with the problem of determining the spectrum of disordered systems, the concepts of a "renormalized" critical temperature T_c (the same as introduced in the Abrikosov-Gor'kov theory) and of a "true" critical temperature T'_c ($T'_c > T_c$) due to fluctuations, are introduced. When $T > T_c$ the superconductivity picture is that of "islets" of the superconducting phase separated by normal regions. The generalized Ginzburg-Landau equation is used to study localized states when $T > T_c$ (superconducting "nuclei"). It is shown that when $c(\mathbf{r})$ is not constant and the concentration is small the equation contains a "potential energy" term that is a linear functional of the impurity concentration.

1. INTRODUCTION

THE study of the properties of superconducting alloys, i.e., superconductors with admixtures of other elements, is one of the urgent problems of modern theory of superconductivity. The theory of superconducting alloys (homogeneous solid solutions) containing both nonmagnetic and paramagnetic impurities was constructed in the well known papers of Abrikosov and Gor'kov [1,2]. The most important result of this theory is the explanation of the dependence of the critical magnetic field (H_{c2}) on the concentration of the nonmagnetic impurities (in particular, the transition to superconductivity of the second kind, occurring at a definite concentration) and the critical temperature (T_c) on the concentration of the paramagnetic impurities (accompanied by a vanishing in the superconductivity at a certain critical concentration).

In the Abrikosov-Gor'kov theory, the impurities are assumed to be distributed over the volume of the superconductor uniformly with a certain average concentration $\bar{c}$. As noted in [3], the results of Abrikosov and Gor'kov can be obtained on the basis of a simplified equivalent Hamiltonian of the interaction between the electrons and the impurities; this Hamiltonian does not contain summation over the random positions of the impurities (and its spins), and takes the form

$$H_1 = \sum_{\mathbf{p}\neq\mathbf{p}'} a_{\mathbf{p}\alpha}^{+}\Gamma_{\alpha\beta}a_{\mathbf{p}'\beta}, \qquad (1.1)$$

where $a^{+}_{\mathbf{p}\alpha}$—operators of creation of an electron in the state $\mathbf{p}$, α ($\mathbf{p}$—momentum, α—spin index), and $\Gamma_{\alpha\beta}$—a certain interaction matrix:

$$\Gamma_{\alpha\beta} = g_n\delta_{\alpha\beta} + g_s\sigma_{\alpha\beta}, \quad \delta_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (1.2)$$

(g_n and g_s—constants expressed in terms of the probabilities of the non-exchange and exchange scattering by the impurities τ_n^{-1} and τ_s^{-1}). This means that within the framework of the model used in [1,2] the real distribution of the impurities is replaced by a simplified uniform (in the mean) distribution, i.e., effects of concentration fluctuations are disregarded. At the same time it is obvious that for certain problems allowance for the inhomogeneity of the impurities is of decisive significance. For example, such problems include the problem of calculating the critical currents in the mixed state of superconductors of the second kind when $\mathbf{j} \perp \mathbf{H}$ (see [4]). Clusters of impurities, produced by fluctuations of their concentration, can play the role of blocking centers for the Abrikosov vortex filaments, leading to a finite value of the critical current j_{crit} in the mixed state.

The purpose of the present paper is to study the influence of the inhomogeneities of the concentration on the properties of superconductors containing paramagnetic impurities, in the absence of an external magnetic field[1]. We consider here the case when the distribution of the impurities in a crystal can be characterized by their average concentration as a function of a point, $c(\mathbf{r})$. This can be done provided the characteristic distance over which the function $c(\mathbf{r})$ varies, and also the distance over which the average superconducting parameters change ($\xi_0 \sim v_0/T_c$), are large compared with their "statistical" dimensions characterizing the microinhomogeneities of the concentration, i.e., actually compared with the average distances between the impurities[2]. At the concentrations of interest, on the order of 1% (see [2]), the latter quantity amounts to several interatomic distances, i.e., it is small com-

---

[1] The influence of the fluctuations of the impurity concentration on the magnetic properties of superconductors, and particularly the problem of the critical currents in the mixed state, are not considered by us in this paper. These questions will be dealt with in a separate article.

[2] The opposite limiting case, when the distance between the impurities is large compared with ξ_0, and therefore the individual impurities can be regarded as isolated defects, were investigated by Suhl et al. [5] and by Fetter [6] (in the case of nonmagnetic impurities).

pared with the correlation length $\xi_0 \sim 10^{-4}$ cm. We shall assume it also to be small compared with the dimensions of the macroscopic inhomogeneities of the concentration δ_0 (the characteristic distances over which a change of the function $c(\mathbf{r})$ takes place), but we shall make no assumption concerning any relation between the parameters ξ_0 and δ_0.

Thus, the term "fluctuation" will mean in the present paper inhomogeneities of the average impurity concentration. Typical examples of systems to which the developed theory applies are, for example, the widely used inhomogeneous alloys produced in the decay of a solid solution quenched from high temperatures, or alloys produced by condensation from a gas mixture on a low-temperature substrate [7] (in particular, "alloys" of mutually insoluble components can be obtained in this manner). As will be shown below, the critical temperature of such systems is essentially determined by the character of the distribution of the impurity concentration over the volume of the sample. Moreover, the very concept of critical temperature calls for a refinement.

Inasmuch as the critical temperature is a function of the concentration, which decreases with increasing c (see [2]), the sections of the alloy in which the concentration exceeds the average value will have decreased values of T_c, and the sections with the lower concentrations will have higher T_c. It is therefore clear that the true critical temperature will be higher than the value corresponding to the average concentration. As was noted in an earlier paper [3], consideration of similar questions leads to a problem analogous to that of calculating the spectrum of disordered systems [8,9], (see also [10] etc.). In the latter case, as shown by I. Lifshitz [8,9], it is possible to introduce in the calculation of the state-density function $\rho(E)$ a "renormalized" end point E_g of the spectrum and a true end point E_g' of the spectrum, which can lie much higher than E_g even at low concentrations. This situation is qualitatively illustrated in Fig. 1a, where the solid curve shows the density of states of a regular crystal (E_{g0}--end point of the band), and the dashed line represents the function of the spectral density of the impurity-containing disordered system.

Figure 1b shows the qualitative dependence of the averaged ordering parameter Δ of the superconductor on the temperature T. The solid curve pertains to the pure superconductor, and the dashed line to a superconducting alloy containing paramagnetic impurities. Without taking the impurity-concentration fluctuations into account (if these impurities have a distribution that is homogeneous in the mean), Δ vanishes at a certain "renormalized" critical temperature T_c, which differs from the transition temperature T_{c0} of the pure superconductor. This is precisely the quantity introduced in the theory of Abrikosov and Gor'kov [2]. In fact, owing to the fluctuations, the superconductivity is conserved up to a certain temperature T_c', exceeding T_c ($T_c' > T_c$), which plays the role of the "true" critical temperature (in principle T_c' may also coincide with T_{c0}). When $T > T_c$, the picture of the superconductivity will consist of "islands" of the superconducting phase, separated by regions of the normal phase [3]. Such a superconductivity cannot be revealed by the vanishing of the resistance, which will be finite, but it can be observed, for example, in experiments on the tunnel effect[3)].

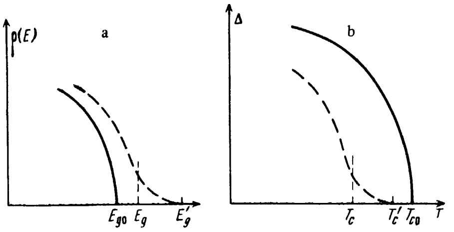

FIG. 1

In the present paper (in Sec. 2) we obtain a generalization of the Abrikosov-Gor'kov equations [2] to include the case of an arbitrary inhomogeneous distribution of the concentration of the paramagnetic impurities $c(\mathbf{r})$ (satisfying the properties indicated above). These equations are solved in Sec. 3 by perturbation theory with the aid of an expansion in powers of the concentration. We obtain for the "renormalized" critical temperature T_c, an expression that coincides with the corresponding expression in the Abrikosov-Gor'kov theory. Finally, in Sec. 4 we investigate the question of calculating the "true" critical temperature T_c' caused by the localized states near regions with decreased concentration of the paramagnetic impurities. T_c' turns out to be here higher than the critical temperature defined in the homogeneous model [2]. To investigate the localized states, we used the modified Ginzburg-Landau equation, which contains a "potential energy" term that is a linear functional of the impurity concentration.

2. CHOICE OF MODEL. ABRIKOSOV-GOR'KOV EQUATIONS IN THE INHOMOGENEOUS CASE

Proceeding to solve our problem, let us discuss first the question of the maximum simplification of the calculation scheme, retaining at the same time the main physical premises which we shall assume to coincide with those on which the Abrikosov-Gor'kov theory [2] is based.

The Hamiltonian of the interaction between the electrons and the impurities is

$$H_1 = \sum_i \int d\mathbf{r}\, \psi_\alpha^+(\mathbf{r})\, [I_i \delta_{\alpha\beta} V_n(\mathbf{r} - \mathbf{R}_i) + (\mathbf{S}_i \boldsymbol{\sigma}_{\alpha\beta}) V_s(\mathbf{r} - \mathbf{R}_i)]\, \psi_\beta(\mathbf{r}), \quad (2.1)$$

where V_n and V_s are the non-exchange and exchange parts of the interaction potential, $\mathbf{R}_i$ are the coordinates of the impurities, $\mathbf{S}_i$ is the spin of the impurity located at the point $\mathbf{R}_i$, $\sigma_{\alpha\beta}$ are the spin matrices of the electron:

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (2.2)$$

and I_i are quantities whose meaning will be explained later.

Following [3], we assume δ-function interactions between the electrons and the impurities, and we put accordingly

---

[3)] Similar considerations can be developed also for the magnetic properties of superconductors with nomagnetic impurities, but we shall confine ourselves henceforth to superconducting alloys with paramagnetic impurities.

$$V_n(\mathbf{r}-\mathbf{R}_i) = V_n\delta(\mathbf{r}-\mathbf{R}_i), \quad V_s(\mathbf{r}-\mathbf{R}_i) = V_s\delta(\mathbf{r}-\mathbf{R}_i). \tag{2.3}$$

The possibility of such a substitution is connected with the fact that although the real interaction has a finite radius of action (on the order of the lattice constant a), nonetheless its magnitude is small compared with the characteristic distances over which the gap in the superconductor varies, i.e., the correlation length ξ_0. The final formulas therefore do not contain detailed features of the interaction potential, but only the scattering amplitudes expressed in terms of the free-path time τ_n and τ_s. Here, as is known from the work of Abrikosov and Gor'kov [2], the change of T_c in a spatially-homogeneous system is expressed (in the Born approximation) only in terms of the spin free path time τ_s. The latter quantity is connected with V_s by the relation [3]

$$1/\tau_s = 2\pi S(S+1)N(0)\bar{c}V_s^2, \tag{2.4}$$

where S—spin of the impurity, c—average concentration of the impurities, and $N(0) = mp_0/2\pi^2$—density of the electron states on the Fermi surface.

The Hamiltonian (2.1), with allowance for (2.3), takes the form

$$H_1 = \sum_i \psi_\alpha^+(\mathbf{R}_i)[I_i\delta_{\alpha\beta}V_n + (\mathbf{S}_i\boldsymbol{\sigma}_{\alpha\beta})V_s]\psi_\beta(\mathbf{R}_i). \tag{2.5}$$

Further simplification consists in the following. Using perturbation theory with respect to the Hamiltonian (2.5), we obtain in first order expressions of the form

$$\int_0^\beta d\tau_1 \sum_i G_{\alpha\gamma}^0(\mathbf{r}-\mathbf{R}_i, \tau-\tau_1)[I_i\delta_{\gamma\delta}V_n + (\mathbf{S}_i\boldsymbol{\sigma}_{\gamma\delta})V_s]G_{\delta\beta}^0(\mathbf{R}_i-\mathbf{r}', \tau_1-\tau') \tag{2.6}$$

Terms of this type should yield zero when averaged over the positions of the impurities [2]. For the second term in the square brackets this is obvious, since the mean value of the spin of the impurity at each side is equal to zero: $S_i = 0$ (the diagrams of higher order containing the product of an odd number of factors S_i vanish in similar fashion). As to the first term in the square brackets of (2.6), in the spatially-homogeneous case it vanishes as a result of renormalization of a chemical potential μ [2,11]. In a spatially-inhomogeneous system, an analogous effect should also take place: in the presence of uncompensated charge of the purities, the electron density is adjusted in such a way that at each point the electroneutrality condition is satisfied, so that the diagrams of odd order vanish. However, this is quite difficult to obtain in a model with a δ-function potential. For this reason we shall consider a model according to which each point at which the impurity atom is located we ascribe a random factor $I_i = \pm 1$ in such a way, that $\overline{I_i} = 0$ at each point. As a result of this, diagrams of odd order vanish when averaged over the configurations of the impurities, for both the exchange and non-exchange parts of the scattering.

We now proceed to determine the critical temperature of the superconducting transition T_c. According to [2], the critical temperature of a superconductor is defined as the largest eigenvalue of the equation

$$\Delta_{\alpha\beta}^*(\mathbf{r}) = |\lambda| T \sum_\omega \int d\mathbf{s}\, \overline{G_{-\omega\gamma\alpha}(\mathbf{s},\mathbf{r})G_{\omega\lambda\beta}(\mathbf{s},\mathbf{r})}\Delta_{\gamma\lambda}^*(\mathbf{s}), \tag{2.7}$$

where $G_{\omega\alpha\beta}(\mathbf{r}, \mathbf{s})$—thermodynamic Green's functions of the electron in the normal metal in the coordinate-frequency representation, $\omega = (2n+1)\pi T$—discrete frequencies,

$$\Delta_{\alpha\beta}^*(\mathbf{r}) = |\lambda|F_{\alpha\beta}^+(\mathbf{r}\mathbf{r}';\ \tau-\tau')|_{\mathbf{r}'\to\mathbf{r},\, \tau'\to\tau}$$

—average values of the ordering parameter of the superconductor, λ—Cooper constant of the interaction. The bar in (2.7) denotes averaging over the random values of the coordinates of the impurities and their spins. It is assumed that there is no correlation between the values of the spins of the impurity at different points.

Just as in the homogeneous case, the matrix structure of the quantities is of the form [2]

$$\Delta_{\alpha\beta}^*(\mathbf{r}) = \Delta^*(\mathbf{r})g_{\alpha\beta}, \quad g_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{2.8}$$

so that Eq. (2.7) can be rewritten in the form

$$\Delta^*(\mathbf{r}) = |\lambda| T \sum_\omega \int d\mathbf{s}\, K_\omega(\mathbf{s},\mathbf{r})\Delta^*(\mathbf{s}), \tag{2.9}$$

where

$$K_\omega(\mathbf{s},\mathbf{r}) = {}^1\!/_2\, g_{\lambda\gamma}\overline{G_{-\omega\gamma\alpha}(\mathbf{s},\mathbf{r})G_{\omega\lambda\beta}(\mathbf{s},\mathbf{r})}\, g_{\beta\alpha} \tag{2.10}$$

(summation over repeated indices).

As seen from the form of (2.7) and (2.9), we neglect, following Abrikosov and Gor'kov, the variation of the ordering parameter near the individual impurities, considering only smooth variations of Δ, due to the inhomogeneities of the average concentration[4]. The dependence of the quantity $G_{-\omega}(\mathbf{s}, \mathbf{r})G_\omega(\mathbf{s}, \mathbf{r})\Delta^*(\mathbf{s})$ on the coordinates consists of a rapid change at distances on the order of the mean distance between the impurities, and a smooth variation at large distances, over which the average impurity concentration changes (δ_0). Smooth changes of Δ can occur only at distances on the order of $\xi_0 \sim v_0/T_c$. Therefore, the indicated quantity can be averaged near each point **r** over a region with characteristic dimensions L, satisfying the condition $L \gg ac^{-1/3}$, but at the same time $L \ll \delta_0$ and $L \ll \xi_0$. After such an averaging, the distribution of the impurities will be described by their concentration $c(\mathbf{r})$, and the summation over the impurities reduces to integration over space with a weight function $c(\mathbf{r})$. In complete analogy with the procedure used in [2], at distances exceeding the mean distance between impurities, Δ can be regarded as a constant and taken outside the averaging sign. The averaged quantity $G_{-\omega}(\mathbf{s}, \mathbf{r})G_{-\omega}(\mathbf{s}, \mathbf{r})G(\mathbf{s}, \mathbf{r})$ will be a function having a characteristic radius $|\mathbf{s}-\mathbf{r}| \sim \xi_0$.

To calculate the kernel $K_\omega(\mathbf{s}, \mathbf{r})$, we shall use the diagram technique developed in [1,2,11]. As already discussed, diagrams of odd order after averaging over the impurities yield zero, and for diagrams of even order, by virtue of the equalities

$$\overline{I_iI_j} = \delta_{ij}, \quad \overline{S_i^pS_j^q} = {}^1\!/_3 S(S+1)\delta_{ij}\delta_{pq} \tag{2.11}$$

(i, j—numbers of points, p, q—Cartesian projections) we find that after averaging there remain only diagrams on which the crosses pertain pairwise to identical impurities (Fig. 2). As a result, for example, the contri-

---

[4] The change of Δ near individual impurities was accounted for in [12], where a small correction to the Abrikosov-Gor'kov expression for T_c was obtained. This effect is not considered in the present article.

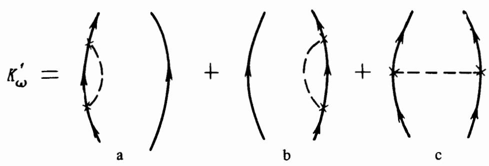

FIG. 2

bution of a diagram shown in Fig. 2c turns out to be (G^0_ω--Green's function of zeroth approximation)

$$[V_n^2 - S(S+1)V_s^2] \int d\mathbf{R}\, c(\mathbf{R}) G_\omega{}^0(\mathbf{s}-\mathbf{R}) G_{-\omega}{}^0(\mathbf{s}-\mathbf{R}) G_\omega{}^0(\mathbf{R}-\mathbf{r}) G_{-\omega}{}^0(\mathbf{R}-\mathbf{r}), \tag{2.12}$$

where

$$c(\mathbf{R}) = \sum_i \overline{\delta(\mathbf{R}-\mathbf{R}_i)} \tag{2.13}$$

is the average concentration of the impurities at the point **R**.

In approximations of order higher than the second, it is necessary to discard all the diagrams with intersecting dashed lines. This follows from the fact that in expressions analogous to (2.12) we should assume that c(**r**) is a slowly varying function (at distances on the order of $p_0^{-1} \sim a$), as a result of which the estimate of these diagrams is carried out in the same manner as when c = const [1, 2, 11], and shows that their contribution is small compared with the contribution of diagrams with non-intersecting lines. The calculation of $K_\omega(\mathbf{s}, \mathbf{r})$ then reduces to summation of diagrams of the "ladder" approximation [11].

A graphic equation for $K_\omega(\mathbf{s}, \mathbf{r})$ is shown in Fig. 3, where the shaded square corresponds to the exact function K_ω. Writing this equation analytically, we get

$$K_\omega(\mathbf{s},\mathbf{r}) = G_\omega(\mathbf{s},\mathbf{r}) G_{-\omega}(\mathbf{s},\mathbf{r}) + [V_n^2 - S(S+1)V_s^2] \int d\mathbf{R}\, c(\mathbf{R}) G_\omega(\mathbf{s},\mathbf{R}) G_{-\omega}(\mathbf{s},\mathbf{R}) K_\omega(\mathbf{R},\mathbf{r}). \tag{2.14}$$

The Green's function $G_\omega(\mathbf{s}, \mathbf{r})$ itself is obtained from an equation summing the "ladder" diagrams shown in Fig. 4 (the thin lines correspond to the unperturbed Green's function, and the heavy ones to Green's functions in the presence of impurities). In accordance with Fig. 4, we have

$$G_\omega(\mathbf{s},\mathbf{r}) = G_\omega{}^0(\mathbf{s},\mathbf{r}) + [V_n^2 + S(S+1)V_s^2] \int d\mathbf{R}\, c(\mathbf{R}) G_\omega{}^0(\mathbf{s}-\mathbf{R}) G_\omega(\mathbf{R},\mathbf{R}) G_\omega(\mathbf{R},\mathbf{r}). \tag{2.15}$$

In the case of constant concentration (c(**R**) = const), Eqs. (2.14) and (2.15) go over into the corresponding equations of Abrikosov and Gor'kov [2] 5). In this case the averaged Green's functions $G_\omega(\mathbf{s}, \mathbf{r})$ and $K_\omega(\mathbf{s}, \mathbf{r})$ depend only on the difference of the arguments **s** – **r**, making it possible to obtain them in explicit form (in the Fourier representation):

$$G_\omega(\mathbf{p}) = (i\omega\eta_\omega - \xi_p)^{-1}, \quad \xi_p = \frac{p^2}{2m} - \mu, \quad \eta_\omega = 1 + \frac{1}{2\tau|\omega|}, \tag{2.16}$$

$$K_\omega(\mathbf{p})|_{\mathbf{p}=0} = \frac{\pi N(0)}{|\omega| + 1/\tau_s}, \tag{2.17}$$

where

$$1/\tau = 2\pi\bar{c}N(0)[V_n^2 + S(S+1)V_s^2], \tag{2.18}$$

and τ_s is determined by formula (2.4). Since in this case Δ = const, substitution of (2.17) in (2.9) leads to the well known equation for the critical temperature [2]:

$$\ln\frac{T_{c0}}{T_c} = \psi\left(\frac{1}{2} + \frac{1}{2\pi\tau_s T_c}\right) - \psi\left(\frac{1}{2}\right), \quad \psi(z) = \frac{d}{dz}\ln\Gamma(z). \tag{2.19}$$

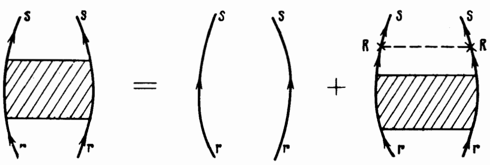

FIG. 3

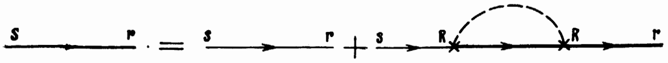

FIG. 4

In the case considered in the present paper, that of variable concentration, the situation turns out to be much more complicated, and Eqs. (2.9), (2.14), and (2.15) cannot be solved in general form. We consider approximate methods for their solution in the following sections.

In order not to interrupt the subsequent exposition, we introduce here certain functions which will be useful in what follows. We define besides $G^0_\omega(\mathbf{R}_1 - \mathbf{R}_2)$ also the functions $S^0_\omega(\mathbf{R}_1 - \mathbf{R}_2)$ and $L^0_\omega(\mathbf{R}_1 - \mathbf{R}_2)$ in accordance with the formulas

$$S_\omega{}^0(\mathbf{R}_1 - \mathbf{R}_2) = \int d\mathbf{r}\, G_\omega{}^0(\mathbf{R}_1 - \mathbf{r}) G_{-\omega}{}^0(\mathbf{r} - \mathbf{R}_2), \tag{2.20}$$

$$L_\omega{}^0(\mathbf{R}_1 - \mathbf{R}_2) = \int d\mathbf{r}\, d\mathbf{s}\, G_\omega{}^0(\mathbf{R}_1 - \mathbf{r}) G_{-\omega}{}^0(\mathbf{r} - \mathbf{s}) G_\omega{}^0(\mathbf{s} - \mathbf{R}_2). \tag{2.21}$$

Using the zeroth-approximation Green's function $G^0_\omega(\mathbf{p})$ (formula (2.16) at $\tau = \infty$), we obtain the values of $S^0_\omega(\mathbf{R}_1 - \mathbf{R}_2)$ and $L^0_\omega(\mathbf{R}_1 - \mathbf{R}_2)$ in the coinciding points:

$$S_\omega{}^0(0) = \frac{\pi N(0)}{|\omega|}, \quad L_\omega{}^0(0) = -\frac{\pi i N(0)}{2\omega^2}\operatorname{sign}\omega. \tag{2.22}$$

The values of the same functions in the case when the difference of the arguments is large compared with the interatomic distance ($R = |\mathbf{R}_1 - \mathbf{R}_2| \gg p_0^{-1}$) are determined by the asymptotic expressions

$$G_\omega{}^0(R) \approx -\frac{m}{2\pi R}\exp(ip_0R\operatorname{sign}\omega)\exp\left(-\frac{|\omega|R}{v_0}\right), \tag{2.23}$$

$$S_\omega{}^0(R) \approx \frac{m}{2\pi R}\sin p_0R \cdot \frac{1}{|\omega|}\exp\left(-\frac{|\omega|R}{v_0}\right), \tag{2.24}$$

$$L_\omega{}^0(R) \approx -\frac{m}{4\pi R}\left[\frac{i\operatorname{sign}\omega}{\omega^2}\sin p_0R + \frac{R}{|\omega|v_0}\exp(ip_0R\operatorname{sign}\omega)\right] \times \exp\left(-\frac{|\omega|R}{v_0}\right). \tag{2.25}$$

The derivation of (2.23) can be found in the book of Abrikosov, Gor'kov and Dzyaloshinskiĭ [11]. The remaining two expressions are obtained in analogy with (2.23).

3. CALCULATION OF THE RENORMALIZED CRITICAL TEMPERATURE (CASE OF LOW CONCENTRATION)

It is obvious that the eigenvalue of Eq. (2.9) with kernel $K_\omega(\mathbf{s}, \mathbf{r})$ defined by (2.14) and (2.15) can be represented at small concentrations by a series in the form

$$T_c = T_{c0} + \delta_1 T + \delta_2 T + \ldots, \tag{3.1}$$

---

5) The function $K_\omega(\mathbf{s}, \mathbf{r})$ introduced by us is connected in this case with the Abrikosov and Gor'kov kernel $K_\omega(\mathbf{p}_1, \mathbf{p}_2)$ by the relation

$$K_\omega(\mathbf{s}, \mathbf{r}) = \sum_{\mathbf{p}_1\mathbf{p}_2} K_\omega(\mathbf{p}_1, \mathbf{p}_2)\exp[i(\mathbf{p}_1 + \mathbf{p}_2)(\mathbf{s} - \mathbf{r})].$$

$$\delta_1 T = \int f_1(\mathbf{R}) c(\mathbf{R}) d\mathbf{R}, \quad \delta_2 T = \int f_2(\mathbf{R}_1, \mathbf{R}_2) c(\mathbf{R}_1) c(\mathbf{R}_2) d\mathbf{R}_1 d\mathbf{R}_2 \ldots, \tag{3.2}$$

and from considerations of homogeneity of space it follows that the function $f_1(\mathbf{R})$ should be a constant independent of $\mathbf{R}$, $f_2(\mathbf{R}_1, \mathbf{R}_2)$ should be a function of the difference $\mathbf{R}_1 - \mathbf{R}_2$, etc. The expansion (3.1) is obtained if it is assumed that $\Delta^*(\mathbf{r})$ can be represented in the form of a series in powers of the concentration

$$\Delta^*(\mathbf{r}) = \Delta_0 + \Delta_1(\mathbf{r}) + \Delta_2(\mathbf{r}) + \ldots, \tag{3.3}$$

where $\Delta i(\mathbf{r}) \to 0$ as $c \to 0$. This means that we seek the upper bound of the unlocalized solutions for the ordering parameter, i.e., the quantity called "renormalized" critical temperature T_c.

Writing the kernel of (2.6) in the form of a series in powers of the concentration:

$$K_\omega(\mathbf{s}, \mathbf{r}) = K_\omega{}^0(\mathbf{s} - \mathbf{r}) + K_\omega{}^1(\mathbf{s}, \mathbf{r}) + \ldots \tag{3.4}$$

and substituting (3.3) and (3.4) in (2.9), we get the successive approximations

$$\Delta_0 - |\lambda| T_{c0} \sum_\omega \int d\mathbf{s}\, K_\omega{}^0(\mathbf{s} - \mathbf{r}) \Delta_0 = 0, \tag{3.5}$$

$$\Delta_1(\mathbf{r}) - |\lambda| T_{c0} \sum_\omega \int d\mathbf{s}\, K_\omega{}^0(\mathbf{s} - \mathbf{r}) \Delta_1(\mathbf{s})$$
$$= -N(0)|\lambda| \frac{\delta_1 T}{T_{c0}} \Delta_0 + |\lambda| T_{c0} \sum_\omega \int d\mathbf{s}\, K_\omega{}^1(\mathbf{s}, \mathbf{r}) \Delta_0, \tag{3.6}$$

etc. We took account here of the fact that in the sums over the frequencies T is $T_{c0} + \delta_1 T + \ldots$ The first equation of (3.5) defines the critical temperature of the pure superconductor T_{c0}. Rewriting K_ω^0 on the basis of (2.23) in the form

$$K_\omega{}^0(R) = G_\omega{}^0(R) G_{-\omega}{}^0(R) = \left(\frac{m}{2\pi R}\right)^2 \exp\left(-\frac{2|\omega| R}{v_0}\right), \tag{3.7}$$

cancelling Δ_0 out of (3.5), integrating over space, and summing over the frequencies, we obtain (with allowance for the fact that the summation should be terminated at the Debye frequency ω_D) the well known expression

$$T_{c0} = \frac{2\gamma}{\pi} \omega_D \exp\left(-\frac{1}{N(0)|\lambda|}\right), \quad \ln \gamma = C = 0.577.$$

To find the correction to the critical temperature $\delta_1 T = T_c - T_{c0}$, we turn to Eq. (3.6). The right side of this equation should be orthogonal to the solution of the corresponding homogeneous equation, i.e., Δ_0. This leads to a relation

$$N(0) \frac{\delta_1 T}{T_{c0}} = T_{c0} \sum_\omega \int K_\omega{}^1(\mathbf{s}, \mathbf{r})\, d\mathbf{s}\, d\mathbf{r}. \tag{3.8}$$

The quantity $K_\omega^1(\mathbf{s}, \mathbf{r})$ is obtained with the aid of Eqs. (2.14) and (2.15). It is represented in the form of a sum of the three diagrams shown in Fig. 2, and accordingly we have

$$K_\omega{}^1(\mathbf{s}, \mathbf{r}) = S(S+1) V_s{}^2 \int d\mathbf{R}\, c(\mathbf{R}) \cdot$$
$$\cdot \{G_\omega{}^0(\mathbf{R} - \mathbf{s}) G_{-\omega}^0(\mathbf{s} - \mathbf{r}) G_\omega{}^0(\mathbf{r} - \mathbf{R}) G_\omega{}^0(\mathbf{R} - \mathbf{R})$$
$$+ G_{-\omega}^0(\mathbf{R} - \mathbf{s}) G_\omega{}^0(\mathbf{s} - \mathbf{r}) G_{-\omega}^0(\mathbf{r} - \mathbf{R}) G_{-\omega}^0(\mathbf{R} - \mathbf{R})$$
$$- G_\omega{}^0(\mathbf{s} - \mathbf{R}) G_{-\omega}^0(\mathbf{s} - \mathbf{R}) G_\omega{}^0(\mathbf{R} - \mathbf{r}) G_{-\omega}^0(\mathbf{R} - \mathbf{r})\}. \tag{3.9}$$

We note that the non-exchange part of the scattering has dropped out of this expression (compare with [2]). Recognizing that we are interested in the sum $T \sum_\omega K_\omega^1$, we can replace ω by $-\omega$ in one of the first two terms of (3.9), after which the corresponding contributions coincide. Integrating with respect to $\mathbf{s}$ and $\mathbf{r}$, we obtain, with allowance for (2.20) and (2.21)

$$\delta_1 T = S(S+1) V_s{}^2 \int d\mathbf{R}\, c(\mathbf{R}) T_{c0}^2 \sum_\omega [2G_\omega{}^0(0) L_\omega{}^0(0) - (S_\omega{}^0(0))^2]. \tag{3.10}$$

Finally, using (2.22) and summing over the frequencies, we get

$$\delta_1 T = -\frac{\pi^2}{2} S(S+1) V_s{}^2 N(0) \int c(\mathbf{R}) d\mathbf{R} = -\frac{\pi^2}{2} S(S+1) N(0) \bar{c} V_s{}^2. \tag{3.11}$$

The obtained expression coincides with the result of Abrikosov and Gor'kov [2]. Introducing the spin free path time τ_s in accordance with formula (2.4) we represent (3.11) in the form

$$\delta_1 T = -\pi / 4\tau_s. \tag{3.12}$$

Thus, the "renormalized" critical temperature introduced by us coincides (in first order in the concentration) with the critical temperature of Abrikosov and Gor'kov. In this sense, the foregoing analysis yields nothing new compared with the "homogeneous" model [2], if we are interested in the "average" transition temperature T_c. However, as will be shown in the next section, for the "true" critical temperature T_c', significant corrections appear even in first order in the concentration, and are connected with possible existence of localized solutions for $\Delta^*(\mathbf{r})$, which cannot be represented in the form $\Delta_0 + \Delta_1(\mathbf{r})$ with small Δ_1 (even if $c \to 0$). The quantity T_c is the limit separating the regions of existence of nonlocalized and localized solutions for the ordering parameter $\Delta^*(\mathbf{r})$.

It is not difficult to calculate the next higher terms of expansions such as (3.1) and (3.2). It must be remembered, however, that T_c is an effective quantity determining the limit of the existence of the nonlocalized solutions for Δ^* of the type (3.3). In the next higher approximations in the concentration there appears not only a shift but also a smearing of this boundary, i.e., an expansion of the type (3.3) becomes incorrect, strictly speaking. A similar situation, as shown by I. Lifshitz [8,9], occurs also for spectra of disordered systems: in the higher approximation, the "renormalized" boundary of the spectrum, which separates the region of the existence of nonlocalized states, can, generally speaking, not be determined exactly. Only the concept of the "true" boundary of the spectrum has a rigorously strict meaning, and in our case this holds for the analogous concept of "true" critical temperature.

4. THE GINZBURG-LANDAU EQUATION AND LOCAL STATES

The purpose of this section is to investigate localized solutions for the ordering parameter (2.9) near the "true" critical temperature T_c'. We confine ourselves to the case of first order in the concentration, i.e., we put $c(\mathbf{r}) \to 0$. The critical temperature differs little in this case from T_{c0}—the critical temperature of a true superconductor. Because of this, it becomes possible to reduce the integral equation (2.9) to a differential equation of the Ginzburg-Landau type [13,14]. The localized states which occur near the minima of the function $c(\mathbf{r})$ will have a larger radius $\sim 1/\sqrt{T_{c0} - T}$, which becomes much larger than the BCS parameter ξ_0 when $T \rightarrow T_{c0}$. In this case the kernel of (2.9) is a rapidly varying function compared with the ordering parameter $\Delta^*(\mathbf{s})$, so that we can write the expansion

$$\Delta^*(\mathbf{s}) = \Delta^*(\mathbf{r}) + \frac{\partial\Delta^*}{\partial r_i}(s_i - r_i) + \frac{1}{2}\frac{\partial^2\Delta^*}{\partial r_i\,\partial r_k}(s_i - r_i)(s_k - r_k) + \ldots \tag{4.1}$$

The rest of the derivation essentially duplicates the derivation of the Ginzburg-Landau equation from the microtheory presented by Gor'kov [14]. Writing $K_\omega(\mathbf{s}, \mathbf{r})$ in the form

$$K_\omega(\mathbf{s}, \mathbf{r}) = K_\omega{}^0(\mathbf{s}, \mathbf{r}) + K_\omega{}^1(\mathbf{s}, \mathbf{r}), \tag{4.2}$$

where $K^0_\omega(\mathbf{s}, \mathbf{r}) = G^0_\omega(\mathbf{s} - \mathbf{r})G^0_{-\omega}(\mathbf{s} - \mathbf{r})$, and $K^1_\omega(\mathbf{s}, \mathbf{r})$ is determined by formula (3.9), and substituting (4.1), (4.2), and (2.9), we obtain

$$\Delta^*(\mathbf{r})\Big[1 - |\lambda| T \sum_\omega \int d\mathbf{R}\, K_\omega{}^0(\mathbf{R})\Big] - \frac{1}{6}|\lambda| T_{c0} \sum_\omega \int d\mathbf{R}\, R^2 K_\omega{}^0(R)\frac{\partial^2\Delta^*}{\partial\mathbf{r}^2}$$
$$- |\lambda| S(S+1) V_s{}^2 T_{c0} \sum_\omega \int d\mathbf{R}\, c(\mathbf{R}) Q_\omega{}^0(\mathbf{R} - \mathbf{r})\Delta^*(\mathbf{r}) = 0. \tag{4.3}$$

The quantity $Q^0_\omega(R)$ is defined by the relation (see (2.20))

$$Q_\omega{}^0(R) = 2G_\omega{}^0(0)G_\omega{}^0(R)S_\omega{}^0(R) - S_\omega{}^0(0)G_\omega{}^0(R)G_{-\omega}{}^0(R). \tag{4.4}$$

The asymptotic form of $Q^0_\omega(R)$ at $R \ll p_0^{-1}$ takes, on the basis of (2.23) and (2.24), the form

$$Q_\omega{}^0(R) \approx -2\pi N(0)\left(\frac{m}{2\pi R}\right)^2 \frac{\exp(-2|\omega|R/v_0)}{|\omega|}\left[1 - {}^1\!/_2 \exp(2ip_0 R \operatorname{sign}\omega)\right] \tag{4.5}$$

(we note that the oscillating term in the square bracket can be immediately discarded, since we shall be interested in what follows in integrals of $Q^0_\omega(R)$ in which this term yields terms of the order of T_{c0}/μ relative to the first).

The expression in the square brackets in (4.3)) vanishes when $T = T_{c0}$, and is therefore proportional to the difference $T - T_{c0}$:

$$1 - |\lambda| T \sum_\omega \int K_\omega{}^0(R)\, d\mathbf{R} = N(0)\,|\lambda|\,\frac{T - T_{c0}}{T_{c0}}. \tag{4.6}$$

The coefficient of $\partial^2\Delta^*/\partial\mathbf{r}^2$ (in 4.3) is calculated in the R-representation and turns out to be [11,14]

$$T_{c0}\sum_\omega \int K_\omega{}^0(R) R^2\, d\mathbf{R} = \frac{3p_0}{4\pi^2}\eta, \qquad \eta = \frac{7\zeta(3)\mu}{6(\pi T_{c0})^2}. \tag{4.7}$$

Finally, the coefficient of $\Delta^*(\mathbf{r})$ can be calculated with the aid of the asymptotic expression (4.5). As a net result we obtain the following equation for $\Delta^*(\mathbf{r})$:

$$-\frac{1}{4m}\frac{\partial^2\Delta^*}{\partial\mathbf{r}^2} + \frac{S(S+1)V_s{}^2}{\eta}\int d\mathbf{r}' Q(\mathbf{r} - \mathbf{r}') c(\mathbf{r}')\Delta^*(\mathbf{r}') = \frac{T_{c0} - T}{\eta T_{c0}}\Delta^*(\mathbf{r}), \tag{4.8}$$

where $Q(R)$ is given by

$$Q(R) = \frac{m^2}{2\pi R^2} T_{c0} \sum_\omega \frac{\exp(-2|\omega|R/v_0)}{|\omega|}. \tag{4.9}$$

Summing over the frequencies, we get

$$Q(R) = \frac{m^2}{2\pi^2 R^2} \ln \operatorname{cth} \frac{\pi T_{c0} R}{v_0}. \tag{4.10}$$

Relation (4.8) is the Schrodinger equation for a particle (Cooper pair) with mass $2m$ situated in the field of the "potential"

$$V(\mathbf{r}) = \frac{S(S+1)V_s{}^2}{\eta}\int Q(\mathbf{r} - \mathbf{r}') c(\mathbf{r}')\, d\mathbf{r}'. \tag{4.11}$$

The problem of determining the critical temperature T_c' reduces to a determination of the smallest energy eigenvalue

$$E = (T_{c0} - T) / \eta T_{c0} \tag{4.12}$$

of this equation

$$T_c' = T_{c0} - \eta T_{c0} E_{min}. \tag{4.13}$$

Supplementing (4.8) with a term proportional to Δ^3, we obtain the Ginzburg-Landau equation [13,14], generalized to include the case of an inhomogeneous impurity concentration $c(\mathbf{r})$. It is clear here that in the case of small concentrations the cubic term can be taken in the same form as when $c = 0$.

Let us consider some particular cases of (4.8).

If the impurity concentration changes slowly from point to point, so that the characteristic distances over which this change takes place are large compared with the "radius" of the nucleus $Q(\mathbf{r})$, i.e., with the parameter ξ_0, we can take $c(\mathbf{r})$ in (4.11) outside the integral sign. Integrating then the expression (4.9), we arrive at the relation

$$V(\mathbf{r}) = \frac{\pi^2}{2\eta T_{c0}} N(0) S(S+1) V_s{}^2 c(\mathbf{r}), \tag{4.14}$$

which shows that in the case of slow variation of $c(\mathbf{r})$ the role of the "potential" in (4.8),is played directly (accurate to a constant factor) by the impurity concentration $c(\mathbf{r})$. We note that at constant $c(\mathbf{r})$ Eq. (4.8) again leads to formula (3.11) for the decrease of the critical temperature in first order in the concentration[6]. In the case when the impurity concentration depends only on one coordinate (x), the potential V in (4.8) is likewise one-dimensional, and in this case it can be represented in the form

$$V(x) = \frac{\pi^2}{2\eta T_{c0}} N(0) S(S+1) V_s{}^2 c^*(x), \tag{4.15}$$

where $c^*(x)$ is the effective concentration, defined by

$$c^*(x) = \int_{-\infty}^{\infty} q(x - x') c(x')\, dx', \tag{4.16}$$

with a kernel $q(x)$ normalized in accordance with the condition

$$\int_{-\infty}^{\infty} q(x)\, dx = 1. \tag{4.17}$$

As can be readily shown on the basis of (4.9) and (4.11), the function $q(x - x')$ is given by

$$q(x - x') = \frac{8T_{c0}}{\pi v_0}\sum_{n=0}^{\infty}\frac{1}{2n+1}\left|\operatorname{Ei}\left(-(2n+1)\frac{2\pi T_{c0}}{v_0}|x - x'|\right)\right|, \tag{4.18}$$

where $\operatorname{Ei}(x)$ is the integral exponential function. The characteristic radius $q(x - x')$ is $|x - x'| \sim v_0/T_{c0} \sim \xi_0$.

The local states near the minima of the function $c^*(x)$, corresponding to T_c', result from the one-dimensional nature of the problem at arbitrarily small depth of the "potential well" $c^*(x)$. Writing $c^*(x)$ near the minimum in the form

$$c^*(x) = c_0 + {}^1\!/_2 k x^2, \tag{4.19}$$

where k is conveniently represented in the form (δ_0--

---

[6] Actually the region of applicability of Eq. (4.8) is not limited to various concentrations. Thus, for example, according to [15], at concentrations up to 0.8 c_{cr}, the deviation from the linear dependence of T_c on the concentration is <10% (for a homogeneous distribution of the impurity).

effective width of the well, c_{cr}--critical concentration introduced in the Abrikosov-Gor'kov theory [2])

$$k=\frac{c_{\rm cr}}{\delta_0^2},\qquad c_{\rm cr}=\frac{T_{c0}}{4\gamma N(0)S(S+1)V_s^2},\tag{4.20}$$

we see that the condition for the smallness of the concentration signifies $c_0 \ll c_{cr}$ and $x_0 \ll \delta_0$, where x_0--radius of local state.

Solving the Schrödinger equation

$$-\frac{1}{4m}\frac{d^2\Delta^*}{dx^2}+\frac{\pi^2}{2\eta T_{c0}}N(0)S(S+1)V_s^2c^*(x)\Delta^*(x)=E\Delta^*(x),\tag{4.21}$$

we obtain for the minimum value of E

$$\Delta^*(x)=\text{const}\cdot\exp\left(-\frac{x^2}{2x_0^2}\right),\qquad x_0^2=\xi_0\delta_0\sqrt{\frac{7\zeta(3)}{3\gamma}},$$
$$\xi_0=\frac{\gamma v_0}{\pi^2T_{c0}},\tag{4.22}$$

$$T_c'=T_{c0}-\frac{\pi^2}{2}N(0)S(S+1)V_s^2c_0-\frac{1}{2^4}\sqrt{\frac{7\zeta(3)}{3\gamma}}\frac{v_0}{\delta_0}.\tag{4.23}$$

Consequently, the radius of the local state $x_0 \sim \sqrt{\xi_0\xi_0}$ will be small compared with δ_0 when $\delta_0 \gg \xi_0$ (the condition $x_0 \gg \xi_0$ is then satisfied automatically). In this case the value of T_c' is determined by the character of the behavior of the concentration near the minimum of (4.19), and not by the value of c(x) far from the minimum point.

In order for a local state to occur in the three-dimensional case, it is necessary that the value of the perturbation exceed a certain critical value. If the radius of the concentration "well" δ_0 is small compared with ξ_0, we can assume that the addition to the concentration $c'(\mathbf{r})$ in (4.11) is proportional to a δ-function, so that the variable part of $V(\mathbf{r})$ assumes the form

$$V'(R)\approx\int\limits_{\delta_0}c'(\mathbf{r})\,d\mathbf{r}\cdot\frac{m^2S(S+1)V_s^2}{2\pi^2\eta R^2}\ln\operatorname{cth}\frac{\pi T_{c0}R}{v_0}.\tag{4.24}$$

Such a potential corresponds to the "incidence" of a particle on a scattering center [16] (if $\int c'(\mathbf{r})d\mathbf{r} < 0$), since $V'(R)$ has a singularity stronger than $R^{-2} \to 0$. It should be remembered, however, that actually formula (4.10), together with expression (4.24), is valid only when $R \gg v_0/\omega_D$, when the summation over the frequencies in (4.9) can be extended to infinity.

If the fluctuation radius δ_0 is large compared with ξ_0, then we can use formula (4.14). As can be readily shown, in this case the condition for the existence of the local state assumes the form (we omit all the dimensionless factors of order of unity)

$$|c'|\gtrsim c_{\rm cr}\xi_0^2/\delta_0^2,\tag{4.25}$$

where c'--characteristic value of the deviation of the concentration from the mean value (in the region with characteristic dimensions $\sim\delta_0$). Naturally, c' must be negative in order for a local state to occur.

The foregoing analysis pertained to the case of small concentration ($\tau \gg \Delta_0^{-1}$). The non-exchange part of the scattering has then dropped out from the final expressions. In the case when the concentration is not small, the parameters of the generalized Ginzburg-Landau equation will depend on the total free path $l = v_0\tau$[14]. The radius of the local state will then be determined also by the length of the free path relative to scattering without spin flip, and thus, a mechanism arises whereby the nonmagnetic impurities influence the critical temperature of superconductors containing paramagnetic impurities [3]. However, this more complicated case, an analysis of which can also be carried out with the aid of Eqs. (2.14) and (2.15), is beyond the framework of the present article.

In conclusion, we take the opportunity to thank A. A. Abrikosov for interest in the work and useful remarks, and to I. M. Lifshitz for a discussion of the work of valuable advice.

---

[1] A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **35**, 1558 (1958); **36**, (1959) [Sov. Phys.-JETP **8**, 1090 (1959); **9**, 220 (1959)].

[2] A. A. Abrikosov and L. P. Gor'kov, ibid. **39**, 1781 (1960) [**12**, 1243 (1961)].

[3] I. O. Kulik, Ekvivalentnyĭ gamil'tonian vzaimodeĭstivya elektronov s primesami v sverkhprovodnikakh (Equivalent Hamiltonian of Electron-impurity Interaction in Superconductors), VINITI, No. 112, 1967.

[4] I. O. Kulik, Zh. Eksp. Teor. Fiz. **50**, 1617 (1966) [Sov. Phys.-JETP **23**, 1077 (1966)].

[5] H. Suhl, D. R. Fredkin, J. S. Langer, and B. T. Matthias, Phys. Rev. Lett. **9**, 63 (1962).

[6] A. Fetter, Phys. Rev. **140**, A1921 (1965).

[7] M. A. Woolf and F. Reif, Phys. Rev. **137**, A557 (1965).

[8] I. M. Lifshitz, Usp. Fiz. Nauk **83**, 617 (1964) [Sov. Phys.-Usp. **7**, 549 (1965)]; Adv. in Phys. **13**, 483 (1964).

[9] I. M. Lifshitz, Zh. Eksp. Teor. Fiz. **53**, 743 (1967) [Sov. Phys.-JETP **26**, 462 (1968)].

[10] B. J. Halperin and M. Lax, Phys. Rev. **148**, 722 (1966).

[11] A. A. Abrikosov, L. P. Gor'kov and I. E. Z Dzyaloshinskiĭ, Metody kvantovoi teorii polya v statisticheskoi fizike, Fizmatgiz, 1962 (Quantum Field Theoretical Methods in Statistical Physics) [Pergamon, 1965].

[12] T. Tsuzuki and T. Tsuneto, Progr. Theor. Phys. **37**, 1 (1967).

[13] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950).

[14] L. P. Gor'kov, ibid. **36**, 1918 (1959); **37**, 1407 (1959) [Sov. Phys.-JETP **9**, 1364 (1959), **10**, 998 (1969)].

[15] S. Skalski, O. Betbeder-Matibet, and P. R. Weiss, Phys. Rev. **136**, A1500 (1964).

[16] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Fizmatgiz 1963 (Quantum Mechanics) [Addison Wesley, 1958].

Translated by J. G. Adashko
25