

DESTRUCTION OF SUPERCONDUCTIVITY BY A CURRENT

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The structure of the intermediate state produced in a cylindrical superconducting sample under the action of an electric current that exceeds the critical value is elucidated. The interfaces between the normal and superconducting phases are paraboloids of rotation or cone-like surfaces, depending on whether the Hall coefficient of the normal phase is large or small. The structure period and velocity along the sample axis are calculated. Corrections to the critical current and the resistance due to the finiteness of the period are found. In very pure metals the superconducting region in the main part of the volume should have the shape of thin toroid-like threads.

THE first quantitative theory of the process of destruction of superconductivity by electric current was proposed by London^[1]. He has shown that a cylindrical sample of radius R, through which a current exceeding a critical value $cH_C R/2$ flows (H_C - critical magnetic field), goes over into an intermediate state. The sample then acquires an electric resistance, which, however, is lower than the resistance in the normal state. In the structure proposed by London, the intermediate state is a system of alternating stationary layers of normal and superconducting phases, arranged perpendicular to the current.

Gorter^[2] considered another possible structure, in which the interfaces between the phases are coaxial cylinders. The major difference between such a structure and the London structure is the presence of continuous motion of the interphase boundaries towards the sample axis.

Sharvin^[3] has demonstrated the possibility of realizing periodic structures moving as a unit with constant velocity in the direction of the current. It follows directly from the results of Sharvin and the author^[4] that there exists an infinite one-parameter family of such structure, including the two aforementioned structures as limiting cases. If we use the macroscopic description of the intermediate state, i.e., neglect effects connected with the surface tension on the phase separation boundaries, then all these structures correspond to identical values of such quantities as resistance, magnetic energy, etc. With the aid of the macroscopic theory it is therefore impossible to ascertain which of the structures should actually be realized. In this paper we consider this question from the microscopic point of view. It turns out as a result that the most convenient is one of the structures intermediate between those of London and of Gorter.

1. MACROSCOPIC DESCRIPTION

In the macroscopic description, the intermediate state of a superconductor is regarded as a medium in which the absolute magnitude of the magnetic intensity vector H is always equal to the critical field H_C . If we choose a cylindrical system of coordinates (r, φ, z) such that the z axis coincides with the axis of the sample, then it follows from symmetry considerations

that the only nonzero component of H is $H_\phi = H_C$. The electric current density $j = (c/4\pi) \text{curl H}$ has in this case only a component along the z axis, equal to $cH_C/4\pi r$.

As shown in^[5], regardless of the concrete of the intermediate-state structure, the current component perpendicular to the magnetic field is connected with the vector E/x_n (x_n - concentration of the normal phase) by the same relation as in a bulky normal metal placed in a field H. Taking the Hall effect into account, we can then write

$$\frac{\sigma}{x_n} E = j + \frac{\lambda}{H_C} [Hj], \tag{1}^*$$

where σ - normal conductivity and $\lambda/\sigma H_C$ - Hall constant.

Taking into account the condition of constancy of E_z , we get from (1):

$$E_r = \lambda E_z, \quad E_\phi = 0, \quad x_n = r/r_0, \tag{2}$$

where $r_0 = cH_C/4\pi\sigma E_z$ is a quantity determining the position of the separation boundary of the region occupied by the intermediate state ($r < r_0$) and the region occupied by the purely normal phase ($r > r_0$). When the total current through the sample is close to the critical value, r_0 practically coincides with the sample radius R. When the current increases, r_0 decreases monotonically.

We see therefore that the problem of macroscopic electrodynamics, i.e., the problem of determining the averaged quantities E, H, and x_n , has a unique solution. This solution, however, can correspond to entirely different layer configurations. It is shown in^[4] that in stationary conditions the possible layer configurations are determined as solutions of the following system of equations:

$$\begin{aligned} (n\nabla)n + \nabla V/V &= n(n\nabla V/V), \\ nH = 0, \quad V &= \frac{c}{H_C^2 x_n} n[EH], \end{aligned} \tag{3}$$

where n - unit vector normal to the phase separation boundaries and V - velocity of boundary motion. In the geometry under consideration, when all the quantities depend only on the coordinate r, the system (3)

*[Hj] ≡ H × j.

can be readily integrated and its general solution is

$$n_r = \frac{\alpha r/r_0 - \lambda}{[1 + (\alpha r/r_0 - \lambda)^2]^{1/2}}, \quad n_z = -\frac{1}{[1 + (\alpha r/r_0 - \lambda)^2]^{1/2}}, \quad n_\varphi = 0, \\ V = -\frac{c}{4\pi\sigma r_0} \frac{\alpha}{[1 + (\alpha r/r_0 - \lambda)^2]^{1/2}} \quad (4)$$

Here α is an arbitrary constant, which cannot be calculated within the framework of the macroscopic theory. When $\alpha = 0$ (and $\lambda = 0$), we obtain $n_z = -1$, $n_r = 0$, and $V = 0$, i.e., the static London structure. In the limit as $\alpha \rightarrow \infty$, we obtain the Gorter structure, corresponding to $V = -c/4\pi\sigma r$, $n_r = 1$, and $n_z = 0$.

The velocity of motion as a whole in the direction of the z axis, for a structure with given α , is equal to

$$u = V/n_z = c^2\alpha/4\pi\sigma r_0. \quad (5)$$

2. PERIODIC STRUCTURES

We shall confine ourselves throughout to the most interesting case, when the current through the sample exceeds the critical value only slightly, and therefore $r_0 \approx R$. The point is that if the current is appreciably larger than critical, then the intermediate state exists only deep in the sample, and an experimental observation of its structure is practically impossible.

By virtue of the homogeneity of the problem in the current direction, all the structure with finite value of α should be periodic. The Gorter structure is homogeneous and therefore calls for a special consideration.

Let $z = z_0(r)$ be the equation of the surface passing through the middle of some normal layer. Inasmuch as the concentration of the normal phase, as seen from (2), is proportional to r , the equations for the layer boundaries are of the form

$$z = z_\pm(r) \equiv z_0(r) \pm rd/2R,$$

where d is the period of the structure.

It is clear from symmetry consideration that the only nonvanishing component of the magnetic field is $H_\varphi(r, z - ut) \equiv H$. In the normal phase, i.e., when $z_- < z < z_+$, it satisfies the equation¹⁾

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \frac{\partial H}{\partial r} - \frac{H}{r^2} + \frac{\partial^2 H}{\partial z^2} - \frac{2\lambda}{Hc} H \frac{\partial H}{\partial z} = 0, \quad (6)$$

where we have omitted the term with the time derivative, inasmuch as the velocity u is such that it can be neglected.

We shall seek the solution in the form of an expansion in the small parameter $(z - z_0)/r$:

$$H = f_0(r) + \frac{[z - z_0(r)]^2}{2r^2} f_2(r).$$

Substituting in (6), we find

$$f_2 = \frac{1}{1 + z_0'^2} \left\{ f_0 - r \frac{df_0}{dr} - r^2 \frac{d^2 f_0}{dr^2} \right\},$$

where $z_0' \equiv dz_0/dr$.

On the boundary between the normal and superconducting phases, i.e., at $z = z_\pm$, the field H should be

equal to H_c . Therefore

$$f_0 + \frac{d^2}{8R^2} \frac{1}{1 + z_0'^2} \left\{ f_0 - r \frac{df_0}{dr} - r^2 \frac{d^2 f_0}{dr^2} \right\} = H_c.$$

When d/R is small, the solution of the last equation is

$$f_0 = H_c \left\{ 1 - \frac{d^2}{8R^2(1 + z_0'^2)} \right\}$$

and thus

$$H = H_c \left\{ 1 + \frac{(z - z_0)^2 - (rd/2R)^2}{2r^2(1 + z_0'^2)} \right\}. \quad (7)$$

At a specified value of the current through the sample, the thermodynamic potential $\tilde{\mathcal{F}}$ should tend to its minimum possible value; the density of this potential is $\tilde{F} = F - H \cdot B/4\pi$ (see^[6]). It is assumed that the conductivity σ is not too small, first, to be able to regard the temperature as constant, and second, for the electric field intensity to be much smaller than the magnetic one. The need for the latter condition is connected with the fact that in the presence of a current in the conductor it is impossible to introduce the concept of electric energy, owing to dissipation.

If we reckon F from its value in the superconducting phase, i.e., we put $F_S = 0$, then in the normal phase at $H = 0$ we have $F_n = H_c^2/8\pi$. When $H \neq 0$ we obviously have $F_n = (H_c^2 + H^2)/8\pi$. Since in a superconductor we always have $B = 0$, the potential \tilde{F}_S also vanishes. In the normal phase B and H practically coincide, and therefore $\tilde{F}_n = (H_c^2 - H^2)/8\pi$. The magnetic energy per unit length of the sample is

$$\tilde{\mathcal{F}}_1 = \frac{1}{d} \int_0^R 2\pi r dr \int_{z_-(r)}^{z_+(r)} dz \frac{H_c^2 - H^2}{8\pi} = \frac{H_c^2 d^2}{24R^3} \int_0^R \frac{r^2 dr}{1 + z_0'^2}, \quad (8)$$

where we have used the expression (7) from the magnetic field in the normal layer and took into account the fact that $d/R \ll 1$. Here and below we shall not write out in the expression for $\tilde{\mathcal{F}}$ the term connected with integration over the volume outside the sample, since this term is a constant that does not depend on the state of the sample.

The derivative $z_0'(r)$ for the structure with given value of the constant α , as seen from (4), equals $\alpha r/R - \lambda$. Substituting this in (8), we get

$$\tilde{\mathcal{F}}_1 = \frac{1}{24} H_c^2 d^2 I_1(\alpha, \lambda), \quad (9)$$

where

$$I_1(\alpha, \lambda) = \int_0^1 \frac{x^2 dx}{1 + (\alpha x - \lambda)^2} = \frac{1}{\alpha^2} + \frac{\lambda}{\alpha^3} \ln \frac{1 + (\alpha - \lambda)^2}{1 + \lambda^2} \\ + \frac{\lambda^2 - 1}{\alpha^3} [\arctg(\alpha - \lambda) + \arctg \lambda].$$

In addition, we should take into account the energy connected with the surface tension on the phase separation boundary. Writing down the coefficient of surface tension, as usual, in the form $H_c^2 \Delta/8\pi$, we obtain

$$\tilde{\mathcal{F}}_2 = \frac{H_c^2}{8\pi} \Delta \frac{2}{d} \int_0^R 2\pi r dr [1 + z_0'^2]^{1/2} = \frac{H_c^2 \Delta R^2}{2d} I_2(\alpha, \lambda), \quad (10)$$

where

$$I_2(\alpha, \lambda) = \int_0^1 x dx [1 + (\alpha x - \lambda)^2]^{1/2} = \frac{1}{\alpha^2} \left\{ \frac{1}{3} [(1 + (\alpha - \lambda)^2)^{3/2} - (1 + \lambda^2)^{3/2}] + \lambda \left[\frac{\alpha - \lambda}{2} (1 + (\alpha - \lambda)^2)^{1/2} + \frac{\lambda}{2} (1 + \lambda^2)^{1/2} \right] \right\}$$

¹⁾We emphasize that we deal here and throughout with a "microscopic" magnetic field. In the preceding section we used the same letter to denote the magnetic intensity in the averaged description.

$$+ \frac{1}{2} \ln \frac{(1 + (\alpha - \lambda)^2)^{1/2} + \alpha - \lambda}{(1 + \lambda^2)^{1/2} - \lambda} \Big] \Big\}.$$

From the condition that the sum of the expressions (9) and (10) be a minimum, we obtain the equilibrium value of the period of a structure with given α :

$$d(\alpha) = (6I_2/I_1)^{1/2} R^{1/2} \Delta^{1/2}. \quad (11)$$

The value of the total thermodynamic potential is in this case

$$\tilde{\mathcal{F}}(\alpha) = \tilde{\mathcal{F}}_1 + \tilde{\mathcal{F}}_2 = 1/8 \cdot 6^{3/2} (I_1 I_2^2)^{1/2} H_c^2 R^{1/2} \Delta^{3/2}. \quad (12)$$

The parameter α corresponding to the equilibrium structure should be determined by minimizing the function $\tilde{\mathcal{F}}(\alpha)$, that is to say, by minimizing the product $I_1 I_2^2$. We confine ourselves to a consideration of limiting cases of small and large values of λ . The former is realized under conditions when the electron mean free path is much smaller than the Larmor radius in the critical magnetic field, and the latter in the case when the mean free path is larger in metals with unequal numbers of electrons and holes.

Substituting $\lambda = 0$ in the foregoing formulas, we get

$$I_1 I_2^2 = \frac{(\alpha - \arctg \alpha)[(1 + \alpha^2)^{3/2} - 1]^2}{9\alpha^7}.$$

This function, which is even in α , has a minimum value at $\alpha \approx \pm 1.3$, i.e., two different structures have the same energy. It is easy to see, however, that allowance for the small terms proportional to λ leads to a lower energy at $\alpha \approx -1.3\lambda/|\lambda|$. The value of the function $\tilde{\mathcal{F}}(\alpha)$ at the minimum is

$$\tilde{\mathcal{F}}(1.3) \approx 0.178 H_c^2 R^{1/2} \Delta^{3/2}. \quad (13)$$

From (11) and (5) we get the following expressions for the period of the equilibrium structure and its velocity

$$d \approx 2.9 R^{1/2} \Delta^{1/2}, \quad u \approx -0.1 \frac{\lambda}{|\lambda|} \frac{c^2}{\sigma R}. \quad (14)$$

The following remark is in order. When $\alpha = 0$, i.e., for a London structure, $\tilde{\mathcal{F}}(\alpha)$ has a maximum. This means that the London structure is unstable, inasmuch as on going to structures with small α there arises a "force" $-\partial \tilde{\mathcal{F}}/\partial \alpha$ that tends to increase $|\alpha|$. However, since the potential of the London structure

$$\tilde{\mathcal{F}}(0) = 1/8 \cdot 3^{1/2} H_c^2 R^{1/2} \Delta^{3/2} \approx 0.180 H_c^2 R^{1/2} \Delta^{3/2}$$

exceeds the potential of the equilibrium structure (13) by only approximately 1%, this force is rather small. For this reason, an appreciable role may be played by friction effects arising when the separation boundaries between the phase move, and due for example to the inhomogeneities of the sample, to defects of the crystal lattice, etc. The friction "force" tends to decrease $|\alpha|$, since it should obviously lead to a decrease of the velocity of the structure. It can therefore be assumed that a London structure should be realized in sufficiently "dirty" samples.

If $|\lambda| \gg 1$, then we get, putting $\gamma = -\lambda/\alpha$,

$$I_1 I_2^2 = \frac{1}{9} \left(\frac{1+2\gamma}{1+\gamma} - 2\gamma \ln \frac{1+\gamma}{\gamma} \right) \left(1 + \frac{3}{2}\gamma \right)^2.$$

It is assumed here that $\gamma > 0$ or $\gamma < -1$. This function has a minimum value equal approximately to 0.079 when $\gamma \approx 1.1$.

In the case $-1 < \gamma < 0$, the main contribution to the integral I_1 is made by the small vicinity of the point $x = \lambda/\alpha$, and as a result we have $I_1 \sim 1/|\lambda|$. The integral I_2 has an order of magnitude $|\lambda|$, and therefore $I_1 I_2^2 \sim |\lambda|$, which is certainly larger than the value given above.

Thus, when $|\lambda| \gg 1$, the equilibrium structure is the one with $\alpha \approx -0.9\lambda$. The corresponding values of the thermodynamic potential, of the period of the structure, and of the velocity are:

$$\begin{aligned} \tilde{\mathcal{F}}(-0.9\lambda) &\approx 0.176 H_c^2 R^{1/2} \Delta^{3/2}, \\ d &\approx 3.4 |\lambda| R^{1/2} \Delta^{1/2}, \quad u \approx -0.072 c^2 \lambda / R\sigma. \end{aligned} \quad (15)$$

3. GORTER STRUCTURE

The Gorter structure is homogeneous in z , and therefore the magnetic field H depends only on the coordinate r . The dependence of H on the time can be neglected here, as above. Let $r = \bar{r} \pm a_n/2 \equiv r_{\pm}$ be the boundaries of some normal layer. We seek the magnetic field H in the form

$$H = H(\bar{r}) + 1/2 (r - \bar{r})^2 A,$$

where A is a constant. Substituting in (6), we find $A = H(\bar{r})/\bar{r}^2$. From the condition that the magnetic field at $r = r_{\pm}$ be equal to the critical field H_c , we get

$$H(\bar{r}) = H_c(1 - a_n^2/8\bar{r}^2),$$

which leads to the following distribution of the field over the thickness of the layer:

$$H = H_c \left\{ 1 + \frac{(r - \bar{r})^2 - (a_n/2)^2}{2\bar{r}^2} \right\}. \quad (16)$$

The total thermodynamic potential of one layer can be readily found with the aid of (16):

$$f(\bar{r}) = f_1 + f_2,$$

where

$$\begin{aligned} f_1 &= 2\pi\bar{r} \int_{r_-}^{r_+} dr \frac{H_c^2 - H^2}{8\pi} = \frac{H_c^2 a_n^3}{24\bar{r}}, \\ f_2 &= \frac{H_c^2}{8\pi} \Delta \cdot 2 \cdot 2\pi\bar{r} = \frac{H_c^2 \Delta \bar{r}}{2}. \end{aligned}$$

The thermodynamic potential $\tilde{\mathcal{F}}$ of the entire sample is obtained from $\tilde{f}(\bar{r})$ by dividing by the local period of the structure along the r axis, equal to $d(\bar{r}) = a_n/x_n = a_n R/\bar{r}$, and integrating with respect to \bar{r} :

$$\tilde{\mathcal{F}} = \frac{H_c^2}{2R} \int_0^R \bar{r} d\bar{r} \left\{ \frac{a_n^3}{12\bar{r}} + \Delta \bar{r} \right\}. \quad (17)$$

To find the $a_n(r)$ dependence we should trace the variation of the thickness of some normal layer as it moves towards the axis of the sample. From the condition of conservation of the magnetic flux and formula (16) it is seen that, with the specified accuracy, a_n does not depend on the time, and therefore $a_n(\bar{r}) = \text{const}$. From (17) we get in this case

$$\tilde{\mathcal{F}} = \frac{H_c^2}{6} \left\{ \frac{a_n^3}{4} + \frac{\Delta R^2}{a_n} \right\}$$

from which we obtain the equilibrium thickness of the normal layers $a_n = (2\Delta)^{1/3} R^{2/3}$ and the thermodynamic

potential of the Gorter structure

$$\tilde{\mathcal{F}} = 2^{-1/2} H_c^2 R^{1/2} \Delta^{3/2} \approx 0.198 H_c^2 R^{1/2} \Delta^{3/2}. \quad (18)$$

Comparison of (18) with (13) and (15) shows that the energy of the Gorter structure greatly exceeds the minimum value realized for periodic structures.

4. CRITICAL CURRENT AND RESISTANCE

The formula obtained by London^[4]

$$W/W_n = 1/2 [1 + (1 - J_{c0}^2/J^2)^{1/2}]$$

for the ratio of the resistance of the sample W to the resistance W_n in the normal state (under the condition $\lambda \rightarrow 0$, which is assumed satisfied in this section) is valid only in the limit when the period of the structure of the intermediate state tends to zero. The same pertains to the value of the critical current $J_{c0} = cH_c R/2$. In the present section we calculate the corrections to the resistance and the critical current, due to the finite period of the structure.

To this end we need an expression for the magnetic field in the region occupied by the purely normal phase, i.e., when $r > r_0$, where $r_0 = R[J/J_{c0} - (J^2/J_{c0}^2 - 1)^{1/2}]$ (see^[1]). By virtue of the periodicity of the structure, the magnetic field is a periodic function of z . We write it in the form $H = \bar{H} + H'$, where $\bar{H}' = 0$, the bar denoting averaging over z . The value of H' , as can be readily seen from (6), decreases exponentially on penetrating into the region $r > r_0$, and differs noticeably from zero only at $r - r_0 \sim d$. From the formulas obtained below it follows that even when $J = J_c \neq J_{c0}$ the thickness $R - r_0$ of the region occupied by the normal phase greatly exceeds the period of the structure d . Therefore we can assume that $h = \bar{H}$ practically wherever $r > r_0$.

Averaging (6) over z we get

$$\frac{d}{dr} \left(r \frac{d\bar{H}}{dr} \right) - \frac{\bar{H}}{r} = 0,$$

whence $\bar{H} = A/r + Br$, where A and B are constants. One equation for their determination is obtained from the equality of the field on the sample surface to $2J/cR$:

$$A/R + BR = 2J/cR; \quad (19)$$

the second is obtained from the condition of the continuity of \bar{H} when $r = r_0$. When $r < r_0$ the average value of the field can be readily determined with the aid of (6), if account is taken of the fact that $H = 0$ in the superconducting phase. We have:

$$\begin{aligned} \bar{H} &= \frac{H_c}{d} \int_{z_-}^{z_+} \left\{ 1 + \frac{(z - z_0)^2 - (rd/2r_0)^2}{2r^2(1 + z_0^2)} \right\} dz \\ &= H_c \frac{r}{r_0} \left\{ 1 - \frac{d^2}{12r_0^2(1 + z_0^2)} \right\}. \end{aligned}$$

The continuity condition yields

$$\frac{A}{r_0} + Br_0 = H_c \left\{ 1 - \frac{d^2}{12r_0^2(1 + \alpha^2)} \right\}. \quad (20)$$

Solving the system of equations (19) and (20) we get, if $d/r_0 \ll 1$,

$$A = H_c \frac{r_0}{2} \left\{ 1 - \frac{R^2}{R^2 - r_0^2} \frac{d^2}{6r_0^2(1 + \alpha^2)} \right\},$$

$$B = \frac{H_c}{2r_0} \left\{ 1 + \frac{r_0^2}{R^2 - r_0^2} \frac{d^2}{6r_0^2(1 + \alpha^2)} \right\} \quad (21)$$

The constant B is connected with the z component of the electric field by the relation $B = 2\pi\sigma E_z/c$, from which we get

$$\frac{W}{W_n} = \frac{1 + (1 - J_{c0}^2/J^2)^{1/2}}{2} + \frac{J_{c0}}{12J} \frac{R}{R^2 - r_0^2} \frac{d^2}{r_0(1 + \alpha^2)}. \quad (22)$$

The contribution of the region $r > r_0$ to the thermodynamic potential at $J - J_{c0} \ll J_{c0}$ equals

$$\tilde{\mathcal{F}}^{(0)} = \int_{r_0}^R \frac{r dr}{4} (H_c^2 - H^2) = -\frac{H_c^2 R^2}{12} \left(2 \frac{J - J_{c0}}{J_{c0}} \right)^{3/2},$$

where we have neglected the terms proportional to d^2 , inasmuch as they are much smaller than the analogous terms arising during the integration over the region $r < r_0$.

The sum of the potential $\tilde{\mathcal{F}}^{(0)}$ and of the potential calculated in Sec. 2 and equal to $0.178 H_c^2 R^{4/2} \Delta^{2/3}$, is the total thermodynamic potential of the structure when $J - J_{c0} \ll J_{c0}$. Since the energy is reckoned by us in such a way that the thermodynamic potential of a sample that is entirely in the superconducting state is zero, the critical current is determined from the condition that the sum $\tilde{\mathcal{F}}^{(0)} + 0.178 H_c^2 R^{4/3} \Delta^{2/3}$ vanish. From this we get

$$(J_c - J_{c0})/J_{c0} = 0.83(\Delta/R)^{1/2}. \quad (23)$$

From (23) and (22) we get the following expression for the jump of the resistance W_c when $J = J_c$:

$$W_c/W_n = 1/2 + 0.64(\Delta/R)^{1/2}. \quad (24)$$

We note that since the values of the thermodynamic potential calculated in Sec. 2 for structures with different α practically coincide, formulas (23) and (24) are valid regardless of which of these structures is actually realized.

Inasmuch as (24) contains a rather low degree of the small parameter Δ/R , the ratio W_c/W_n for values of R customarily used in experiment noticeably exceeds $1/2$. Formula (23) for the critical current contains a higher power of Δ/R . Therefore the relative corrections to the critical current are much smaller than those to the resistance jump. This agrees with the experimental results^[7-9]. In addition, formula (24) is in good quantitative agreement with Meissner's data^[9] on the dependence of W_c on the radius of the sample (see^[10]).

5. STRUCTURES IN VERY PURE METALS

A unique situation arises in pure metals in which the electron mean free path l for scattering by impurities is much larger than the sample dimensions R . The simple layered structures discussed above cannot be realized in this case, as can be seen from the following considerations.

The formula $x_n = 4\pi\sigma E_z r/cH_c$, where σ is the conductivity of bulky normal metal, holds true in the region occupied by the layered structure, regardless of the relation between l and R . This is connected with the character of the reflection of the electronic excitations of the normal metal ("electrons" and

“holes”) from the boundary with the superconducting phase. Namely, this reflection is such that the conductivity σ enters in the equations of the electrodynamics of the intermediate state, from which the aforementioned formula is obtained even under conditions when $l \gtrsim R$ (see^[11]).

On the other hand, in the region $r > r_0$, where r_0 is the boundary between the intermediate state and the purely normal phase, the effective conductivity equals $\sigma^* \sim \sigma R/l \ll \sigma$. From Maxwell's equation $\text{curl } \mathbf{H} = (4\pi/c)\mathbf{j}$ we then obtain

$$\left. \frac{dH}{dr} \right|_{r=r_0+0} = \frac{4\pi\sigma^*}{c} E_z - \frac{H}{r_0} = \frac{H_c}{r_0} \left\{ \frac{\sigma^*}{\sigma} x_n(r_0) - 1 \right\} \approx -\frac{H_c}{r_0}, \quad (25)$$

where we have used the condition of continuity of E_z and H , and also the equality $H = H_c$ in the intermediate state. The derivative dH/dr is such that in the region occupied by the normal phase the magnetic field is lower than the critical one, and furthermore by an amount on the order of H_c . It is clear that such a structure cannot be realized.

The only possibility lies in the following. When $r_1 \ll r < r_0$ the intermediate state is such that the concentration of the superconducting phase $x_S \ll 1$ and the superconducting regions represent thin torus-like filaments coaxial with the sample. When $r \lesssim r_1$, where by definition, $x_S \sim 1$, we have a layered structure. The diameter of the superconducting filaments d should be connected with x_S by a condition ensuring equality of the electron mean free path l_0 , characterizing the scattering by the filaments, to the characteristic dimension of the sample R . Indeed, if $l_S \ll R$, then we again return to the situation described above. On the other hand, if $l_S \gg R$, then the effective conductivity equals σ^* and we get the equality $x_n = 4\pi\sigma^* E_z r / c H_c$, from which we see that when $r \sim r_0$ the concentration is $x_S \sim 1$. But when $x_S \sim 1$ the value of l_S cannot exceed R .

The conductivity is now a continuous function of r , and when $r = r_0$, as seen from (25), we get $dH/dr = (H_c/r_0)(x_n - 1) \leq 0$. In order to have $H > H_c$ throughout the normal phase, $x(r_0)$ should equal unity and $dH/dr = 0$. From these conditions

$$H(R) = \frac{2J}{cR}, \quad H(r_0) = H_c, \quad \left. \frac{dH}{dr} \right|_{r_0} = 0$$

we determine the two constants in the expression for H ($H = A/r + Br$) as well as the value of r_0 . It is clear that as a result we obtain for r_0 the previous expression $r_0 = R [J/J_{C0} - (J_2/J_{C0}^2 - 1)^{1/2}]$. In addition, we determine from this the total resistance of the sample, which equals in order of magnitude the resistance of the normal metal with conductivity σ^* .

At small values of r , where there is a layered structure, the effective conductivity equals σ and $x_n = 4\pi\sigma E_z r / c H_c$. From this we get that $r_1 \sim (\sigma^*/\sigma) r_0$.

The diameter of the superconducting filaments d can be determined from the condition that the thermo-

dynamic potential be a minimum. To this end we note that the magnetic field in the normal phase situated in the space between the filaments differs from H_c by an amount on the order of $\delta H \sim H_c L^2/R^2$, where L is the distance between filaments (cf. (7)). The number of filaments per unit area is equal to $n \sim x_S/d^2$, therefore $L \sim n^{-1/2} \sim dx_S^{-1/2}$. The corresponding contribution to the thermodynamic potential is

$$\tilde{\mathcal{F}}_1 \sim H_c \delta H R^2 \sim H_c^2 d^2 / x_S.$$

The thermodynamic potential due to surface tension on the phase boundary is

$$\tilde{\mathcal{F}}_2 \sim H_c^2 \Delta \cdot R d \cdot n R \sim H_c^2 R^2 \Delta x_S / d.$$

In addition, in this case an important role is played by the fact that the magnetic field on the curved boundary between phases differs from H_c , owing to the surface tension, by an amount on the order of $H_c \Delta/d$. This gives a contribution to the thermodynamic potential

$$\tilde{\mathcal{F}}_3 \sim H_c (H_c \Delta / d) R^2 \sim H_c^2 R^2 \Delta / d.$$

The ratio $\tilde{\mathcal{F}}_2/\tilde{\mathcal{F}}_3$ is of the order of $x_S \ll 1$, and therefore $\tilde{\mathcal{F}}_2$ can be neglected. From the condition that the sum $\tilde{\mathcal{F}}_1 + \tilde{\mathcal{F}}_2$ be a minimum we get $d \sim R^{2/3} \Delta^{1/3}$. As shown above, the mean free path l_S should be of the order of R . On the other hand, $l_S \sim (nd)^{-1} \sim d/x_S$, whence $d \sim x_S R$. We thus have

$$d \sim \sqrt{\Delta R}, \quad x_S \sim \sqrt{\Delta/R}.$$

The distance between filaments is then $L \sim \Delta^{1/3} R^{3/4} \ll R$.

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