

EXCITATION OF TRANSVERSE ZERO SOUND IN LIQUID He³

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The problem of the excitation of the transverse mode of zero sound during tangential oscillations of the boundary between liquid He³ and a solid is solved. The energy flux is calculated under conditions of totally diffuse reflection of the quasiparticles from the boundary. The magnitude of the sound damping is found and the conditions under which it may be observed are considered.

AS Landau has shown^[1], zero sound of different types can propagate in a Fermi liquid. It is customary to denote the departure of the distribution function of quasiparticles from the equilibrium value n_0 by δn , and separate the angular part of this deviation $\nu(\theta, \chi)$: $\delta n = \nu(\theta, \chi) \cdot \partial n_0 / \partial \epsilon$. Then, in the various sounds, $\nu \propto \exp(\pm im\chi)$ ($m = 0, 1, 2, \dots$). Sound with $m = 0$ was recently observed experimentally;^[3] for its excitation, the plane in contact with the liquid He³ was vibrated along the normal to the boundary. Sound of the type $m = 1$ can also exist in He³. It is natural to call this sound transverse; however, it has not yet been observed. It is shown in the present work that for tangential oscillation of the boundary of a solid with liquid He³, transverse oscillation of considerable intensity can be excited. Conditions are discussed under which such a sound could in principle be observed.

We shall write the function $F(\varphi)$, which describes the interaction of quasiparticles in the Landau theory, in the form $F(\varphi) = F_0 + F_1 \cos \varphi$; in this approximation, we get for a transverse sound wave of specific polarization^[1]

$$\nu(\theta, \chi) = A \frac{\sin \theta \cos \theta}{\eta - \cos \theta} \cos \chi, \tag{1}$$

the angle θ here is measured from the direction of propagation of the wave, χ is the azimuthal angle, the factor $\exp i(\mathbf{k} \cdot \mathbf{r} - \omega t)$ is omitted; the constant A plays the role of the amplitude of the wave, while η is the wave velocity in units of the Fermi velocity v . It is determined from the equation

$$(\eta^2 - 1)w(\eta) = \frac{F_1 - 6}{3F_1}; \quad w(\eta) = \frac{\eta}{2} \ln \frac{\eta + 1}{\eta - 1} - 1. \tag{2}$$

Equation (2) has real roots when the right side is positive. According to the latest data,^[3] $F_1 = 6.25$ for a pressure of 0.28 atm and 14.4 for 27 atm. Transverse sound can exist throughout this interval. In what follows, we shall be interested in the energy flux in the wave (1), the derivation of the expression for which is given in the Appendix. The direction of the flux is the same as the direction of propagation of the wave; its magnitude is

$$Q = \frac{2\pi p_0^2 A^2}{(2\pi\hbar)^3} \eta [(3\eta^2 - 1)w - 1], \tag{3}$$

p_0 is the Fermi momentum.

We shall now solve the problem of the excitation produced in He³ by a tangentially vibrating solid. Such a problem has already been considered by Bekarevich and Khalatnikov in connection with the construction of the

theory for the Kapitza discontinuity on the boundary between He³ and a solid;^[4] however, the value of $F_1 = 1.3$ assumed at that time led to the impossibility of the existence of transverse zero sound. Moreover, in contrast with^[4], we shall not be interested in the total energy flux from the wall to the He³, but only in that part of it due to the sound.

Let the z axis be perpendicular to the boundary between He³ and the solid, and let the oscillations take place along the x axis with the speed $u = u_0 e^{-i\omega t}$. The kinetic equation for the function $\nu(\theta, \chi, z)$ is then written as:^[4]

$$-i\omega\nu + \nu \cos \theta \frac{\partial}{\partial z} \left[v + F_0 \int \frac{v' d\theta'}{4\pi} + F_1 \int v' \cos \theta' \frac{d\theta'}{4\pi} \right] = I(\nu), \tag{4}$$

$$\cos \theta = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\chi - \chi').$$

We shall express the collision integral in a simplified form which, however, guarantees the satisfaction of the conservation laws; this allows us to find subsequently the value of the sound damping:

$$I(\nu) = -\frac{1}{\tau} (\nu - \bar{\nu} - 3\nu \cos \theta \cos \theta$$

$$- 3\nu \sin \theta \cos \chi \sin \theta \cos \chi - 3\nu \sin \theta \sin \chi \sin \theta \sin \chi).$$

The bar here indicates averaging over the angle; τ is a constant having the meaning of the average time between collisions of the quasiparticle. It is convenient to introduce the function

$$\varphi = \nu + F_0 \int \frac{v' d\theta'}{4\pi} + F_1 \int v' \cos \theta' \frac{d\theta'}{4\pi}. \tag{6}$$

We shall assume the reflection on the boundary to be diffuse, which leads to the condition^[4]

$$\varphi(z=0) \equiv \Psi(\theta, \chi) = -p_0 u \sin \theta \cos \chi, \quad 0 < \theta < \pi/2. \tag{7}$$

The form of (4)-(6) is such that for the boundary condition (7), $\varphi \sim \cos \chi$ and $\nu \sim \cos \chi$; therefore, $\bar{\nu} = 0$, $\nu \cos \theta = 0$, $\nu \sin \theta \sin \chi = 0$, with account of what has been pointed out, the equation for φ will be

$$(1 - i\omega\tau)\varphi + \cos \theta \frac{\partial \varphi}{\partial z'} = 3 \sin \theta \cos \chi \left[1 - \frac{i\omega\tau F_1}{3 + F_1} \right] \varphi_{11}. \tag{8}$$

For brevity in writing, we introduce the notation $\varphi_{11} = \varphi \sin \theta \cos \chi$ and $z' = z/v\tau$. Multiplying (8) by $e^{-sz'}$ and integrating over z' , we obtain the Laplace transforms

$$\varphi(\theta, \chi, s) = \int_0^\infty \varphi(\theta, \chi, z') e^{-sz'} dz'; \quad \varphi_{11}(s) = \int_0^\infty \varphi_{11}(z') e^{-sz'} dz', \tag{9}$$

$$(s \cos \theta + 1 - i\omega\tau)\varphi - \Psi(\theta, \chi) \cos \theta = 3 \left[1 - \frac{i\omega\tau F_1}{3 + F_1} \right] \varphi_{11} \sin \theta \cos \chi. \tag{10}$$

In order to obtain an equation for φ_{1t} , we multiply (10) by $\sin \theta \cos \chi / (s \cos \theta + 1 - i\omega\tau)$ and integrate over the solid angle. We get, with account of (7),

$$\varphi_{1t}\Delta(s) = \frac{p_0 u}{2s} \left\{ \frac{1}{3} - \left[\left(\frac{i\omega\tau - 1}{s} \right)^2 - 1 \right] w \left(\frac{i\omega\tau - 1}{s} \right) \right\} \quad (11)$$

$$+ \int_0^{2\pi} d\chi \int_{\pi/2}^{\pi} \frac{\sin^2 \theta \cos \theta \cos \chi [\psi(\theta, \chi) + p_0 u \sin \theta \cos \chi] d\theta}{s \cos \theta + 1 - i\omega\tau}.$$

Here,

$$\Delta(s) = 1 - \frac{3}{2(i\omega\tau - 1)} \left(1 - \frac{i\omega\tau F_1}{3 + F_1} \right) \left\{ \left[\left(\frac{i\omega\tau - 1}{s} \right)^2 - 1 \right] \times w \left(\frac{i\omega\tau - 1}{s} \right) - 1 \right\}. \quad (12)$$

Equation (11) differs from Eq. (2.4) of [4] in that $\Delta(s)$ now has roots, which we denote by $\pm s_0$ ($\text{Re } s_0 > 0$); therefore, in the solution of (11), in addition to $\Delta(s)$, we must bring in the function $G(s)$ according to the formula

$$\Delta(s) = \frac{s^2 - s_0^2}{s^2 - (1 - i\omega\tau)^2} G(s). \quad (13)$$

In the remaining discussion, we repeat the corresponding division of [4]; we shall only put down the final result:

$$\varphi_{1t}(s) = -\frac{p_0 u}{3s} \frac{3 + F_1}{F_1 + 3/(1 - i\omega\tau)} \left[1 - \frac{s_0(s + 1 - i\omega\tau)}{(s + s_0)(1 - i\omega\tau)} \frac{g_-(s)}{g_-(s_0)} \right]. \quad (14)$$

The function $g_-(s)$ is determined, together with the function $g_+(s)$, by the relation

$$g_{\pm}(s) = \exp \left\{ \frac{1}{2\pi i} \int_{\pm\beta - i\infty}^{\pm\beta + i\infty} \frac{\ln G(u)}{u - s} du \right\}, \quad 0 < \beta < 1; \quad (15)$$

$g_+(s)$ is analytic for $\text{Re } s < \beta$ and $g_-(s)$ for $\text{Re } s > -\beta$. In the region $-\beta < \text{Re } s < \beta$,

$$G(s) = g_+(s) / g_-(s). \quad (16)$$

Up to this point, we have made no assumptions on the value of $\omega\tau$. In what follows, with the exception of the case specially discussed, we shall assume that $\omega\tau \gg 1$ and carry out the corresponding simplifications.

Knowledge of the Laplace transform $\varphi_{1t}(s)$ allows us to determine the asymptotic behavior of $\varphi_{1t}(z')$ for large z . For this purpose, we use the formula for the inverse Laplace transform

$$\varphi_{1t}(z') = \frac{1}{2\pi i} \int_{\delta - i\infty}^{\delta + i\infty} \varphi_{1t}(s) e^{sz'} ds \quad (17)$$

and note that $\varphi_{1t}(s)$ has singularities in the left half of the complex plane of the variable s . The first pole is at the point $-s_0$ and the second is the logarithmic branch at $s = i\omega\tau - 1$. One can establish this fact by noting that $g_-(s) = g_+(s)/G(s)$ and $G(s)$ has such a singularity. We make the cut as shown in the drawing. We extend the integration contour in (17) to the left to infinity; there remains here the residue at the pole and the integral over the cut

$$\varphi_{1t}(z') = \text{Res}[\varphi_{1t}(-s_0)] e^{-s_0 z'} + \frac{1}{2\pi i} \int_C \varphi_{1t}(s) e^{sz'} ds. \quad (18)$$

We compute the first term of φ_{1tR} . As is seen, it is dominant for large z :

$$\varphi_{1tR} = \frac{p_0 u}{3} \frac{3 + F_1}{F_1} \frac{1 - \eta}{\eta} \frac{g_+(-s_0)}{G(-s_0)g_-(0)} e^{-s_0 z'}. \quad (19)$$

In order to find s_0 from the equation

$$\Delta(s) = 0, \quad (20)$$

we make the substitution $s = (1 - i\omega\tau)/\xi$, where $\xi = \eta + i\zeta$. Assuming that ζ is small, we expand Δ in ζ . Then the real part of (20) gives Eq. (2) for η and the imaginary part enables us to find ζ :

$$\frac{\zeta}{\eta} = \frac{1}{\omega\tau} \frac{3}{F_1} \frac{1 - (\eta^2 - 1)w}{(3\eta^2 - 1)w - 1}. \quad (21)$$

The exponent in (19) has the form ikz , where $k = \omega/\eta + i\gamma$, i.e., at infinity the perturbation of the distribution function transforms to a plane wave propagating with velocity ηv , while $\varphi \sim \cos \chi$; therefore, the wave will be zero sound with $m = 1$. The coefficient of sound damping is expressed in terms of ζ :

$$\gamma = \frac{1}{\eta v \tau} \left(1 - \frac{\zeta}{\eta} \omega\tau \right). \quad (22)$$

Since the expansion in $\zeta/(\eta - 1)$ is carried out in the solution of (29), the region of application of the resulting formulas is limited by the requirement of the smallness of this quantity:

$$\omega\tau \gg \frac{3}{F_1} \frac{\eta}{\eta - 1} \frac{1 - (\eta^2 - 1)w}{(3\eta^2 - 1)w - 1}. \quad (23)$$

We can consider the other limiting case $\omega\tau \ll 1$; we then get from (20)

$$s = \frac{1 - i}{\sqrt{2}} \sqrt{\frac{15\omega\tau}{3 + F_1}}.$$

Substituting this expression in (19), we see that

$$\varphi_{1t} \sim \exp \left\{ \frac{i - 1}{\sqrt{2}} \sqrt{\frac{15\omega\tau}{v^2 \tau (3 + F_1)}} z \right\},$$

i.e., the ordinary viscous wave is obtained in this case, as was to be expected.

We can find the sound amplitude A by means of (19). For this purpose, we must compute φ_{1t} , using its definition and (1) and (6), and then compare the resultant expression with (19); in other words,

$$A = -p_0 u \frac{\eta - 1}{\eta} \frac{g_+(-s_0)}{g_-(0)G(-s_0)}. \quad (24)$$

To compute the value of $g_+(-s_0)$ here, we express it in the form

$$g_+(-s_0) = \exp \left\{ -\frac{1}{\pi} \int_1^{\infty} \frac{\text{arctg } S(x)}{x + 1/\eta} dx \right\},$$

$$S(x) = \frac{3\pi F_1}{4x(3 + F_1)} \left(1 - \frac{1}{x^2} \right) \left\{ 1 - \frac{3F_1}{2(3 + F_1)} \times \left[1 - \frac{x^2 - 1}{x^2} \left(1 + \frac{1}{2x} \ln \frac{x - 1}{x + 1} \right) \right] \right\}^{-1}. \quad (25)$$

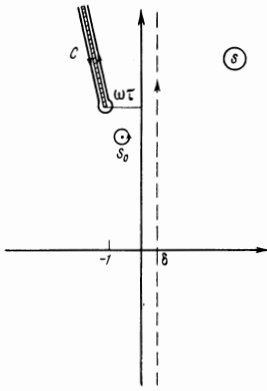
as was done in [4]. When F_1 varies from 6.0 to 14.4, η changes from 1.0 to 1.2. Numerical integration gives the corresponding values 0.47 and 0.37 for $[g_+(-s_0)]^2$ in these cases. $G(-s_0)$ is computed by going to the limit, in the definition of (13):

$$G(-s_0) = \frac{3F_1(\eta^2 - 1)}{4(3 + F_1)} [(3\eta^2 - 1)w - 1], \quad (26)$$

while for the calculation of $g_-(0)$ we use the parity property $g_+(-s) = 1/g_-(s)$, where, by means of (16), we easily obtain

$$\left[\frac{1}{g_-(0)} \right]^2 = G(0) = \frac{3\eta^2}{3 + F_1}. \quad (27)$$

Substituting (24)–(27) in (3), we find the flux in the sound



The dotted line indicates contour of integration in Eq. (17), and the solid line corresponds to Eq. (18).

wave:

$$Q = \frac{4}{3} \frac{(3 + F_1)^2 [g_+(-s_0)]^2}{F_1^2 (\eta + 1)^2 [(3\eta^2 - 1)w - 1]} \rho c u^2. \quad (28)$$

Here s is the He^3 density and $c = \eta v$ is the velocity of transverse zero sound.

The coefficient of proportionality between Q and $\rho c u^2$ (we shall denote it by α) serves as the natural measure of the effectiveness of the sound excitation. For ordinary sound, α is close to unity.^[5] The results of the calculation by (28) for various values of F_1 are given in the table. It has been assumed that $[g_+(-s_0)]^2 = 0.47$ for all values of F_1 .

We now compute the integral over the cut in Eq. (18).

Substituting (14) for φ_{1t} and transforming to integration over the variable $t = s/(i\omega\tau - 1)$, we get

$$\varphi_{11c}(z') = \int_1^{\infty} e^{-iz'(1-i\omega\tau)(t-1)} f(t) dt, \quad (29)$$

where

$$f(t) = -\frac{\rho_0 u}{4\eta g_-(0)} \frac{(t + 1/\eta)}{t^4} g_+[t(i\omega\tau - 1)] \left(\left\{ 1 + \frac{3F_1}{2(3 + F_1)} \right. \right. \\ \left. \left. \times \left[\frac{t^2 - 1}{t^2} \left(1 - \frac{1}{2t} \ln \frac{t+1}{t-1} \right) - 1 \right] \right\}^2 + \left[\frac{3\pi F_1}{2(3 + F_1)} \frac{t^2 - 1}{t^3} \right]^2 \right)^{-1}.$$

The asymptotic expression for the integral in (29) can be found for large z by taking the integral twice by parts:

$$\varphi_{11c} \sim f(1) \frac{e^{-z'(1-i\omega\tau)}}{(z'\omega\tau)^2} = \frac{\rho_0 u}{\eta g_-(0)} \\ \times \frac{(1 + 1/\eta) g_+(i\omega\tau - 1) (3 + F_1)^2}{(F_1 - 6)^2} \left(\frac{v}{z\omega} \right)^2 e^{-z'(1-i\omega\tau)/v\tau}. \quad (30)$$

This part of φ_{1t} is connected with the quasiparticles traveling freely after collision at the wall; it falls off more rapidly than the sound part because of the difference in the value of the exponent and because of the factor $(v/z\omega)^2$. To separate the sound from the freely traveling quasiparticle, it is necessary to guarantee such conditions also for which the contribution to the cut would be small in comparison with the contribution of the pole, i.e., that the condition

$$\frac{(3\eta^2 - 1)w - 1}{4\eta[(\eta - 1)w]^2} \frac{e^{-z'(1-\gamma v\tau)}}{(z'\omega\tau)^2} \ll 1. \quad (31)$$

be satisfied. As is seen from the values of γ given in the table, the sound signal is extinguished at distances of the order of the path length; therefore, detection of the signal should be accomplished at $z' \sim 1$; satisfaction of the condition (31) can then be guaranteed because

F_1	6.24	6.4	6.7	8.5	10.4	14.3
η	1.003	1.005	1.010	1.050	1.10	1.20
$\gamma \cdot v\tau$	0.87	0.85	0.81	0.70	0.61	0.52
α	0.09	0.11	0.13	0.24	0.31	0.41
$(\omega\tau)^2 \gg$	$1.9 \cdot 10^4$	$6.1 \cdot 10^3$	$1.7 \cdot 10^3$	92	26	7.6

of the value of $\omega\tau$. In the last column of the table, the values of $(\omega\tau)^2$ are given for which the contributions to φ_{1t} of the pole and the cut are equal in magnitude over one path length.

It is seen from the table that the conditions for the observation of the transverse mode of zero sound are improved with increase of F_1 , i.e., with increase in the pressure; however, observation of this mode becomes a complicated experimental problem with increase in pressure. An added difficulty in comparison with the observation of the longitudinal mode arises because of the greater damping of the transverse sound, which leads to the necessity of locating the receiving and radiating crystals very close to one another. The distance between them should not be much greater than the path length of the excitation, and even at 10^{-3} °K, it is of the order of 10^{-3} cm.

If we keep one more term in the expansion of the function $F(\vartheta)$ in Legendre polynomials, i.e., we take it to be of the form

$$F(\vartheta) = F_0 + F_1 \cos \vartheta + F_2 P_2(\cos \vartheta), \quad (32)$$

then the equation for the velocity of transverse sound will have the form

$$(\eta^2 - 1)w = \frac{(F_1 - 6)(1 + F_2/5) + 3F_2\eta^2}{3F_1(1 + F_2/5) + 9F_2\eta^2}. \quad (33)$$

Measurement by experiment of the value of η would probably permit us to draw some conclusions as to F_2 .

Besides liquid He^3 , degenerate solutions of He^3 in He^4 are also suitable for the study of Fermi liquids. However, the values of F_1 computed by Bardeen, Baym, and Pines^[6] for solution with concentrations of 1, 3, and 5% are such that the transverse nodes of zero sound should not exist in them.

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APPENDIX

The energy flux in a Fermi liquid is defined by the expression^[7]

$$Q = \int n \epsilon \frac{\partial \epsilon}{\partial \mathbf{p}} d\tau. \quad (A.1)$$

We represent the distribution function for quasiparticles in the form $n = n_0(\epsilon) + \delta n$. By ϵ here we mean the energy of the quasiparticle with account of interaction. The function δn is connected with the deviation of the distribution function from the equilibrium δn_0 by the relation

$$\overline{\delta n} = \delta n - \frac{\partial n_0}{\partial \epsilon} \int f(\mathbf{p}, \mathbf{p}') d\tau'.$$

The function $n_0(\epsilon)$ changes the integral of (A.1) to zero,

as it is not difficult to see. Replacing ϵ by $\epsilon - \mu + \mu$ in the integrand of (A.1), we get

$$Q = \int \overline{\delta n}(\epsilon - \mu) \frac{\partial \epsilon}{\partial \mathbf{p}} d\tau + \int \overline{\delta n \mu} \frac{\partial \epsilon}{\partial \mathbf{p}} d\tau. \tag{A.2}$$

The second component in the given formula represents the energy carried by the particle flow. It is not necessary to compute it, since, according to the meaning of the problem considered, the number of particles in any volume does not change in the mean. In other words, physical interest attaches to the potential flow $E - \mu N$. With accuracy to second order in $\delta \bar{n}$,

$$Q_z = \int \left[\delta n - \frac{\partial n_0}{\partial \epsilon} \int f(\mathbf{p}, \mathbf{p}') \delta n' d\tau' \right] \left[v(p - p_0) + \int f(\mathbf{p}, \mathbf{p}'') \delta n'' d\tau'' \right] v \cos \theta d\tau - \int \left[v(p - p_0) + \int f(\mathbf{p}, \mathbf{p}') \delta n' d\tau' \right] \frac{\partial n_0}{\partial \epsilon} v \cos \theta \left(\int f(\mathbf{p}, \mathbf{p}'') \delta n'' d\tau'' \right) d\tau \tag{A.3}$$

For the chosen form of the interaction $f(\mathfrak{g}) = f_0 + f_1 \cos \mathfrak{g}$, the last of the described integrals vanishes because of the properties of parity of the integrand and because of the presence of the δ function in them. The contributions of the remaining two integrals are easily computed. We have

$$Q_z^{(1)} = v^2 \int (p - p_0) \cos \theta \delta n d\tau = \frac{2p_0^2 v^2}{(2\pi\hbar)^3} \int \cos \theta d\omega \int_{p_0}^{p_0 - v} (p - p_0) dp = \frac{p_0^2}{(2\pi\hbar)^3} \int v^2(\theta, \chi) \cos \theta d\omega.$$

Then, substituting $v(\theta, \chi)$ from (1), we get

$$Q_z^{(1)} = \frac{\pi p_0^2 A^2}{(2\pi\hbar)^3} \int_{-1}^{+1} \frac{5t^4 - 3t^2}{\eta - t} dt.$$

The contribution of the second integral to (A.3) is equal

to

$$Q_z^{(2)} = v \int f(\mathbf{p}, \mathbf{p}') \delta n \delta n' \cos \theta d\tau d\tau' = \frac{2p_0^2}{(2\pi\hbar)^3} \int \cos \theta v(\theta, \chi) F(\theta) v(\theta', \chi') \frac{d\omega d\omega'}{4\pi} = \frac{2\pi p_0^2 A^2}{(2\pi\hbar)^3} \frac{F_1}{4} \left(\int_{-1}^{+1} \frac{t(1-t^2)}{\eta - t} dt \right) \left(\int_{-1}^{+1} \frac{t^2(1-t^2)}{\eta - t} dt \right).$$

Noting that

$$\int_{-1}^{+1} \frac{t(1-t^2)}{\eta - t} dt = \frac{4}{F_1}$$

by virtue of (2), we get

$$Q_z^{(2)} = \frac{2\pi p_0^2 A^2}{(2\pi\hbar)^3} \int_{-1}^{+1} \frac{t^2(1-t^2)}{\eta - t} dt.$$

Combining $Q_z^{(1)}$ and $Q_z^{(2)}$, we finally get

$$Q_z = \frac{\pi p_0^2 A^2}{(2\pi\hbar)^3} \int_{-1}^{+1} \frac{3t^4 - t^2}{\eta - t} dt = \frac{2\pi p_0^2 A^2}{(2\pi\hbar)^3} \eta [(3\eta^2 - 1)\omega - 1]. \tag{A.4}$$

¹L. D. Landau, Zh. Eksp. Teor. Fiz. **32**, 59 (1957) [Sov. Phys.-JETP **5**, 101 (1957)].

²W. R. Abel, A. C. Anderson and J. C. Wheatley, Phys. Rev. Lett. **17**, 74 (1956).

³J. C. Wheatley, Quantum Fluids Proceedings of Sussex University Symposium, Amsterdam, 1966.

⁴I. L. Bekarevich and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **39**, 1699 (1960) [Sov. Phys.-JETP **12**, 1187 (1961)].

⁵J. Gavoret, Phys. Rev. **137**, A721 (1965).

⁶J. Bardeen, G. Baym and D. Pines, Phys. Rev. **156**, 207 (1967).

⁷L. D. Landau, Zh. Eksp. Teor. Fiz. **30**, 1058 (1956) [Sov. Phys.-JETP **3**, 920 (1956)].

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