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STRESS TENSOR FOR A PLASMA IN A HIGH FREQUENCY ELECTROMAGNETIC FIELD WITH ACCOUNT OF COLLISIONS

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An expression is derived for the time-averaged stress tensor for a plasma in a high frequency electromagnetic field. The initial equations are the kinetic equations in which collisions are taken into account. Weakly ionized and totally ionized plasmas are considered. It is shown that the stress tensor depends significantly on the nature of the collisions and cannot be expressed in terms of the dielectric constant.

INTRODUCTION

THERE are no known general expressions for the time-averaged stress tensor that determines the forces acting on an absorbing medium in an alternating electric field. It is therefore of interest to derive such expressions for the case when the stress tensor can be obtained from the microscopic equations. The present paper is devoted to the solution of this problem for a plasma.

For transparent media, the stress tensor was found by Pitaevskiĭ[1]. Let us discuss the limits of applicability of the expression obtained by him when it comes to a plasma. The starting point was the expression for the energy of the field in a dispersive transparent medium[2]:

$$U=\frac{1}{8\pi}\frac{d(\omega\varepsilon)}{d\omega}\langle E^2\rangle. \tag{1}$$

We have retained here only the term with the electric field, and the angle brackets denote averaging over the fast oscillations. Repeating the derivations that lead to expression (1), but assuming that the medium is absorbing, we get (we disregard the terms with the magnetic field)

$$-\left\langle\frac{d}{dt}\frac{E^2}{8\pi}+\operatorname{div}\mathbf{S}\right\rangle=\frac{1}{4\pi}\left\langle\mathbf{E}\frac{d\mathbf{D}}{dt}\right\rangle$$
$$=\frac{\omega}{4\pi}\varepsilon''\langle E^2\rangle+\frac{d}{dt}\left\{\frac{1}{8\pi}\frac{d[\omega(\varepsilon'-1)]}{d\omega}\langle E^2\rangle\right\}. \tag{2}$$

Here S is the Poynting vector, and $\epsilon = \epsilon' + i\epsilon''$ is the dielectric constant (the term containing $\epsilon'' d/dt$ has been discarded). Consequently, the change of the energy of matter in the field per unit time can be regarded as equal to the second term of the right hand side of (2) only if the characteristic time t_1 of the variation of $\langle E^2\rangle$ satisfies the condition

$$t_1\ll\frac{1}{\omega\varepsilon''}\frac{d[\omega(\varepsilon'-1)]}{d\omega}. \tag{3}$$

If $\langle E^2\rangle$ varies more slowly than is allowed by the condition (3), then the change of the field energy is not determined by the second term of the right hand side of (2), and the expression for the field energy (1) is meaningless. For a plasma, the criterion (3) is of the form

$$t_1\ll 1/\nu,$$

where ν is the collision frequency. Thus, expression (1), together with Pitaevskiĭ's result[1], is valid in the case of a plasma if the characteristic time of variation of the field is much shorter than the free-path time.

In the present paper we consider the opposite limiting case and assume, in addition, that the characteristic dimension of the inhomogeneity of the field is much larger than the free path length. Under these conditions we can solve the kinetic equation for the plasma particles by a method analogous to the Chapman-Enskog method[3]. It turns out that our result differs from Pitaevskiĭ's even when $\omega \gg \nu$, and depends strongly on the form of the differential scattering cross section (but not on its magnitude).

We note that we use here the temperature T in its kinetic definition (so that 3T/2 is the average kinetic energy of disordered motion of one particle). It is precisely in terms of this quantity that we express the stress tensor. Introduction of the temperature by strictly thermodynamic means is impossible, since the absorbing medium is not in an equilibrium state in the electromagnetic field. We note, however, that a redefinition of the temperature (and consequently of the pressure) can change the form of only that part of the stress tensor which is proportional to $\delta_{\alpha\beta}$.

1. FUNDAMENTAL EQUATIONS

We start from the kinetic equations for the different species of particles (electrons, ions, atoms):

$$\frac{\partial f^a}{\partial t}+v_\alpha\frac{\partial f^a}{\partial x_\alpha}+\frac{e_a}{m_a}\left\{\tilde{E}_\alpha+\frac{1}{c}[\mathbf{v}\tilde{\mathbf{H}}]_\alpha\right\}\frac{\partial f^a}{\partial v_\alpha}=\sum_b S_{ab}(f^a,f^b). \tag{4*}$$

Here $f^a(\mathbf{r}, \mathbf{v}, t)$—distribution function of the particles of species a, e_a—particle charge, m_a—particle mass, S_{ab}—collision integral:

$$S_{ab}(f^a,f^b)=\int[f^a(\mathbf{v}')f^b(\mathbf{v}_1')-f^a(\mathbf{v})f^b(\mathbf{v}_1)]$$
$$\times\,\sigma(|\mathbf{v}-\mathbf{v}_1|,\vartheta)\,|\mathbf{v}-\mathbf{v}_1|\,d^3v_1\,d\Omega; \tag{5}$$

σ—differential cross section for the collision of particles a and b; $\mathbf{v}$, $\mathbf{v}_1$ and $\mathbf{v}'$, $\mathbf{v}_1'$—velocities of these particles before and after collision; $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$—intensities of the high-frequency electric and magnetic fields, which we represent in the form

*$[\mathbf{vH}] = \mathbf{v} \times \mathbf{H}$.

$$\widetilde{\mathbf{E}} = \mathrm{Re}\,\mathbf{E}e^{-i\omega t}, \quad \widetilde{\mathbf{H}} = \mathrm{Re}\,\mathbf{H}e^{-i\omega t}. \tag{6}$$

Here E and H are the amplitudes of the fields, which depend little on the coordinates and on the time. Summation over the repeated Greek indices is implied in (4) and further.

The distribution functions will be sought in the form of their expansion in harmonics of the frequency ω:

$$f^a = f_0^a + {}^1\!/_2[f_1^a e^{-i\omega t} + f_1^{a*} e^{i\omega t}] + \ldots \tag{7}$$

It is assumed that f_0^a and f_1^a depend little on the coordinates and on the time. We substitute f_1^a, determined by formula (7), into the system (4). Averaging the obtained equations over the fast oscillations, we get

$$\frac{\partial f_0^a}{\partial t} + v_\alpha \frac{\partial f_0^a}{\partial x_\alpha} + \frac{1}{2}\mathrm{Re}\left\{\frac{e_a}{m_a}\left(E_\alpha^* + \frac{1}{c}[\mathbf{v}\mathbf{H}^*]_\alpha\right)\frac{\partial f_1^a}{\partial v_\alpha}\right\} = \sum_b S_{ab}(f_0^a, f_0^b) + \frac{1}{2}\mathrm{Re}\sum_b S_{ab}(f_1^a, f_1^{b*}). \tag{8}$$

Equations (8) determine s_0^a (the values of the functions f^a averaged over the period of the high-frequency field) accurate to E^2.

We shall solve these equations by a method analogous to the Chapman-Enskog method[3]. A criterion for the applicability of the employed approximation will be discussed below. In the zeroth approximation, the system (8) reduces to the conditions

$$\sum_b S_{ab}(f_0^a, f_0^b) = 0, \tag{9}$$

which are satisfied by Maxwellian distribution functions. Accordingly, we put

$$f_0^a = \Phi^a(w) + \psi^a(\mathbf{w}), \tag{10}$$

where $\mathbf{w} = \mathbf{v} - \mathbf{u}$, u—hydrodynamic plasma velocity, w—random particle velocity;

$$\Phi^a(w) = n_a\left(\frac{m_a}{2\pi T}\right)^{3/2}\exp\left\{-\frac{m_a w^2}{2T}\right\} \tag{11}$$

—Maxwellian distribution functions of the particles, ψ^a—corrections to them, and T—temperature in energy units, which is the same for all species of particles.

It is assumed that the parameters n_a, T, and u depend slowly on the coordinates and on the time. We impose on the function ψ^a the additional conditions:

$$\int \psi^a d^3w = 0, \quad \sum_a \int \psi^a m_a w_\alpha d^3w = 0, \quad \sum_a \frac{m_a}{2}\int \psi^a w^2 d^3w = 0, \tag{12}$$

which mean that the functions ψ^a make no contribution to the particle concentration, the total momentum, and the total energy per unit volume of the plasma.

Multiplying (8) by m_a, and $m_a v_\alpha$, integrating over the velocities, and summing over the particle species, we obtain in the usual manner the equation of continuity and the equations of motion

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_\alpha}(\rho u_\alpha) = 0, \tag{13}$$

$$\rho\frac{du_\alpha}{dt} = \frac{\partial P_{\alpha\beta}}{\partial x_\beta}, \tag{14}$$

where

$$P_{\alpha\beta} = -p\delta_{\alpha\beta} + T_{\alpha\beta} + T_{\alpha\beta}^{(0)} \tag{15}$$

is the total stress tensor, $P_{\alpha\beta} = P_{\beta\alpha}$, and

$$T_{\alpha\beta}^{(0)} = \frac{1}{16\pi}[-|E|^2\delta_{\alpha\beta} + E_\alpha^* E_\beta + E_\alpha E_\beta^* - |H|^2\delta_{\alpha\beta} + H_\alpha^* H_\beta + H_\alpha H_\beta^*] \tag{16}$$

is the Maxwellian stress tensor; $p = \Sigma_a n_a T$ is the total pressure; $\rho = \Sigma_a m_a n_a$ is the density; $d/dt = \partial/\partial t + u_\alpha \partial/\partial x_\alpha$. The tensor $T_{\alpha\beta}$ is expressed in terms of the functions ψ^a as follows:

$$T_{\alpha\beta} = -\sum_a m_a \int \psi^a w_\alpha w_\beta d^3w. \tag{17}$$

If we confine ourselves in the equation of motion to terms that are linear in the gradients, then the functions ψ^a must be sought without account of the gradients. In this approximation, using formulas (4), (7), (8) and (10), we obtain equations for f_1^a and ψ^a:

$$-i\omega f_1^a + \frac{e_a E_\alpha}{m_a}\frac{\partial \Phi^a}{\partial w_\alpha} = \sum_b [S_{ab}(\Phi^a, f_1^b) + S_{ab}(f_1^a, \Phi^b)], \tag{18}$$

$$\frac{\partial \Phi^a}{\partial t} + \frac{1}{2}\frac{e_a}{m_a}\mathrm{Re}\,E_\alpha^*\frac{\partial f_1^a}{\partial w_\alpha} - \frac{1}{2}\mathrm{Re}\sum_b S_{ab}(f_1^a, f_1^{b*}) = \sum_b [S_{ab}(\psi^a, \Phi^b) + S_{ab}(\Phi^a, \psi^b)]. \tag{19}$$

In (18) we used in lieu of the functions f_0^a their zeroth approximations Φ^a. This can be done because the functions f_1^a are needed in the first approximation in E. In the left side of (19) we have left out the term $\partial\psi^a/\partial t$. This is equivalent to assuming that all the changes in the system during the free-path time are small.

The conditions for the solvability of the system (19) lead in the usual manner to the zeroth-approximation hydrodynamic equations

$$\frac{\partial n_a}{\partial t} = 0, \quad \frac{\partial}{\partial t}\sum_a m_a n_a \mathbf{u} = 0, \quad \frac{\partial}{\partial t}\sum_a \frac{3}{2}n_a T = \frac{1}{2}\mathrm{Re}\,\mathbf{j}\mathbf{E}^*, \tag{20}$$

where

$$\mathbf{j} = \sum_a e_a \int f_1^a \mathbf{w} d^3w. \tag{21}$$

Accordingly, the quantities $\partial\Phi^a/\partial t$ in (19) should be written in the form

$$\frac{\partial \Phi^a}{\partial t} = \frac{\partial \Phi^a}{\partial T}\cdot\frac{1}{3}\left(\sum_a n_a\right)^{-1}\mathrm{Re}\,\mathbf{j}\mathbf{E}^*. \tag{22}$$

The solutions of (19) can be sought in the form

$$\psi^a = G_{\alpha\beta}^a(w^2)w_{\alpha\beta} + F^a(w^2) + c_1^a\Phi^a + c_{2\alpha}w_\alpha m_a\Phi^a + c_3 w^2 m_a\Phi^a, \tag{23}$$

where

$$w_{\alpha\beta} = w_\alpha w_\beta - {}^1\!/_3 w^2\delta_{\alpha\beta}. \tag{24}$$

The functions $G_{\alpha\beta}^a$ and F^a should be determined from (19), and the constants c_1, $c_{2\alpha}$, and c_3 should be determined from the supplementary conditions (12).

To determine the stress tensor we need only the functions $G_{\alpha\beta}^a$. In fact, according to (15), (17), and (12), we can write

$$T_{\alpha\beta} = -\sum_a m_a \int \psi^a w_{\alpha\beta}\,d^3w = -\sum_a m_a \int G_{\gamma\delta}^a w_{\gamma\delta} w_{\alpha\beta}\,d^3w. \tag{25}$$

We shall determine below the functions $G_{\alpha\beta}^a$ and the stress tensor for certain particular cases.

2. STRESS TENSOR FOR WEAKLY IONIZED PLASMA

In a weakly ionized plasma it is possible to neglect the collisions of the charged particles with one another. The deviations from Maxwellian distribution for the heavy particles are much smaller than for electrons, and they can be neglected. Then Eq. (18) for the electrons takes the form

$$-i\omega f_1^e + \frac{e}{m} E_\alpha \frac{\partial \Phi^e}{\partial w_\alpha} = S_{ea}(f_1^e, \Phi^a). \tag{26}$$

The index e pertains here to electrons and the index a to atoms; e and m are the electron charge and mass. We seek f_1^e in the form $f_1^e = E_\alpha w_\alpha \varphi(w)$. Accurate to terms of zero order with respect to m/M (M—mass of the atom), we can write

$$S_{ea}(f_1^e, \Phi^a) = -\nu(w) f_1^e. \tag{27}$$

Here

$$\nu(w) = n_a w \int \sigma(w, \vartheta)(1 - \cos\vartheta)\, d\Omega \tag{28}$$

is the frequency of the electron-atom collisions. Now Eq. (26) yields

$$f_1^e = -\frac{e}{m} \frac{1}{\nu - i\omega} E_\alpha \frac{\partial \Phi^a}{\partial w_\alpha}. \tag{29}$$

We turn now to Eq. (19). Here, too, we can neglect all the deviations of the atomic distribution function from equilibrium. In addition, we can resolve the inhomogeneity into two components—one containing the tensor $w_{\alpha\beta}$ with zero trace, and one depending only on the modulus of the velocity. Accordingly, the function ψ^e can be sought in the form

$$\psi^e = \psi_1^e + \psi_2^e, \tag{30}$$

where ψ_1^e is proportional to the tensor $w_{\alpha\beta}$, and ψ_2^e depends only on the modulus of the velocity. Then the equation for ψ_1^e takes the form

$$-\frac{1}{2} \frac{e^2}{m^2} \operatorname{Re} \left\{ E_\alpha^* E_\beta \left[\frac{\partial}{\partial w_\alpha} \left(\frac{1}{\nu - i\omega} \frac{\partial \Phi^e}{\partial w_\beta} \right) - \frac{1}{3} \delta_{\alpha\beta} \frac{\partial}{\partial w_\gamma} \left(\frac{1}{\nu - i\omega} \frac{\partial \Phi^e}{\partial w_\gamma} \right) \right] \right\} = S_{ea}(\psi_1^e, \Phi^a). \tag{31}$$

Assuming that ψ_1^e is proportional to $w_{\alpha\beta}$, we can easily obtain, in analogy with formula (27),

$$S_{ea}(\psi_1^e, \Phi^a) = -\nu_1^*(w) \psi_1^e, \tag{32}$$

where

$$\nu_1^*(w) = {}^3/_2 n_a w \int \sigma(w, \vartheta) \sin^2\vartheta\, d\Omega. \tag{33}$$

Thus

$$\psi_1^e = -\frac{e^2}{4mT} \frac{1}{\nu_1^*} \frac{w_{\alpha\beta}}{w} B_{\alpha\beta} \frac{\partial}{\partial w} \left(\frac{\nu \Phi^e}{\nu^2 + \omega^2} \right), \tag{34}$$

where

$$B_{\alpha\beta} = E_\alpha^* E_\beta + E_\alpha E_\beta^*. \tag{35}$$

The tensor $T_{\alpha\beta}$ is obtained with the aid of formula (25). Using, in addition, the relation

$$\frac{1}{4\pi} \int B_{\alpha\beta} w_{\alpha\beta} w_{\gamma\delta}\, d\Omega = \frac{2}{15} \left(B_{\gamma\delta} - \frac{1}{3} \delta_{\gamma\delta} B_{\alpha\alpha} \right) w^4, \tag{36}$$

we obtain after integrating by parts

$$T_{\alpha\beta} = -\frac{e^2}{30T} \left(B_{\alpha\beta} - \frac{1}{3} \delta_{\alpha\beta} B_{\gamma\gamma} \right) \int \Phi^e \frac{\nu}{\nu^2 + \omega^2} \frac{1}{w^2} \frac{\partial}{\partial w} \left(\frac{w^5}{\nu^*} \right) d^3 w. \tag{37}$$

From (37) we see that even at low pressures, when $\nu \ll \omega$, the stress tensor depends strongly on the character of the collisions between the electrons and the atoms. Using Eq. (18) written out for atoms, as well as expression (29), we can find the function f_1^a and estimate the term $S_{ea}(\Phi^e, f_1^a)$ which was discarded in (26). It turns out that it differs from the retained term $S_{ea}(f_1^e, \Phi^a)$ by a factor $\alpha/(i\omega - \alpha)$ where $\alpha = mn_e\nu/Mn_a$. Thus, our results become incorrect only at such low frequencies that the condition $\omega \gg \alpha$ is no longer satisfied. The terms connected with the deviation of the atomic distribution function from Maxwellian in the equation for ψ^e are smaller in the ratio $(n_e\nu/n_a\nu_a)(m/M)^2$, where ν_a is the frequency of the collisions of the atoms with each other.

So far we have assumed that the temperature of the electrons and of the atoms are the same. There is no need, however, to make this assumption. If the electrons are heated by the field, then the electron distribution function should be sought in the form

$$f_0^e = f_{00} + \psi_1^e,$$

where f_{00} is the isotropic part of the distribution function, which is determined, with allowance for the electron heating, and ψ^e is proportional to the tensor $w_{\alpha\beta}$. Then the expressions (34) for ψ_1^e and the (37) for the tensor $T_{\alpha\beta}$ would retain the same form, the only difference being that they would contain f_{00} in lieu of Φ^e. We note that the determination of the isotropic part of the distribution function in a heating field has been the subject of a number of papers[4]. If the electron temperature is much higher than the temperature of the heavy particles, then ψ_1^e is small compared with f_{00}^e in a ratio m/M. It should be born in mind that in this case the electron pressure p_e, which enters in expression (15) for the total stress tensor, is given by

$$p_e = {}^1/_3 \int f_{00} m w^2\, d^3 w = n_e T_e. \tag{38}$$

3. STRESS TENSOR FOR A PLASMA AT LOW PRESSURES

In the case of low pressures, when $\omega\tau \gg 1$ (τ—time between collisions), we can solve the equation for f_1^a (18) by iteration, assuming the collision integrals to be small. Then we obtain in first approximation

$$f_1^a = f_1^{a0} + \frac{1}{i\omega} \sum_b [S_{ab}(f_1^{a0}, \Phi^b) + S_{ab}(\Phi^a, f_1^{b0})], \tag{39}$$

where

$$f_1^{a0} = \frac{1}{i\omega} \frac{e_a}{m_a} E_\alpha \frac{\partial \Phi^a}{\partial w_\alpha}. \tag{40}$$

The zeroth approximation f_1^{a0} makes no contribution to the second term of the left side of (19), and in the third term for f_1^a we can confine ourselves only to the zeroth approximation. We then obtain in lieu of (19)

$$\frac{\partial \Phi^a}{\partial t} + \frac{1}{2} \frac{e_a}{m_a} \operatorname{Re} \frac{1}{i\omega} E_\alpha^* \sum_b \frac{\partial}{\partial w_\alpha} [S_{ab}(\Phi^a, f_1^{b0}) + S_{ab}(f_1^{a0} \Phi^b)] - \frac{1}{2} \operatorname{Re} \sum_b S_{ab}(f_1^{a0}, f_1^{b0*}) = \sum_b [S_{ab}(\psi^a, \Phi^b) + S_{ab}(\Phi^a, \psi^b)]. \tag{41}$$

The functions ψ^a can be sought in the form (23), and we put

$$G_{\alpha\beta}{}^a = \frac{1}{8\omega^2}\frac{e^2}{T^2}B_{\alpha\beta}\Phi^a\left(\frac{e_a{}^2}{e^2}+\chi^a\right), \tag{42}$$

where χ^a is a new unknown function. We chose this form of $G^a_{\alpha\beta}$ because $\chi^a = 0$ corresponds to the result of Pitaevskiĭ[1] (provided the temperature is suitably redefined).

In order to simplify the equations for χ^a, it is convenient to use the identities

$$\frac{\partial}{\partial v_\alpha}S_{ab}(\varphi^a,\varphi^b) = S_{ab}\left(\frac{\partial\varphi^a}{\partial v_\alpha},\varphi^b\right)+S_{ab}\left(\varphi^a,\frac{\partial\varphi^b}{\partial v_\alpha}\right) \tag{43}$$

$$S_{ab}\left(\Phi^a,\frac{\partial\Phi^b}{\partial v_\alpha}\right)+S_{ab}\left(\frac{\partial\Phi^a}{\partial v_\alpha},\Phi^b\right)=0, \tag{44}$$

$$S_{ab}\left(\frac{\partial^2\Phi^a}{\partial v_\alpha\partial v_\beta},\Phi^b\right)+S_{ab}\left(\frac{\partial\Phi^a}{\partial v_\alpha},\frac{\partial\Phi^b}{\partial v_\beta}\right) + S_{ab}\left(\frac{\partial\Phi^a}{\partial v_\beta},\frac{\partial\Phi^b}{\partial v_\alpha}\right)+S_{ab}\left(\Phi^a,\frac{\partial^2\Phi^b}{\partial v_\alpha\partial v_\beta}\right)=0. \tag{45}$$

The first of these identities, in which φ^a and φ^b are arbitrary functions of the velocity, follows from the invariance of the collision integral with respect to the Galilean transformations[1)]. The other two identities are obtained by differentiating the equation $S_{ab}(\Phi^a, \Phi^b) = 0$. Using formulas (39) and (42) and the foregoing identities, we transform the system (41) into

$$-\sum_b\left(\frac{e_a}{m_a}-\frac{e_b}{m_b}\right)^2\frac{m_b{}^2}{e^2}S_{ab}(\Phi^a,\Phi^b w_{\alpha\beta}) = \sum_b[S_{ab}(\Phi^a,\Phi^b\chi^b w_{\alpha\beta})+S_{ab}(\Phi^a\chi^a w_{\alpha\beta},\Phi^b)]. \tag{46}$$

For simplicity we confine ourselves here to the case of a fully ionized plasma. We use the indices e and p for electrons and ions, respectively. Then, neglecting the collision integral, which contains χ^p (it can be shown that it contains the electron/ion mass ratio m/M), we obtain the following equation for χ^e:

$$-\frac{M^2}{m^2}S_{ep}(\Phi^e,\Phi^p w_{\alpha\beta}) = S_{ep}(\Phi^e\chi^e w_{\alpha\beta},\Phi^p) + S_{ee}(\Phi^e,\Phi^e\chi^e w^{\alpha\beta})+S_{ee}(\Phi^e\chi^e w_{\alpha\beta},\Phi^e). \tag{47}$$

Neglecting terms containing the ratio m/M, we can obtain

$$S_{ep}(\Phi^e\chi^e w_{\alpha\beta},\Phi^p) = -\nu_1{}^* w_{\alpha\beta}\Phi^e\chi^e, \tag{48}$$

$$\left(\frac{M}{m}\right)^2 S_{ep}(\Phi^e,\Phi^p w_{\alpha\beta}) = -w_{\alpha\beta}\Phi^e\left[\nu_1{}^*-2\nu_1+2\frac{T}{m}\frac{1}{w}\frac{\partial\nu_1}{\partial w}\right], \tag{49}$$

where ν_1 and ν_1^* are determined by formulas (28) and (32), in which σ should be taken to mean the electron-ion collision cross section, and n_a should be replaced by the ion concentration n_p. In the case under consideration it is necessary to choose for σ the Rutherford cross section and to employ the usual procedure of cutting off the collision integral at small scattering angles. We can then obtain

$$\nu_1 = 4\pi n_p w\left(\frac{Ze^2}{mw^2}\right)^2\lambda, \qquad \nu_1{}^* = 3\nu_1, \tag{50}$$

where Ze is the ion charge and λ is the Coulomb logarithm.

The simplest result is obtained in the case of the so-called Lorentz gas, when the electron-electron collisions are neglected (this can be done, strictly speaking, if Z is sufficiently large). In this case we get

$$\chi^e = -1+\frac{2}{\nu_1{}^*}\left(\nu_1-\frac{T}{m}\frac{1}{w}\frac{\partial\nu_1}{dw}\right), \tag{51}$$

and we obtain for the tensor $T_{\alpha\beta}$ the expression

$$T_{\alpha\beta} = -\frac{e^2n_e}{4m\omega^2}\left(B_{\alpha\beta}-\frac{1}{3}\delta_{\alpha\beta}B_{\gamma\gamma}\right)\cdot\frac{16}{15}. \tag{52}$$

This expression differs only by a factor 16/15 from the expression that follows from the Pitaevskiĭ formulas (if we redefine the temperature in such a manner that the term proportional to $\delta_{\alpha\beta}$ is included in the pressure).

At small values of Z, when it is necessary to take into account the electron-electron collisions, Eq. (47) can be solved only numerically. We have performed such a calculation for the case Z = 1 by the Chapman-Enskog method. The function χ^e was expanded in Laguerre polynomials

$$\chi^e = \sum_{k=0}^{\infty} a_k L_k{}^{5/2}\left(\frac{mw^2}{2T}\right). \tag{53}$$

To determine the coefficients a_k, Eq. (47) was multiplied by $L_n^{5/2}(mw^2/2T)w_{\alpha\beta}$, and integrated with respect to the velocities. The obtained infinite system of algebraic equations for the coefficients a_k was solved by successive approximations. The n-th approximation corresponds to retaining n + 1 terms in the expansion (53). In the first approximation the result is $a_0 = 0.39$, in the second $a_0 = 0.10$, and in the third $a_0 = 0.12$. The fourth and fifth approximations do not differ, at the indicated accuracy, from the third, we therefore used $a_0 = 0.12$. With this value, the tensor $T_{\alpha\beta}$ is given by

$$T_{\alpha\beta} = -\frac{e^2n_e}{4m\omega^2}\left(B_{\alpha\beta}-\frac{1}{3}\delta_{\alpha\beta}B_{\gamma\gamma}\right)\cdot 1{,}12. \tag{54}$$

One of the main assumptions used above was that the change of the field is small during the free-path time. Otherwise, collisions can be neglected. Then the electron kinetic equation in the zeroth approximation with respect to the gradients is

$$\frac{\partial f}{\partial t}+\frac{e}{m}\tilde{E}_\alpha\frac{\partial f}{\partial v_\alpha}=0.$$

An exact solution of this equation is

$$f = \mathcal{F}(\mathbf{r},\mathbf{v}-\mathbf{V}(t)),$$

where V(t) satisfies the equation

$$m\frac{d\mathbf{V}}{dt} = e\tilde{\mathbf{E}},$$

and $\mathcal{F}$ is an arbitrary function. Using formula (6) for $\tilde{\mathbf{E}}$, we get

$$\mathbf{V} = \mathrm{Re}\frac{e\mathbf{E}}{-i\omega m}e^{-i\omega t}.$$

In order to eliminate the arbitrariness in the choice of

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1) This property can be readily obtained also from the explicit form of the collision integral (5). Replacing v by v + δv and the integration variable $\mathbf{v}_1$ by $\tilde{\mathbf{v}}_1-\delta\mathbf{v}$, we get

$$S_{ab}(\mathbf{v}+\delta\mathbf{v}) = \int[\varphi^a(\tilde{\mathbf{v}}'+\delta\mathbf{v})\varphi^b(\tilde{\mathbf{v}}_1'+\delta\mathbf{v})-\varphi^a(\mathbf{v}+\delta\mathbf{v})\varphi^b(\tilde{\mathbf{v}}_1+\delta\mathbf{v})] \times\sigma(|\mathbf{v}-\tilde{\mathbf{v}}_1|,\vartheta)|\mathbf{v}-\tilde{\mathbf{v}}_1|d^3\tilde{\mathbf{v}}_1 d\Omega,$$

where $\tilde{\mathbf{v}}'$ and $\tilde{\mathbf{v}}'_1$ are the velocities of the particles after the collision, if the velocities were v and $\tilde{\mathbf{v}}_1$ before the collision. Expanding both parts in terms of δv, we obtain relation (43).

the function $\mathcal{F}$, we shall assume that the plasma was at equilibrium at $t \to -\infty$, and then the field $\tilde{E}$ was adiabatically turned on.

Thus, we must put

$$f = \Phi(\mathbf{w} - \mathbf{V}(t)), \quad \Phi(w) = n\left(\frac{m}{2\pi T_0}\right)^{3/2} \exp\left(-\frac{mw^2}{2T_0}\right)$$

and assume that ω has a small imaginary part $i\delta$, where δ is the adiabatic-switching parameter. Hence

$$-p_e\delta_{\alpha\beta} + T_{\alpha\beta} = -m\int f w_\alpha w_\beta\, d^3w = -nT_0\delta_{\alpha\beta} + \frac{\varepsilon - 1}{16\pi} B_{\alpha\beta}, \quad \varepsilon - 1 = -\frac{4\pi ne^2}{m\omega^2}, \qquad (55)$$

which coincides with the result of Pitaevskiĭ. Here T_0 is the temperature prevailing prior to the adiabatic switching on of the field. It is connected with the kinetic temperature

$$T = \left\langle \frac{2}{3n}\int \frac{mw^2}{2} f\, d^3w \right\rangle$$

by the relation

$$T = T_0 + \frac{e^2}{12m\omega^2} B_{\gamma\gamma}.$$

Formula (55) can be written in a form that is convenient for a comparison with formulas (52) and (54):

$$-p_e\delta_{\alpha\beta} + T_{\alpha\beta} = -nT\delta_{\alpha\beta} - \left(B_{\alpha\beta} - \frac{1}{3}\delta_{\alpha\beta}B_{\gamma\gamma}\right)\frac{ne^2}{4m\omega^2}.$$

If we retain in the left side of (19) the term $\partial\psi^a/\partial t$ and use the fact that in the case of adiabatic switching we have

$$\partial\psi^a / \partial t = 2\delta\psi^a,$$

then we can verify that the collision integrals can be neglected only if $\delta \gg \nu$.

We note in conclusion that the nondiagonal part of the average electromagnetic stress tensor $T_{\alpha\beta} + T^0_{\alpha\beta}$ with allowance for collisions does not have automatically the property of continuity of the normal components (unlike the case when the tensor $T_{\alpha\beta}$ is determined by formula (55)). This means that the plasma cannot be at rest on the interface between two media if the field is not perpendicular to the boundary and is not parallel to it. Continuity of the normal components of the total stress tensor should be ensured by the viscosity.

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