

# AN EXPERIMENTAL INVESTIGATION OF THE DOPPLER EFFECT AT THE ELECTRON CYCLOTRON FREQUENCY

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The Doppler shift of the electron gyro frequency is studied experimentally. The measurements are carried out with an electron beam moving along the axis of a multimode resonator in the 10-cm range. Both a nonrelativistic and a relativistic Doppler shift of the electron cyclotron frequency  $\omega_c$  are detected. The possibility of applying the Doppler effect at  $\omega_c$  in an investigation of the electron velocity distribution is discussed.

## 1. INTRODUCTION

ELECTRON cyclotron resonance (ECR) has been used extensively to heat plasmas and to contain them in cavities by means of magnetic mirror geometry,<sup>[1-3]</sup> to accelerate particles,<sup>[4]</sup> to generate microwaves,<sup>[5,6]</sup> and to diagnose plasmas.<sup>[7]</sup> Despite the fundamental simplicity of ECR, under actual conditions it is often complicated considerably by various factors, such as collisions, electron velocity spread, nonlinear effects, random oscillations etc., which it is difficult to take into account simultaneously. It is therefore desirable to investigate the more subtle effects associated with ECR (the Doppler effect, nonlinear effects etc.) under relatively simple conditions at first. Since the high-frequency properties of plasmas are determined only by their electron component, a plasma can be modeled suitably by an electron beam with known parameters. We have previously used this model to enhance the accuracy of electron concentration measurements by means of resonators.<sup>[8]</sup> In the present work we are concerned mainly with the application of ECR to plasma diagnosis, and particularly, following the suggestion of Ya. B. Faïnberg, to investigate the possibility of determining an electron velocity distribution from the Doppler effect at an electron cyclotron frequency.

The registration of ECR is discussed in Sec. 2. The considered ECR observation scheme was found to be quite suitable and sensitive. We registered ECR by means of microwave power absorption, the sign reversal of the resonator natural-frequency shift, and the reduction of resonator  $Q$ .

The Doppler effect at the electron cyclotron frequency  $\omega_c$  is investigated in Sec. 3. Despite complete theoretical clarity, especially in connection with the investigation of plasma absorption and emission at natural frequencies,<sup>[9]</sup> there have been almost no experimental studies of this effect (the Doppler effect at ion cyclotron frequency in a moving plasma was observed in<sup>[10]</sup>). This situation evidently results from the fact that it is difficult to observe the Doppler effect at natural frequencies in real plasmas, because the ordered motion of the particles usually relaxes rapidly into thermal motion. In an electron beam modeling a plasma, the Doppler shift of  $\omega_c$  was observed very clearly, includ-

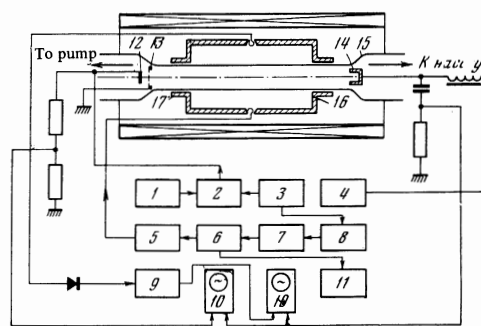


FIG. 1. Experimental scheme. 1 – 15-kV rectifier, 2 – pulse-shaping line, 3 – trigger, 4 – 600-V rectifier, 5 – 10-db attenuator, 6 – power divider, 7 – 10-cm klystron oscillator, 8 – delay line, 9 – amplifier, 10 – dual-beam oscillographs, 11 – heterodyne frequency meter, 12 – cathode, 13 – anode, 14 – collector, 15 – quartz tube, 16 – resonator, 17 – cutoff waveguides

ing a relativistic correction.

Section 4 discusses the possibility of using the Doppler shift of  $\omega_c$  to determine the distribution of longitudinal electron velocities. We obtain the here required dependence of microwave power absorption on electron density and velocity and on the magnetic field strength.

## 2. REGISTRATION OF ELECTRON CYCLOTRON RESONANCE

We studied ECR resulting from the interaction of a pulsed electron beam with a multimode resonator in the 10-cm range. Figure 1 is a diagram of the experimental setup. The electron beam (accelerating voltage  $U = 1-12$  kV, current  $I = 0-3$  A, diameter  $2a = 10$  mm, pulse length  $\tau = 25$   $\mu$ sec) came from a gun consisting of an indirectly heated LaB<sub>6</sub> cathode and grid anode; the gun was immersed in a uniform magnetic field ( $H = 0-1500$  Oe,  $\pm 2\%$  inhomogeneity). The beam was directed along the axis of a quartz tube of 30-mm diameter, evacuated to  $p \sim 10^{-6}$  Torr. A 10-cm multimode resonator (i.d.  $2b = 102$  mm, height  $h = 300$  mm, diameter of end apertures  $2d = 33$  mm, length of cutoff waveguides  $l = 40$  mm) was aligned axially with the tube. The resonator was excited by a coupled klystron oscilla-

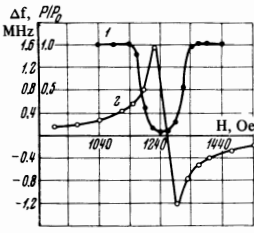


FIG. 2. (1) Transmitted microwave power and (2) frequency shift for the  $H_{011}$  mode as functions of the magnetic field ( $f = 3424$  MHz,  $Q_0 = 6000$ ,  $\tau_{\text{vhf}} = 5$   $\mu\text{sec}$ ,  $p = 3 \times 10^{-6}$  Torr,  $I = 2$  A,  $U = 10$  kV,  $n_0 = 2.7 \times 10^9$   $\text{cm}^{-3}$ ,  $H = 1230$  Oe).

tor ( $W \sim 10$  mW,  $\tau_{\text{vhf}} = 3-10$   $\mu\text{sec}$ ,  $f = 1700-3800$  MHz). The microwave pulse could be shifted in time relative to the beam current pulse by means of a delay line. The following modes were excited separately in the resonator:  $E_{01n}$  ( $n = 0.2, \dots, 6$ ),  $H_{11}$  ( $n = 1, 3, 5$ ),  $H_{21n}$  ( $n = 1, 3, 5$ ), and  $H_{011}$ . The modes depended on the spatial distribution of the microwave fields, which were determined by a perturbation method.

The ECR was registered, as the magnetic field was varied near the resonance frequency  $f_c = 2.82$  H (MHz), by means of the relative reduction of microwave power transmitted through the resonator (for E and H modes), as well as by the sign reversal of the natural-frequency shift and the reduction of the resonator Q (for H modes). We know that the natural-frequency shift of a resonator for  $a \ll b$  in the case of  $E_{01n}$  modes is represented near the resonator axis ( $E_{\parallel}^2 \gg E_{\perp}^2$ ) by

$$\frac{\Delta\omega}{\omega} = 1.85 \frac{\omega_0^2 a^2}{\omega^2 b^2} \left(1 - \frac{c^2}{v_{\text{ph}}^2}\right),$$

and is thus independent of the magnetic field. For H modes near the axis we have  $E_{\perp}^2 \gg E_{\parallel}^2$ , and the frequency shift depends strongly on the magnetic field. Specifically, for the  $H_{01n}$  modes with  $v_{\text{ph}}^2 \gg c^2$

$$\frac{\Delta\omega}{\omega} = 3.1 \left(\frac{a}{b}\right)^2 \frac{\omega_0^2}{\omega^2 - \omega_c^2} [J_1^2(ka) - J_0(ka)J_2(ka)], \quad \omega = kc$$

where J represents Bessel functions.

Figure 2 represents the relative microwave power transmitted through the resonator ( $P/P_0$ , curve 1) and the frequency shift ( $\Delta f$ , curve 2) for the  $H_{011}$  mode as functions of the magnetic field. Similar curves were obtained for the  $H_{111}$  and  $H_{211}$  modes. We note that for these modes ( $n = 1$ ) the phase velocity considerably exceeds the beam velocity ( $v_{\text{ph}} = \omega h / \pi n \gg v_0$ ) and the resolving power of the setup (determined by the electron transit time, magnetic field inhomogeneity etc., see below) does not permit clear detection of the Doppler effect at  $\omega_c$ .

The foregoing scheme for registering ECR is most sensitive to the microwave power absorption. The relative absorption  $\Delta P_{\text{min}}/P_0 \approx 5\%$  was registered reliably;

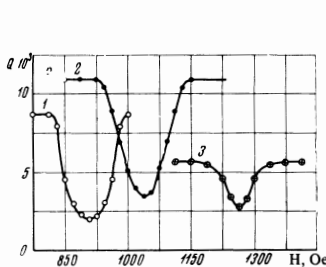
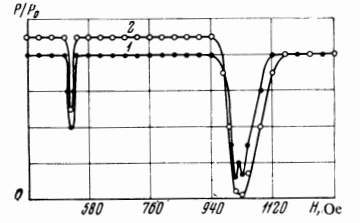


FIG. 3. Loaded Q of resonator versus magnetic field ( $p = 2 \times 10^{-6}$  Torr,  $U = 10$  kV,  $\tau_{\text{vhf}} = 4$   $\mu\text{sec}$ ). Curve 1— $H_{115}$ ,  $f = 2898$  MHz,  $I = 0.27$  A,  $n_0 = 3.5 \times 10^8$   $\text{cm}^{-3}$ ,  $v_{\text{ph}} = 3.5 \times 10^{10}$  cm/sec,  $H_c^{(1)} = H_0(1 - v_0/v_{\text{ph}}) = 860$  Oe; resonance of  $H_c^{(2)} = H_0(1 + v_0/v_{\text{ph}})$  is not shown); 2— $H_{211}$ ,  $f = 2862$  MHz,  $I = 1.3$  A,  $n_0 = 1.8 \times 10^9$   $\text{cm}^{-3}$ ,  $H = 1020$  Oe; curve 3 =  $H_{011}$ ,  $f = 3424$  MHz,  $I = 0.27$  A,  $n_0 = 3.5 \times 10^9$   $\text{cm}^{-3}$ ,  $H = 1230$  Oe.

FIG. 4. Transmitted microwave power versus magnetic field strength for the  $H_{211}$  mode ( $f = 2870$  MHz,  $U = 10$  kV,  $I = 0.4$  A,  $n_0 = 5.3 \times 10^8$   $\text{cm}^{-3}$ ). 1— $p = 1 \times 10^{-4}$  Torr,  $n \sim 10^{10}$   $\text{cm}^{-3}$ ; 2— $p = 3 \times 10^{-6}$  Torr,  $n \sim 10^7$   $\text{cm}^{-3}$ .



$\Delta P_{\text{min}}$  was limited by instability of the measuring conditions. Sensitivity with respect to the minimum electron concentration depends on the mode and Q of the resonator, the transit time etc. As a relative criterion of this sensitivity we take the electron concentration in the beam ( $U = 10$  kV, beam diameter 10 mm) for which the microwave power transmitted through the resonator is reduced by the factor e. We obtained the following values of  $n_{\text{min}}$  for several modes:

$Q_0$	$H_{113}$	$H_{011}$	$H_{213}$	$E_{014}$
$n_{\text{min}}, \text{cm}^{-3}$	$\sim 9000$	$\sim 6000$	$\sim 10^4$	$\sim 4000$
	$\sim 3 \cdot 10^6$	$\sim 10^7$	$\sim 5 \cdot 10^8$	$\sim 10^9$

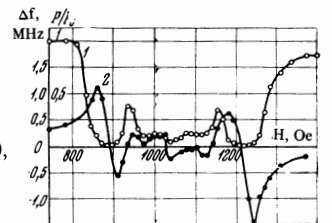
With increase of Q to  $15-20 \times 10^3$  (vacuum resonator) and reduction of the magnetic field inhomogeneity to 0.5% in the cases of the  $H_{11n}$  and  $H_{011}$  modes, it was possible to observe the reduction of  $P/P_0$  to 0.5–0.1 in the thermoelectrons emitted by the cathode at  $U = 0$  (the cathode temperature corresponded to the beam current  $I = 1$  A at  $U = 10$  kV); here the calculations yielded  $n_{\text{min}} \lesssim 10$   $\text{cm}^{-3}$ .

Figure 3 shows the change of Q for the beam-loaded resonator as a function of the magnetic field. Far from ECR, for the H modes the loaded factor Q and the unloaded value  $Q_0$  differ very little, but Q decreases by a factor of several times unity at the passage through resonance. It must be noted that the mean statistical error of measurements in the given region far from ECR is  $\sim 10\%$ . Near ECR, where there is considerable microwave absorption and a reduction of Q, the relative error of  $P/P_0$  and of the frequency shift rises to about 30%. On the other hand, the accurate locating of ECR depends on its half-width:

$$Q_{\text{eff}} \sim 10 - 50, \\ \Delta H_c / H_c \sim 1 / 10 Q_{\text{eff}} \sim 10^{-2} - 10^{-3}.$$

In addition to ECR we investigated the possibility of observing resonances at harmonics and subharmonics of  $\omega_c$  in the following ranges of the parameters:  $f = 1700-3800$  MHz,  $n_0 = 10^7-4 \times 10^9$   $\text{cm}^{-3}$ ,  $H = 400-1500$  Oe (the lower limit of H was determined by the stability of the beam). We observed resonance at  $\omega = 2\omega_c$  in  $H_{21n}$  modes ( $n = 1, 3, 5$ ). Figure 4 shows how the transmitted microwave power in the  $H_{211}$  mode depends on the magnetic field strength for two different

FIG. 5. (1) Transmitted microwave power and (2) frequency shift in the  $H_{115}$  mode versus magnetic field strength ( $p = 3 \times 10^{-6}$  Torr,  $U = 10$  kV,  $I = 0.27$  A,  $n_0 = 3.5 \times 10^8$   $\text{cm}^{-3}$ ,  $f = 2898$  MHz,  $Q_0 = 9000$ ,  $\tau_{\text{vhf}} = 4$   $\mu\text{sec}$ ,  $H_0 = 1030$  Oe,  $v_{\text{ph}} = 3.5 \times 10^{10}$  cm/sec,  $H_c^{(1)} = 860$  Oe,  $H_c^{(2)} = 1200$  Oe).



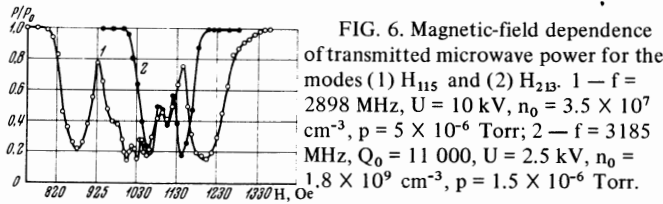


FIG. 6. Magnetic-field dependence of transmitted microwave power for the modes (1)  $H_{115}$  and (2)  $H_{213}$ . 1 —  $f = 2898$  MHz,  $U = 10$  kV,  $n_0 = 3.5 \times 10^7$   $\text{cm}^{-3}$ ,  $p = 5 \times 10^{-6}$  Torr; 2 —  $f = 3185$  MHz,  $Q_0 = 11000$ ,  $U = 2.5$  kV,  $n_0 = 1.8 \times 10^9$   $\text{cm}^{-3}$ ,  $p = 1.5 \times 10^{-6}$  Torr.

residual gas pressures. It is reasonable to assume that resonance at  $2\omega_c$  results from small helicity of the beam. It is known<sup>[5,6,11]</sup> that the interaction of a helical beam with an electromagnetic field leads to absorption and the generation of oscillations near the frequencies  $\omega = l\omega_c$ , where  $l$  is the azimuthal periodicity of the field ( $l = 2$  in our case). Our hypothesis is confirmed by the fact that the shape and depth of the  $2\omega_c$  resonance, unlike ECR, is independent of pressure, with absorption depending on the electrons of the beam but not the plasma. Furthermore, by perturbing the static magnetic field near the electron gun the degree of beam helicity could be changed.  $P/P_0$  then varied from unity to near zero. Strong absorption corresponded to the generation of oscillations with  $\sim 1$  mW of power in this frequency region.

### 3. THE DOPPLER EFFECT IN ELECTRON CYCLOTRON RESONANCE

For the purpose of observing the Doppler effect at  $\omega_c$  it is sufficient, without changing the experimental setup, to reduce the phase velocity of the mode, i.e., to excite in the resonator a mode having a sufficiently large number of half-waves spanning the height of the resonator. Since a standing wave can be resolved into two running waves, for an electron beam moving along the resonator axis Doppler splitting of the gyro frequency will be observed; the two frequencies will pertain to the waves traveling parallel and antiparallel to the electrons:

$$\omega_c^{(1,2)} = \omega_{c0} \frac{1 \pm v_0/v_{ph}}{(1 - v_0^2/c^2)^{1/2}}, \quad \omega_{c0} = \frac{eH_0}{mc}$$

This effect is illustrated in Fig. 5, where the relative amount of transmitted microwave power and the shift in the natural frequency of the resonator for the  $H_{115}$  mode are shown as functions of the magnetic field. In addition to  $\omega_c^{(1,2)}$ , resonance is observed at  $\omega_c^{(3)} = \omega_{c0}$ , which is determined by the electrons of the plasma that is pro-

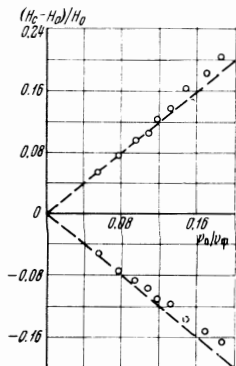
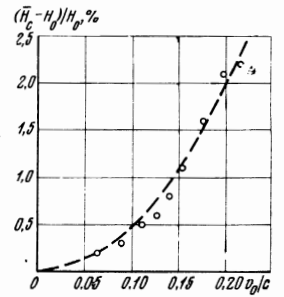


FIG. 7. Doppler shift of the resonance magnetic field versus  $v_0/v_{ph}$  ( $H_{115}$ ,  $f = 2898$  MHz,  $Q_0 = 9000$ ,  $U = 10$  kV,  $n_0 = 3.5 \times 10^7$   $\text{cm}^{-3}$ ,  $p = 2 \times 10^{-6}$  Torr).

FIG. 8. Transverse Doppler effect versus  $v_0/v_{ph}$  (with experimental conditions as for Fig. 7).



duced when the beam ionizes the residual gas. It is interesting that when a microwave pulse is applied immediately after the beam current pulse resonances are not observed at  $\omega_c^{(1,2)}$ , but the resonance at  $\omega_c^{(3)}$  remains because the plasma cannot decay. Each of the three resonances corresponds to a sign reversal of the resonator frequency shift. The shift near a resonance can be used to determine the electron concentrations in the beam and in the plasma separately. The beam electron concentration can also be determined from the current density and the velocity of beam electrons:  $n_0 = 3.5 \times 10^8$   $\text{cm}^{-3}$ , while the concentration of plasma electrons is about one order of magnitude smaller.

In Fig. 5 we observe unusual shapes of resonance that are characteristic of saturation. For the purpose of obviating saturation it is sufficient either to reduce the electron concentration or to select a mode that is only weakly coupled to the beam (i.e., with a relatively small  $E_{\perp}$  component of the microwave field in the beam region). This situation is illustrated in Fig. 6. For  $H_{115}$  modes the  $E_{\perp}$  field is maximal on the axis, while for  $H_{213}$  we have  $E_{\perp} \rightarrow 0$  on the axis; therefore the optimal electron concentrations for these modes differ by about two orders. When the resolution of the measuring circuit is enhanced (by selecting the optimal electron concentration and reducing the phase velocity) a fine structure is observed on the curve of microwave power absorption (curve 1 in Fig. 6) which can be attributed to ordered velocity of the secondary electrons from ionization of the residual gas.

Figure 7 shows relative Doppler shifts of the cyclotron-resonance magnetic field as a function of the ratio between electron velocity and the phase velocity. The dashed lines represent the nonrelativistic expression

$$\frac{H_c^{(1,2)} - H_0}{H_0} = \frac{\omega_c^{(1,2)} - \omega_{c0}}{\omega_{c0}} = \pm \frac{v_0}{v_{ph}}, \quad \omega = \omega_{c0} = \frac{eH_0}{mc}$$

The magnetic field strength was measured in the midplane of the resonator by means of a Hall probe and an oscillator pickup.<sup>[12]</sup>

The absolute magnetic field and its inhomogeneity were measured with  $\sim 2\%$  accuracy. The accuracy of the measurements of relative changes  $\Delta H/H$  in the magnetic field was  $\sim 10^{-3}$ . The accuracy with which the relative Doppler shift was measured was determined from the ECR half-width (with  $Q_{eff} \approx 50$  in this case), and equaled  $\Delta H_c/H_c \sim 10^{-3}$ .

Figure 7 shows that with the increase of  $v_0/v_{ph}$  the experimental values deviate from the calculated values in the direction of higher  $H_c$ . This effect can be accounted for by a relativistic correction of the Doppler shift:

$$\frac{\omega_c^{(1,2)} - \omega_c^{(0)}}{\omega_c^{(0)}} = \pm \frac{v_0}{v_{ph}}, \quad \omega_c^{(0)} = \frac{\omega_c^{(1)} + \omega_c^{(2)}}{2} = \frac{\omega}{(1 - v_0^2/c^2)^{1/2}},$$

which could be observed at relatively low electron velocities since the measurements were sufficiently accurate.

Figure 8 shows the relative magnitude of the relativistic correction for  $\omega_c^{(0)}$  (the relative magnitude of the transverse relativistic Doppler effect) as a function of  $v_0/v_{ph}$ . The dashed curve was calculated from

$$\frac{\omega_c^{(0)} - \omega_{c0}}{\omega_{c0}} = \frac{\bar{H}_c - H_0}{H_0} = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} - 1 \approx \frac{1}{2} \frac{v_0^2}{c^2},$$

where

$$\bar{H}_c = 1/2(H_c^{(1)} + H_c^{(2)}).$$

Quite good agreement between theory and experiment is evident.

#### 4. ON THE POSSIBILITY OF MEASURING THE VELOCITY DISTRIBUTION

The Doppler effect at electron cyclotron frequencies can be used, in principle, to derive the longitudinal-velocity distribution of plasma electrons. The relationship between the profile of the cyclotron absorption line and the velocity distribution for ordinary and extraordinary waves in a plasma has been investigated in [13]. The present work considers the simpler case of an electromagnetic wave that is weakly perturbed by a plasma ( $\omega^2 \gg \omega_0^2$ ). An elementary calculation of hf power absorption is performed in the single-particle approximation.

An electron moving with a velocity  $v_{||}$  along a static magnetic field  $H$  subject to a perpendicular alternating electric field  $E_r = E_{r0} \cos(\omega t + k_3 z)$  near ECR ( $|\omega^2 - \omega_c^2| \ll \omega^2$ ) acquires the transverse energy

$$\mathcal{E} = \frac{e^2 E_{r0}^2}{8m} \frac{1}{(\omega - \omega_{c0} \pm k_3 v_{||})^2 + 1/\tau^2}, \quad (1)$$

where  $\tau$ , which is the time during which the electron interacts with the microwave field, is assumed to be considerably longer than the period of the oscillations.

In (1) the resonance width is determined by  $\tau$ . Under actual conditions, an additional contribution to the resonance width that is especially important for large  $\tau$  will introduce spatial inhomogeneity and time-dependent instability of the experimental parameters (such as magnetic field inhomogeneity, instability of the oscillator frequency etc.). These factors can be taken into account by a series expansion in terms of small parameters near resonance:

$$\mathcal{E} = \frac{e^2 E_{r0}^2}{8m} \left[ \left( \omega^{(0)} - \omega_{c0}^{(0)} \pm k_3^{(0)} v_{||} - \frac{\partial \omega_{c0}^{(0)}}{\partial z} \Delta z + \frac{\partial \omega_{c0}^{(0)}}{\partial t} \Delta t + \dots \right)^2 + \frac{1}{\tau^2} \right]^{-1/2}.$$

We shall assume, for simplicity, that the additional broadening effects can be neglected:

$$\frac{1}{\tau} \gg \frac{\partial \omega_{c0}^{(0)}}{\partial z} \Delta z, \quad \frac{\partial \omega_{c0}^{(0)}}{\partial t} \Delta t, \dots \quad (\Delta z \sim h, \Delta t \sim \tau).$$

Two more assumptions are now made: 1) The range of electron velocities that fulfill the resonance condition

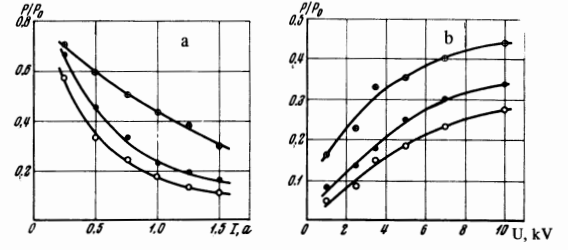


FIG. 9. Dependence of microwave power transmitted in ECR on the (a) current and (b) energy of a beam for the  $H_{2/3}$  mode ( $f = 3185$  MHz,  $Q_0 = 11\,000$ ,  $\tau_{vhf} = 5\mu\text{sec}$ ,  $p = 3 \times 10^{-6}$  Torr). For the beam:  $\omega_c^{(1,2)} = \omega \pm k_3 v_0$  for the lower and middle curves, with (a)  $U = 10$  kV,  $n_0 = 1.3 \times 10^9 \text{ cm}^{-3}$ , (b)  $n_0 = 1.8 \times 10^9 \text{ cm}^{-3}$ ; for the plasma:  $\omega = \omega_{c0}$  for the upper curve, and  $n \sim 10^7 - 10^8 \text{ cm}^{-3}$ .

is considerable smaller than the mean "thermal" electron velocity:

$$\Delta v_{||} \sim 1/k_3 \tau \ll v_m.$$

2) Energy absorption at resonance considerably exceeds nonresonance absorption:

$$\frac{e^2 E_{r0}^2}{8m} \frac{f(v_{||})}{\tau^2} \frac{1}{k_3 \tau} \sim \frac{e^2 E_{r0}^2}{8m} \frac{n}{k_3 v_m} \gg \frac{e^2 E_{r0}^2}{8m} \frac{n}{\omega^2}.$$

The two inequalities can be combined as follows:

$$\frac{1}{\omega \tau} \ll \frac{v_m}{v_{ph}} \ll \omega \tau \quad (\omega = k_3 v_{ph}). \quad (2)$$

In this case the microwave power absorbed in unit volume and averaged over the transit time for a given magnetic field  $H_{c0}$  will depend only on the resonant electrons, which in the velocity interval  $v_{||} - v_{||} + \Delta v_{||}$  have the concentration  $n(v_{||}) = f(v_{||}) \Delta v_{||}$ :

$$W(v_{||}) = \frac{e^2 E_{r0}^2}{8m} \tau n(v_{||}), \quad v_{||} = \pm \frac{\omega - \omega_{c0}}{k_3}.$$

The relative microwave power transmitted through the resonator and the losses due to cyclotron absorption are related by

$$\frac{P}{P_0} = \left[ 1 + \frac{\pi Q_0 e^2}{m \omega} \int_{v_{pl}} n(v_{||}) \tau E_{r0}^2 dV \int_{V_r} E_0^2 dV \right]^{-2},$$

where  $Q_0$  is the unloaded resonator  $Q$  (i.e., in the absence of cyclotron loss); the integral in the numerator is taken over the volume of plasma, and that in the denominator is taken over the volume of the resonator;  $E_0$  is the electric field amplitude. Taking  $n(v_{||})$  and  $\tau$  to be independent of the coordinates in the volume of plasma and introducing the coefficient

$$K = \int_{v_{pl}} E_{r0}^2 dV \int_{V_r} E_0^2 dV,$$

we obtain

$$\frac{P}{P_0} = \left[ 1 + \frac{\pi e^2 Q_0 K \tau}{m \omega} n(v_{||}) \right]^{-2}.$$

For a monoenergetic electron beam traversing the resonator we shall have

$$\tau = \tau_{tr} = h/v_0, \quad n = n_0 \delta(v_0 - v), \quad \omega \tau \gg 1, \\ \frac{P}{P_0} = \left\{ 1 + \frac{\pi e^2 Q_0 K h n_0}{m \omega v_0} \left[ \left( k_3 \pm \frac{\omega - \omega_{c0}}{v_0} \right)^2 h^2 + 1 \right]^{-1} \right\}^{-2}. \quad (R)$$

An especially simple relation between  $P/P_0$  and the distribution function is established when  $\tau$  is independent of  $v$ . This can occur, for example, in a plasma where the interaction time  $\tau$  depends on the frequency of collisions between electrons and neutral particles, and where this frequency, in turn, does not depend on the electron velocity:<sup>[7]</sup>  $1/\tau = \nu = \sigma v N_0$ ,  $\sigma \propto 1/v$ . We have the additional necessary requirement that collisional resonance broadening should be considerably exceeded by the Doppler broadening; this condition coincides with the first inequality in (2). In that case the distribution is represented by

$$n(v_{||}) = \frac{m \omega v}{\pi e^2 Q_0 K} \sqrt{\frac{P_0}{P} - 1}. \quad (S)$$

The experimental dependence of absorbed microwave power on the electron concentration and velocity can be obtained by sending an electron beam with known parameters through the resonator. Figure 9 shows the relative microwave power transmitted through the  $H_{213}$  resonator as a function of the electron concentration and velocity; these results agree satisfactorily with the calculations.

Figure 10 shows the microwave power transmitted through the  $E_{015}$  resonator as a function of the magnetic field with the residual gas pressure as a parameter. The observed dependences permit a qualitative account of the process that develops prior to beam-plasma breakdown. In a good vacuum ( $p \sim 10^{-6}$  Torr) we at first observe two sharp peaks of microwave power absorption pertaining to the beam ( $\omega_c = \omega - k_3 v_0$ ) and the plasma ( $\omega_{c0} = \omega$ ); the peak at  $\omega_c = \omega + k_3 v_0$  is not shown. The beam density considerably exceeds the plasma density. With increasing pressure the plasma density is augmented, the absorption peaks (especially that of the plasma) become diffused, and the plasma peak develops a fine structure that evidently results from the ordered motion of some plasma electrons. At the pressure corresponding to the initiation of the beam-plasma breakdown [ $p = (1-2) \times 10^{-4}$  Torr] the plasma density rises rapidly and the peaks are broadened. Considerable noise and plasma density fluctuations arise, impeding the measurements and apparently producing considerable broadening of the resonances (for example, as a result of increased effective frequency of collisions due to electron scattering by random oscillations, or caused by "momentary" shifts of the resonator's natural frequency that are induced by fluctuations of the electron concentration).

Summarizing briefly, we can state that a number of questions must still be answered (particularly in connection with the resonance width) before we can realize the possibility of determining quantitatively the electron velocity distribution in a laboratory plasma. However, the given method is suitable at the present stage for a qualitative investigation of the distribution, and particularly for studying ordered flows of charged particles in a plasma.

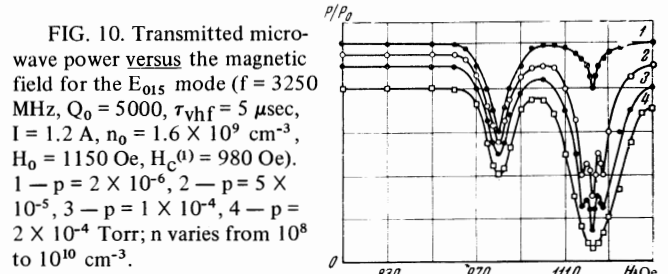


FIG. 10. Transmitted microwave power versus the magnetic field for the  $E_{015}$  mode ( $f = 3250$  MHz,  $Q_0 = 5000$ ,  $\tau_{vhf} = 5$   $\mu$ sec,  $I = 1.2$  A,  $n_0 = 1.6 \times 10^9$  cm<sup>-3</sup>,  $H_0 = 1150$  Oe,  $H_c^{(1)} = 980$  Oe). 1 —  $p = 2 \times 10^{-6}$ , 2 —  $p = 5 \times 10^{-5}$ , 3 —  $p = 1 \times 10^{-4}$ , 4 —  $p = 2 \times 10^{-4}$  Torr;  $n$  varies from  $10^8$  to  $10^{10}$  cm<sup>-3</sup>.

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