CONCERNING AN EXPERIMENTAL VERIFICATION OF THE EQUALITY OF THE INERTIAL AND GRAVITATIONAL MASSES

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It is indicated that the statement that the inertial and gravitational masses are rigorously equal is not a unique logical consequence of the available experimental data, since the results of the experiments of Galileo, Newton, and Eotvos can in principle be explained without this statement. On the other hand, the available direct experiments are insufficient both in coverage and in accuracy.

IN this note we wish to call attention to the fact that the statement that the inertial and gravitational masses are equal is, strictly speaking, not a unique logical consequence of the existing experimental data.

The results of the well known experiments of Galileo and Newton, and also of the Eotvos experiments (which were performed with maximum accuracy), can be physically explained without stating that the masses are equal. On the other hand, the direct experiments, for example, those performed with the Cavendish balance and capable in principle of confirming the indicated equality, pursued only a limited purpose—numerical determination of the gravitational constant, and were therefore performed within a narrow range of masses and for a few types of observed bodies. Different experiments yielded data that differed already in the third significant figure.

We note first of all that it is possible to determine logically the gravitational mass M_g of the body producing a field on the direct basis of the experimental data on the motion of bodies in gravitational fields, by determining the acceleration w of any other body located at a distance L from the first, using the relation

$$w = \gamma M_g / L^2, \tag{1}$$

without resorting beforehand to the concepts of force and inertial mass of the body. The distances are measured in this case directly, and the accelerations are calculated from kinematic considerations. At the same time, the inertial mass M_i of a body is determined by the acceleration imparted to this body by a non-gravitational force F_n:

$$w = F_n / M_i. \tag{2}$$

The force is measured here by a spring dynamometer, and the accelerations are measured in the same manner as for (1).

Thus, to confirm the equality of the masses M_g and M_i which enter in relations (1) and (2), the same body should serve successively (or perhaps simultaneously) as the body producing the gravitational field and the body accelerated by the nongravitational force. It is easy to see that no such verifications were made in the experiments of Galileo, Newton, and Eotvos. Let us turn to the analysis of the results of the aforementioned experiments.

From the point of view formulated above, the quality of the accelerations of different bodies freely falling on earth (the Galileo experiment) follows directly from relation (1), which contains neither the inertial nor the gravitational masses of the falling body; in this sense, the equality of the masses is not confirmed by this experiment. Speaking more rigorously when measuring the acceleration w_f with which the falling body approaches the earth, we take into account not only the acceleration imparted to the body by the earth (relative to the common center of mass), but also the opposing acceleration imparted to the earth by the falling body, and in accordance with the same relation (1) we have

$$w_f = (M_{ge} + M_{gb}) / L^2. \tag{3}$$

However this relation likewise contains only the gravitational masses of the earth M_{ge} and of the body M_{gb}. The inertial masses do not take part in this relation.

The foregoing considerations pertain equally well to the experiments of Newton, in which the equality of the periods of oscillations of a pendulum with different bobs is established. The period of the oscillation of the pendulum is connected with the acceleration of the freely falling body relative to the pendulum, and if the pendulum is fixed on the earth, then this acceleration depends only on the gravitational mass of the earth and on its rotation.

In estimating the results of the Eotvos experiments, it is necessary to take into account two circumstances. First, the centripetal acceleration, necessary for the body to rotate together with the earth, is produced in the Eotvos experiment by a nongravitational pressure force F_n of the rod on the body fastened to it. In accordance with (2), this force is proportional to the inertial mass of the body:

$$F_n = M_i w_{cp}. \tag{4}$$

Second, the weight P, measured by the spring dynamometer, is a nongravitational force that prevents the free motion of the body (fall). The weight imparts to the body an acceleration w (relative to an object that moves freely in the vicinity) and, in accordance with (2), is proportional to the inertial mass of the body:

$$P = M_i w. \tag{5}$$

Representing, further, the resultant vector of the weight as the geometric sum of the weight P_0, which would be applied to the body in the absence of the earth's rotation ($P_0 = M_i g_0$), and the centripetal force F_n in accordance with expression (4), we obtain the angle α of the deflection of the resultant weight vector from the radial direction at the point with geographic latitude φ:

$$\alpha = \operatorname{arc\,tg} \frac{F_n \sin\varphi}{P_0 - F_n \cos\varphi} = \operatorname{arc\,tg} \frac{w_{cp} \sin\varphi}{g_0 - w_{cp} \cos\varphi}. \qquad (6)$$

Here the value of g_0 is determined only by the gravitational mass of the earth, and w_{cp} is determined only by its rotation.

The Eotvos experiment was made under the assumption that the weight is proportional to the gravitational mass of the body. In this case the angle α would be dependent on the ratio of the gravitational mass of the body to its inertial mass. If the indicated ratio were to be different for two unlike bodies, then, by fastening such bodies on a rod and suspending it on a filament, we would observe a torque tending to rotate the rod. The absence of a torque in experiments performed with high accuracy was taken to be proof of the exact equality of the masses. However, expression (6) shows that the angle does not depend on the properties of the bodies, so that no torque can be produced on the rod. Thus, the Eotvos experiment, from the point of view under consideration, cannot serve as proof of the equality of the masses.

Yet, in sufficiently accurate direct experiments, which determine the mass of the bodies in accordance with relation (1) and (2), the inequality of the masses could be observed, and the organization of such experiments is of fundamental significance.

We note in conclusion that in the scheme considered here the law of momentum conservation is satisfied in different forms, depending on the character of the interaction between the bodies. In the case of gravitational interaction of two bodies, this law includes only their gravitational masses, as follows from relation (1) written out for both bodies. In the case of a nongravitational interaction, it includes only inertial masses, as follows analogously from relation (2) and the third law of dynamics. The indicated circumstance can also be used to verify the equality of the masses.

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