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EFFECT OF IMPURITIES ON SUPERCONDUCTIVITY-LIKE PHENOMENA IN ONE-DIMENSIONAL SYSTEMS

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Phase transitions of the superconducting type in a one-dimensional metal in the presence of impurities are investigated on the basis of the Bychkov, Gorkov and Dzyaloshinskiĭ theory. A study of the contribution of "parquet" type diagrams in coordinate space shows that the highest power of the typical logarithmic term occurs at a certain space order of the elementary lattices. It is shown that owing to a special cancellation of the phase shifts the impurities do not affect the transition temperature or excitation spectrum of any possible superconducting state in the quasiclassical approximation.

INTRODUCTION

LITTLE[1] called attention to the possible existence of superconductivity in one-dimensional systems. Such a system was investigated theoretically by Bychkov, Gor'kov, and Dzyaloshinskiĭ (BGD)[2,3]. They have shown that the structure of the superconducting system can be much more complicated in the one-dimensional system than in the three-dimensional system, because in the one-dimensional case there can exist two types of bound states: Cooper pairs with resultant zero momentum, and an electron-hole pair with momentum $2p_0$ (p_0—Fermi momentum).

We investigate in this paper, on the basis of the work of BGD, a one-dimensional superconducting system containing impurities. The employed model is one-dimensional because the potential of the impurity depends on the distance along a straight line. In real systems, the transverse dimension of the chain is determined by the dimension of the atoms. The variation on the potential in the transverse direction is neglected, but this leads only to slight corrections to the final results. An analysis is not applicable to molecules whose electrons can move from one point to the other along different paths.

In the first section we compare the temperature of the possible transition of pure and impurity chains1). The Green's functions are calculated in the quasiclassical approximation (more accurately, in the case when the impurity potential is smaller than the kinetic energy of the electrons at the Fermi level). Particular attention is paid to diagrams whose contribution contains typical logarithmic expressions, which were taken into account in the BGD papers in the determination of the transition temperature. We shall use here coordinate space rather than momentum space. As a result of a careful investigation of the problem of averaging over the impurity locations, it is shown that it is possible to average only in the final result, and that averaging in individual parts of the diagrams leads to erroneous expressions. We find that in our approximation the impurities do not influence the transition temperature, because in the quasi-classical approximation the electron phase shifts connected with impurities cancel each other. This cancellation is realized only in a linear system in which the electrons can move from one point to the other along a single path, and therefore the phase shift has a definite meaning.

In the second section we investigate the superconducting state. We do not investigate the feasibility of the superconducting state in a one-dimensional system. Many objections were raised[4] against the existence of superconductivity in linear systems. These objections are valid only for an infinite system[5], and although we do assume that our system is finite, it is regarded as sufficiently long to be able to neglect the boundary effects. The solution of the equation for the Green's functions, as obtained in the BGD papers, shows that the physical quantities such as the energy spectrum and the coherence length, are not sensitive to the presence of impurities. The influence of impurities is appreciable only in cases when the linear system is investigated by trial particles which do not belong to the system.

1. CALCULATION OF THE FIRST CORRECTION TO VERTEX PART IN THE PRESENCE OF IMPURITIES

BGD have shown that the diagrams of second order give a logarithmic contribution of the type $\ln(\epsilon_0/T)$, where ϵ_0 is the cutoff energy (equal in order of magnitude to the Fermi energy) and T is the temperature. They are the diagrams of the Cooper pair and of the particle-hole pair (of the zero-sound type). We calculate below, on the basis of the results of Gor'kov and Dzyaloshinskiĭ, the contributions of these diagrams, taking the influence of the impurities into account.

The electron interaction is of the form

$$\overset{0}{\Gamma}_{\alpha\beta\gamma\delta}(p_1p_2p_3p_4) = g(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}), \tag{1}$$

where the coupling constant g is small. Such a local

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1)The transition temperature is the temperature at which the vertex part of the normal system diverges. (The contribution of the diagrams is calculated in the logarithmic approximation, and therefore an arbitrary factor appears in the expression for the transition temperature.

FIG. 1.

interaction corresponds to the effective electron-electron interaction proposed by Little[1], and contains the screened Coulomb interaction. Gor'kov and Dzyaloshinskiĭ have shown that the Coulomb interaction is screened, because the entire Bose complex is in this case electrically neutral.

The contribution of the vertex corrections of second order of the Cooper and zero-sound types (Figs. 1a and b) is given by

$$A(x_1, x_2) = T\sum_{\omega} G_{\omega}(x_1, x_2)\, G_{-\omega}(x_1, x_2) \tag{2}$$

and

$$B(x_1, x_2) = T\sum_{\omega} G_{\omega}(x_1, x_2)\, G_{\omega}(x_2, x_1), \tag{3}$$

where G_{ω} is the single-particle temperature Green's function. In these formulas, the spin indices are not written out. The equation for the Green's function is

$$\left\{ i\omega + \frac{1}{2m}\frac{\partial^2}{\partial x_1^2} + \mu - V(x_1) \right\} G_{\omega}(x_1, x_2) = \delta(x_1 - x_2), \tag{4}$$

where $V(x)$ is the potential of the impurities and μ is the chemical potential. The free Green's function (solution for $V(x) \equiv 0$) are written in the form

$$G_{\omega}^{(0)}(x) = G_{\omega, r}^{(0)}(x) + G_{\omega, l}^{(0)}(x), \tag{5}$$

where (in the case when $\omega \ll \mu$)

$$G_{\omega, \alpha}^{(0)}(x) = \frac{e^{-|\omega x|/v}}{2vi}\, e^{\pm i p_0 x}\, [\operatorname{sign}(\omega) \pm \operatorname{sign}(x)] \tag{6}$$

(the plus sign in (6) is taken for $\alpha = r$, and the minus sign for $\alpha = l$). Here $p_0 = mv_0$ is the Fermi momentum and $x = x_1 - x_2$. The separation of the Green's functions into two parts corresponds to the fact that the electrons with momentum $p < 0$ and momentum $p > 0$ (the left and right sides of the momentum space) are investigated separately.

Assuming that $\omega \ll \mu$ and $V \ll \mu$, the solution of (4) is written in the form

$$G_{\omega}(x_1, x_2) = G_{\omega, r}(x_1, x_2) + G_{\omega, l}(x_1, x_2), \tag{7}$$

where

$$G_{\omega, \alpha}(x_1, x_2) = G_{\omega, \alpha}^{(0)}(x_1 - x_2) \exp\left\{ \pm \frac{i}{v} \int_{x_1}^{x_2} V(z)\, dz \right\}$$

$$\sim \exp\left\{ \pm i \Big[p_0(x_1 - x_2) + \frac{1}{v} \int_{x_1}^{x_2} V(z)\, dz \right\}. \tag{8}$$

Expression (8) shows that the wave function of the electrons near the Fermi surface is multiplied only by a phase shift that depends on the coordinates. The separation of the Green's function into left and right parts at energies close to the Fermi energy continues to be meaningful, owing to the assumption $V \ll \mu$.

Calculating the vertex part of the Cooper type, we find that its contribution does not depend on the presence or absence of impurities. Indeed, substituting the Green's function (7)–(8) in (2), we obtain (for example, or $x_1 > x_2$)

$$A(x_1, x_2) = T\sum_{\omega} G_{\omega, r}^{(0)}(x_1 - x_2)\, G_{-\omega, l}^{(0)}(x_1 - x_2). \tag{9}$$

The phase factors, which depend on the potential, cancel each other here. In the case of an electron-hole pair, such a cancellation does not take place (for $x_1 > x_2$):

$$B(x_1, x_2) = T\sum_{\omega > 0} G_{\omega, r}^{(0)}(x_1 - x_2)\, G_{\omega, l}^{(0)}(x_2 - x_1)$$

$$\times \exp\left\{ +2\frac{i}{v} \int_{x_1} V(z)\, dz \right\}$$

$$+ T\sum_{\omega < 0} G_{\omega, l}^{(0)}(x_1 - x_2)\, G_{\omega, r}^{(0)}(x_2 - x_1) \exp\left\{ -2\frac{i}{v} \int_{x_1}^{x_2} V(z)\, dz \right\}. \tag{10}$$

The phase factors can be averaged by assuming a random impurity distribution

$$\overline{V(x)} = 0, \qquad \overline{V(x_1)\, V(x_2)} = \frac{v}{\tau}\, \delta(x_1 - x_2),$$

where τ is the relaxation time. We get

$$\overline{B(x_1, x_2)} = T\sum_{\omega} G_{\omega}^{(0)}(x_1 - x_2)\, G_{\omega}^{(0)}(x_2 - x_1)\, e^{-2|x|/l} \tag{11}$$

where $l = \tau v$.

The typical logarithmic correction $(g^2/2\pi v)\ln(\epsilon_0/T)$ is obtained from (9) by integrating $A(x)$ with respect to x in the interval (r_0, ∞), where r_0 is the small-distance cutoff length. A similar calculation for $\int_{r_0}^{\infty} B(x)\, dx$ yields a similar logarithmic expression for the almost pure chain $(T\tau \gg 1)$, but with negative sign. In the presence of impurities we have

$$\int_{r_0}^{\infty} \overline{B(x)}\, dx = -\frac{g^2}{2\pi v} \ln \varepsilon_0 \tau$$

and this can be smaller than for the pure chain. This raises the question whether it is legitimate to average over the impurity distribution in individual parts of diagrams of the "parquet" type[2-3] on Fig. 2. In the case of averaging in individual parts of the diagrams, the contribution of the zero-sound type can be suppressed by increasing the impurity concentration.

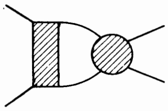

FIG. 2.

Before we answer this question, let us calculate the contribution of the diagrams in the logarithmic approximation, performing the integration in coordinate space.

2. CALCULATION OF DIAGRAMS IN COORDINATE SPACE

BGD[2,3] calculated typical diagrams of the "parquet" type, using momentum variables. The purpose of their calculation was to take into account the logarithmic terms of the highest (of the type $g[g \ln(\epsilon_0/T)]^n$). We investigate the behavior of such terms by integration in coordinate space. Let us consider separately diagrams corresponding to two parts of the Green's function $G_{\omega,r}$ and $G_{\omega,l}$, marking the corresponding lines with the indices r and l. Since $G^{(0)}_{\omega,r}$ and $G^{(0)}_{\omega,l}$ (6) contain the factor $(1/2)[\mathrm{sign}(\omega) - \mathrm{sign}(x)]$, which has values ±1 or 0, depending on the sign of ω and x, it is advantageous to carry out the integration with respect to x separately for the different relative spatial arrangements of the variables. We shall designate the order of the arrangement of the variables of the bare interaction along the horizontal direction of the diagrams. Instead of discussing the general case, we investigated a typical diagram, shown in Fig. 3, which, according to BGD, makes a contribution $\sim g^4 \ln^3(\epsilon_0/T)$. The dashed line separates those parts of the diagram, which give the logarithmic contribution. Figure 4 shows three types of different spatial distributions of the variables. The distances between the elementary vertex parts are marked on the horizontal line under the diagram. All the distances are positive.

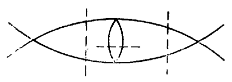

FIG. 3.

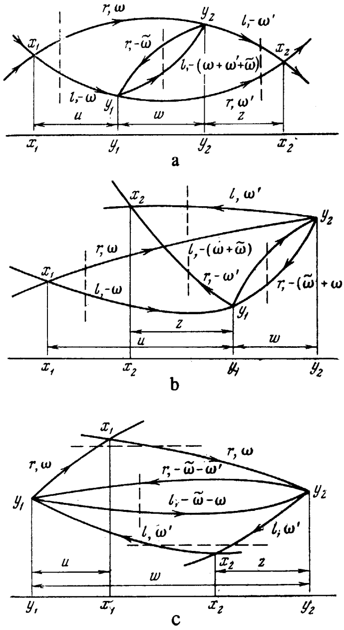

FIG. 4.

We consider first the diagram on Fig. 4a. The energy variables were chosen such that its contribution does not vanish only in the case when all the vertices ω, ω', and $\tilde{\omega}$ are positive. The spatial order of the points y_1 and y_2 is important, for if the order is reversed the contribution vanishes at positive energies. The oscillating exponential factors of the Green's functions (6) cancel each other at the points x_1 and x_2, but, more significantly, they cancel each other also at the points y_1 and y_2. If this were not so, then the integral of the rapidly oscillating factors $\exp(\pm 2ip_0 y)$ or $\exp(\pm 4ip_0 y)$ with limits y_1 and y_2 would approximately vanish.

The contribution of the given order of arrangement of the variables is given by the expression

$$
\begin{aligned}
& g^4 \int_{r_0}^{\infty} dz \int_{r_0}^{\infty} dw \int_{r_0}^{\infty} du \frac{T^3}{v^6} \\
& \times \sum_{\omega>0} \sum_{\omega'>0} \sum_{\tilde{\omega}>0} e^{-2(u+w)\omega/v} \\
& \times e^{-2(w+z)\omega'/v} e^{-2w\tilde{\omega}/v} \\
= & \, g^4 \left(\frac{T}{2v^2}\right)^3 \int dz \int dw \int du \cdot \\
& \times \mathrm{sh}^{-1}\left(2\pi T \frac{u+w}{v}\right) \\
& \times \mathrm{sh}^{-1}\left(2\pi T \frac{w+z}{v}\right) \\
& \times \mathrm{sh}^{-1}\left(2\pi T \frac{w}{v}\right).
\end{aligned}
\tag{12}
$$

We now integrate with respect to u and z, introducing the new variable $s = 2\pi Tw/v$ and retaining only the significant terms having the maximum power of the logarithm. We then obtain

$$
\begin{aligned}
& \frac{g^4}{(2\pi v)^3} \int_{2\pi T\varepsilon_0}^{\infty} ds (\ln s)^2 \frac{1}{\mathrm{sh}\, s} ds \\
& \sim g^4 \left(\ln \frac{\varepsilon_0}{T}\right)^3
\end{aligned}
\tag{13}
$$

A similar result is obtained also in the case shown in Fig. 4b. The situation is entirely different in the case of Fig. 4c. The integrations cannot be carried out completely independently of one another, and yield

$$
\begin{aligned}
& g^4 \left(\frac{T}{v^2}\right)^3 \int_{r_0}^{\infty} dw \int_{r_0}^{w} du \int_{r_0}^{w} dz \, \mathrm{sh}^{-3}\left(2\pi T \frac{w}{v}\right) \\
& \sim \frac{g^4}{(2\pi v)^3} \int_{2\pi T\varepsilon_0}^{\infty} ds \, s^2 \frac{1}{\mathrm{sh}^3 s} \sim g^4 \left(\ln \frac{\varepsilon_0}{T}\right),
\end{aligned}
\tag{14}
$$

where $s = 2\pi Tw/v$.

Thus we see that limiting the integration interval makes the degree of the logarithm expression smaller than without such a limitation.

From this example we see the following.

1. A significant contribution is made only by such arrangements of the internal vertex points, whose exponential factors (for example $\exp[+2ip_0y]$) cancel each other.

2. By investigating diagrams of the "parquet" type in momentum space, BGD found that the highest degree of the logarithmic expression $\ln(\epsilon_0/T)$ is determined by the maximum number of such cuts along two lines of the diagrams, which cause the diagram to break up into two unconnected parts. Three such cuts are shown in Figs. 3 and 4. The integration interval is not bounded when the cut can be drawn vertically. If this cannot be done, then the cut does not lead to the appearance of a logarithmic term (see diagram 4c).

3. INFLUENCE OF IMPURITIES ON THE LOGARITHMIC TERMS AND CRITICAL TEMPERATURE

In the presence of impurities in the Green's function, there appears an additional phase shift containing an integral of the impurity potential (see Eq. (8)). We investigate these new phase factors again in the case shown in Fig. 3. We take an arrangement of variables corresponding to Fig. 4a. Using the Green's function (8), we see that the additional phase factor in the contribution (12) vanishes:

$$\exp\left\{\frac{i}{v}\left[\int_{x_1}^{y_2} V(z)\,dz-\int_{x_1}^{y_1} V(z)\,dz-\int_{y_2}^{x_2} V(z)\,dz+\int_{y_1}^{x_2} V(z)\,dz\right.\right. \tag{15}$$

$$\left.\left.+\int_{y_2}^{y_1} V(z)\,dz-\int_{y_1}^{y_2} V(z)\,dz\right]\right\}=1.$$

The same is obtained in the case of Figs. 4b and c. If we were to substitute in the expression for the contribution of the diagram the average value of the closed loop lying between the vertices y_1 and y_2, then we would obtain, in accordance with (11), the factor $\exp\{-2\,|y_1-y_2|/l\}$, and therefore we would obtain not the correct result $g^4\ln^3(\epsilon_0/T)$, but $g^4\ln^2$ $g^4\ln^2(\epsilon_0/T)\ln\epsilon_0\tau)\ln\epsilon_0\tau$. It follows therefore that it is impossible to average over the distribution of the impurity separately for different parts of the diagram. If this is necessary, then it is possible to average only in the final result.

We now investigate the impurity phase factor for any diagram, without averaging over the impurity distribution. We first investigate the dependence of the phase factor on the coordinates of the internal bare interactions. We see that a significant contribution is obtained from those regions of the intervals of the internal bare interactions, in which the phase factors of the type $\exp(\pm i2p_0y)$ which do not depend on the impurities, vanish. It is easy to establish that the integrals of the impurity potential in the additional phase factors likewise do not depend on the position of the internal vertex parts in the significant interval, since the signs in front of the phase factors containing the momentum p_0 and the impurity potential V are in a unique correspondence. The limits of the integrals in the additional phase factors are the coordinates of the bare interactions lying on the boundary of the diagram. Let $\Gamma_{s_1s_2\ldots s_{2n}}(x_1, x_2, \ldots, x_n\,|\,x_{n+1}, \ldots x_{2n}\,|\,V)$ denote the generalized vertex part with 2n variables, where the indices $s_1, s_2, \ldots, s_{2n}$ denote the indices r and l of the incoming and outgoing lines. The vertex part Γ can be represented in the form of the product

$$\Gamma_{s_1s_2\ldots s_{2n}}(x_1\ldots x_n\,|\,x_{n+1}\ldots x_{2n}\,|\,V)$$

$$=\overset{0}{\Gamma}_{s_1s_2\ldots s_{2n}}(x_1\ldots\, x_n\,|\,x_{n+1}\ldots x_{2n})$$

$$\times\exp\left\{\frac{i}{v}\varphi_{s_1s_2\ldots s_{2n}}(x_1\ldots x_n\,|\,x_{n+1}\ldots x_{2n}\,|\,V)\right\}, \tag{16}$$

where Γ^0 gives the vertex part in the absence of impurities. The phase factor φ contains n integrals of the impurity potential.

It should be mentioned that if it is assumed that the integrals have a positive sign in the phase factor, then the upper limits of integration are the coordinates of the vertex parts on the boundary of the diagram, which are connected with the outgoing lines with index r or with the incoming lines with index l.

Inasmuch as the region of applicability of (16) is broader than the logarithmic approximation, the phase factors that depend on the impurities are separated in the equation for the vertex part of the "parquet" type, and we are left with an equation for the vertex part of the pure system. Therefore the divergence of the vertex part appears at the same temperature in both cases. Thus, if $V \ll \mu$, then the critical temperature does not depend on the impurity concentration.

4. SUPERCONDUCTING STATE

The theory of the possible superconducting state in a one-dimensional system was developed by BGD. We took into account two types of anomalous pairings, viz., pairs of the Cooper and zero-sound type. A divergence appears at the critical temperature in the scattering amplitudes of both pairs. Introducing two anomalous Green's functions, BGD decoupled the equations, in a manner similar to that used in the three-dimensional case. We shall briefly discuss here the case when there are no impurities, writing out the equations in coordinate space. We assume first that the self-energy parts Δ and κ, corresponding to anomalous pairing, are local functions in space (we shall subsequently discard this assumption). As in the normal case, i.e., above the critical temperature, we separate the Green's functions into two parts and determine them from the BGD solution in the Fourier representation:

$$G_\omega^{(0)}(x)=G_{\omega,r}^{(0)}(x)+G_{\omega,l}^{(0)}(x),$$

$$F_\omega^{+(0)}(x)=F_{\omega,r}^{+(0)}(x)+F_{\omega,l}^{+(0)}(x),$$

$$F_\omega^{(0)}(x)=F_{\omega,r}^{(0)}(x)+F_{\omega,l}^{(0)}(x), \tag{17}$$

where

$$G_{\omega,\alpha}^{(0)}(x_1-x_2)=e^{\pm ip_0(x_1-x_2)}\frac{e^{-\sqrt{\omega^2+\delta^2}\,R/v}}{2vi}\left(\frac{\omega}{\sqrt{\omega^2+\delta^2}}+\operatorname{sign}(x)\right)$$

$$F_{\omega,\alpha}^{+(0)}(x_1-x_2)=e^{\pm ip_0(x_1-x_2)}\frac{e^{-\sqrt{\omega^2+\delta^2}\,R/v}}{2v}\,\frac{\Delta^*}{\sqrt{\omega^2+\delta^2}},$$

$$F_{\omega,\alpha}^{(0)}(x_1-x_2)=e^{\mp ip_0(x_1-x_2)}\frac{e^{-\sqrt{\omega^2+\delta^2}\,R/v}}{2v}\,\frac{\Delta}{\sqrt{\omega^2+\delta^2}}$$

(the upper sign in the right sides is taken for the functions with index α = r, and the lower one for functions with index $\alpha = l$).

Further, we introduce two new Green's functions, which take into account production or annihilation of the electron-hole pair with momentum $\pm 2p_0$:

$$G_\omega^{\mp(0)}(x_1, x_2)=e^{\mp ip_0(x_{10}+x_{20})}\frac{e^{-\sqrt{\omega^2+\delta^2}\,R/v}}{2v}\,\frac{\varkappa^*}{\sqrt{\omega^2+\delta^2}}. \tag{18}$$

Here

$$\delta^2=|\varkappa|^2+|\Delta|^2, \tag{19}$$

δ is the total energy gap; $x_{i0} = x_i - x_0$, where x_0 is an arbitrary point, and for simplicity we assume $x_0 \to -\infty$. The Green's functions (17) and (18) are solutions of the decoupled equation, which can be written in coordinate space in the form

$$\left(i\omega+\frac{1}{2m}\frac{\partial^2}{\partial x_1{}^2}+\mu\right)G_{\omega,r}^{(0)}(x_1-x_2)+\Delta F_{\omega,r}^{+(0)}(x_1-x_2)$$

$$+\varkappa e^{2ip_0x_{10}}G_\omega^{-(0)}(x_1, x_2)=\delta(x_1-x_2), \tag{20a}$$

$$\left(i\omega-\frac{1}{2m}\frac{\partial^2}{\partial x_1{}^2}+\mu\right)F_{\omega,r}^{+(0)}(x_1-x_2)+\Delta^* G_{\omega,r}^{(0)}(x_1-x_2)=0, \tag{20b}$$

$$\left(i\omega + \frac{1}{2m}\frac{\partial^2}{\partial x_1{}^2} + \mu\right)G_\omega^{-(0)}(x_1, x_2) + \varkappa^* e^{-2ip_0x_{10}} G^{(0)}_{\omega,\,r}(x_1 - x_2) = 0. \quad \text{(20c)}$$

Equations of the same type are obtained by making the substitutions r ⟷ *l* and + ⟷ –.

A similar system of equations is derived also in the presence of impurities. It can be solved in the quasi-classical approximation. The Hamiltonian is supplemented by the potential energy, and a dielectric gap is introduced with a phase factor, such as to satisfy the self-consistence requirement

$$\varkappa e^{2ip_0x_{10}} \to \varkappa e^{2ip_0x_{10}} \exp\left\{-\frac{2i}{v}\int_{x_0}^{x_1} V(z)\,dz\right\}. \quad (21)$$

The solution of the obtained equations follows from the solution for the pure chain by adding the phase factors discussed in the preceding section. For example,

$$G_{\omega,\,r} \sim \exp\left\{ip_0(x_1 - x_2) + \frac{i}{v}\int_{x_1}^{x_2} V(z)\,dz\right\},$$

$$G_\omega^{\mp} \sim \exp\left\{\mp ip_0(x_{10} + x_{20}) \pm \frac{i}{v}\left[\int_{x_0}^{x_1} V(z)\,dz + \int_{x_0}^{x_2} V(z)\,dz\right]\right\},$$

$$F^{+}_{\omega,\,r} \sim \exp\left\{ip_0(x_1 - x_2) + \frac{i}{v}\int_{x_1}^{x_2} V(z)\,dz\right\}. \quad (22)$$

So far it was assumed that the self-energy part is a local function. We now generalize the results. We assume that the separation of the Green's functions into parts corresponding to the left and right sides of the momentum space is not connected with the locality of the self-energy part. The phase factors of the Green's function are in this case likewise independent of the locality, and only the non-oscillating amplitudes are sensitive to the structure of the self-energy part. Equation (20a) takes the form

$$\left(i\omega + \frac{1}{2m}\frac{\partial^2}{\partial x_1{}^2} + V(x_1) + \mu\right)G_{\omega,\,r}(x_1, x_2) + \int d^2x_3\,\Delta(x_1, x_3)$$
$$\times \exp\left\{ip_0(x_1 - x_3) + \frac{i}{v}\int_{x_3}^{x_1} V(z)\,dz\right\}F^{+}_{\omega,r}(x_3, x_2)$$
$$+ \int d^2x_3\,\varkappa(x_1, x_3)\exp\left\{ip_0(x_{10} + x_{30})\right.$$
$$\left. - \frac{i}{v}\left[\int_{x_0} V(z)\,dz + \int_{x_0}^{x_3} V(z)\,dz\right]\right\}G_\omega(x_3, x_2) = \delta(x_1 - x_2). \quad (23)$$

It can be shown that in such a choice of phases, the self-consistency of the solution is satisfied. Let us investigate, for example, the contribution of the diagram (Fig. 5) to the self-energy part. Substituting (22) in the contribution of this diagram, we can show that the obtained phase is taken into account in (23).

From the foregoing we can draw, in the quasi-classical approximation, the following conclusions.

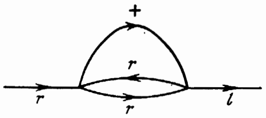

FIG. 5.

1. The amplitude of the self-energy part does not depend on the impurity concentration.

2. The single-particle energy spectrum is likewise independent of this concentration. The spectrum is determined from the single-particle Green's function

$$\rho(E) = -\frac{1}{2\pi i}\int \{G(x, x; E + i\varepsilon) - G(x, x; E - i\varepsilon)\}\,dx,$$

where $G_\omega = G(E = i\omega)$. Substituting the Green's function from (22), we see that $\rho(E)$ actually does not depend on the impurity concentration.

3. In a manner similar to the foregoing, we can prove that the function of the after effect of the current on the vector potential likewise is independent of the impurities. Consequently, in the quasi-classical approximation the coherence length does not depend on the impurities. The results obtained in the three-dimensional case is entirely different.

Thus, we conclude that the physical properties of a one-dimensional system in the quasi-classical approximation did not depend on the concentration of the impurities, if the system is investigated separately from other systems.

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