DYNAMICS OF GENERATION OF A PULSED MODE-LOCKING LASER

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The dynamics of generation of ultrashort pulses in a laser with mode-locking by external loss modulation and Q switching is considered theoretically. The approach is based on an analysis of the behavior of a light pulse moving inside the resonator and varying its duration and intensity as a result of amplification, dispersion of the refractive index, and loss modulation. It is shown that the evolution of the pulse duration consists of two stages. During the first stage the pulses are shortened as a result of the increased number of the locked modes, and during the second it broadens as a result of the dispersion of the medium inside the resonator. The theory explains the results of a number of experiments and points the way towards obtaining ultrashort light pulses of giant power using external loss modulation.

1. INTRODUCTION

MUCH progress has been made recently in the field of generation of ultrashort pulses of light by the method of axial mode locking in a solid-state laser.[1)] In the first experiments they used external periodic modulation of the loss of a laser operating in the spike regime, attaining thereby the locking of approximately 20 modes (pulse duration 5×10^{-10} sec).[2, 3] In cw generation the number of locked modes increases, and the duration decreases by one more order of magnitude ($\sim 4 \times 10^{-11}$ sec).[4] The next significant step consisted[5] of realizing simultaneous locking of a large number of modes and Q-switching of the laser with the aid of a saturable solution. The pulse duration in such a laser reaches $\sim 5 \times 10^{-12}$ sec.[6]

The theory of locking of the large number of modes of a solid-state laser in the stationary generation regime and the limiting duration of the pulses were first presented in [7]. It was shown that an important role is played by the dispersion of the medium inside the resonator in the limitation of the number of locked modes in the continuous operation regime. The estimate obtained for the limiting duration ($\sim 10^{-11}$ sec) agrees with experiment.[4]

An effective method of obtaining ultrashort pulses of giant power is to use a nonstationary regime, such as the regime with instantaneous Q-switching. However, only a total of 20 modes could be locked in the existing experiments with nonstationary generation.[2, 3] The purpose of the present investigation was to consider theoretically the dynamics of generation of ultrashort pulses in a laser with external loss modulation and Q switching. The proposed theory explains the results of the experiments[2, 3] and points the way towards obtaining ultrashort pulses of giant power with external loss modulation. However, the results do not hold for a laser with self-phasing of the modes by a nonlinear absorber.[2)]

It is shown below that the evolution of the pulse duration consists of two stages. During the first stage, the pulse duration is reduced relatively slowly, as a result of the increase in the number of locked modes, and during the second stage the pulses broaden as the result of dephasing of the mode owing to dispersion of the medium inside the resonator. At some instant of time, the pulse duration reaches a minimum. The change of the pulse energy are of the average radiation power and is similar to the change of the radiation intensity in a Q-switched laser. The optimal case occurs when the maximum average power and the minimum pulse duration are reached simultaneously. To obtain ultrashort pulses with maximum power, it is proposed to increase instantaneously the Q at the instant when minimum duration is reached, for a time sufficient to allow the ultrashort pulse to radiate the stored energy.

2. MODEL AND INITIAL EQUATIONS

In [7] we used an oscillatory approach to the analysis of the behavior of each mode out of the aggregate of interacting modes, taking into account the capture or the phasing of the modes even in the presence of a certain detuning (dephasing). Such an approach is perfectly adequate when the stationary regime is considered. When the nonstationary regime is considered, and the duration of the entire generation process amounts to 10^{-5}–10^{-6} sec, the approach used in [7] is not very suitable and, furthermore, mode locking in the presence of a certain detuning, which is characteristic for nonlinear self-oscillating systems, plays here a negligibly slight role. The mode dephasing, for example as a result of the the dispersion of the medium inside the resonator, is most probably cumulative and in final analysis should lead to a gradual increase of the duration of the generated pulses. We therefore used in the present article a different approach to the consideration of the nonstationary regime, based on an analysis of the behavior of a pulse moving inside the resonator and changing its duration and intensity as a result of amplification, dispersion, and loss modulation.

We consider a multimode laser, in whose resonator there moves a single light pulse due to the locking of a large number of axial modes. In fact, usually two pulses move opposite each other inside the resonator. In the

1)The first successful experiment on laser mode locking was performed [1] with a gas laser.

2)A number of questions involved in steady-state laser generation with mode self-locking is considered in [8,9], However, the processes in such a laser are essentially nonstationary, and the time evolution of the pulse duration and the self-Q-switching are interrelated. These effects were considered by us and the results were published briefly [10]. A detailed exposition will be presented later.

case of external loss modulation, their interaction is negligibly small, and it is sufficient to consider the evolution of one pulse. During each passage through the resonator, the pulse is compressed by the external modulation of the transmission of the element inside the resonator, is amplified and broadened in the active dispersive medium, and is attenuated upon reflection from the output mirror as a result of the emergence of the radiation.

The field at the fixed point inside the resonator E(t) is the superposition of the fields in different axial modes:

$$E(t)=\mathrm{Re}\sum_m A_m(t)\exp\{i\omega_0 t+i\varphi_m(t)\}, \tag{1}$$

where A_m and φ_m are the amplitude and phase of the field in the m-th mode, and ω_0 is the frequency of the mode closest to the center of the line. The difference between the mode frequencies is included in the phase of the field. For example, the phase difference of the field in two neighboring modes $\varphi_{m+l}(t)-\varphi_m(t)\approx\Omega t$, where $\Omega = 2\pi v/L$ is the frequency distance between the modes ($v = c(n_0+\omega_0\partial n/\partial\omega)^{-1}$ is the group velocity of the light in the resonator, $n(\omega)$ is the refractive index of the medium in the resonator, L is the resonator length, and c is the speed of light in vacuum).

During each pass of the pulse E(t) through the resonator, the form of the pulse changes as a result of amplification, loss modulation, and dispersion. If the transmission of the modulating element varies like[3)] $\eta(t) = \eta_0(1+\rho\cos\Omega t)$, then the change of the field per pass is:

$$E_1(t)=\eta^{1/2}(t)E(t)=\eta_0^{1/2}(1+{}^1/_2\rho\cos\Omega t)E(t), \tag{2}$$

where $\rho \ll 1$. The approximation $\rho \ll 1$ has no fundamental significance whatever, but greatly simplifies the derivations.

The gain and the radiation output loss change the field in the following manner:

$$E_2^2(t)=K(t)rE^2(t), \tag{3}$$

where K(t) is the radiation gain in the active medium per pass and r is the reflection coefficient of the output mirror. We confine ourselves here to the case when the spectral region of the locked modes is much smaller than the width $\Delta\omega$ of the amplification line. This assumption corresponds fully to the experimental conditions. In this case the amplification of all the modes is practically the same.

Dispersion causes the phase of the m-th mode to shift during one pass by an amount $\psi_m = \omega_m c^{-1}Ln(\omega_m)$, which depends on the mode frequency. As a result the field changes as follows:

$$E_3(t)=\sum_m A_m(t)\exp\{i\omega_0 t+i\varphi_m(t)+i\psi_m\}. \tag{4}$$

The total change of the field per pass is

$$E(t+T)=[K(t)r\eta(t)]^{1/2}\sum_m A_m(t)\exp\{i\omega_0 t+i\varphi_m(t)+i\psi_m\}, \tag{5}$$

where $T = L/v$ is the time required for the pulse to pass through the resonator.

3)The evolution of the pulse at an arbitrary law of periodic modulation will be considered below.

The gain of the active medium is given by the expression $K=\exp\{\sigma_0 N(t)l\}$, where σ_0 is the cross section of the radiative transition between the levels at the frequency ω_0, l is the length of the active medium, and N(t) is the density of the inverted population of the active-particle levels, satisfying the equation

$$\frac{\partial N}{\partial t}+\frac{N-N_0}{T_1}=-\frac{N(t)}{\mathscr{E}_s}\frac{c}{8\pi}\langle E^2(t)\rangle, \tag{6}$$

where T_1 is the spontaneous lifetime of the particles at the upper levels, N_0 is the density of the inverted population in the absence of the field and is determined by the pumping rate, $\langle\ \rangle$ denotes averaging over the optical period, $\mathscr{E}_S$ is the gain saturation energy. For a three-level system (ruby) $\mathscr{E}_S=\hbar\omega_0/2\sigma_0$, and for a four-level system (neodymium glass) $\mathscr{E}_S=\hbar\omega_0/2\sigma_0$ when $\tau_0 \gg \tau_S$ or $\mathscr{E}_S=\hbar\omega_0/\sigma_0$ when $\tau_0 \ll \tau_S$, where τ_0 is the lifetime of the particle at the lower working level and τ_S is the characteristic time during which the gain saturation, determined by the average radiation power, takes place.

If the self-excitation condition

$$r\eta_0(1+\rho)K(0)\geqslant 1 \tag{7}$$

is satisfied at the initial instant of time t = 0, then the lasing regime sets in.

3. EQUATIONS FOR THE AMPLITUDES AND PHASES OF THE MODES

The finite-difference equation (5) can be reduced to two differential equations for the amplitudes and phases of the modes. To this end, we transform the left side of (5) as follows:

$$\begin{aligned}E(t+T)&=\sum_m A_m(t+T)\exp\{i\omega_0(t+T)+i\varphi_m(t+T)\}\\&=e^{i\omega_0 t}\sum_m A_m(t+T)\exp\{i\varphi_m(t+T)\},\end{aligned} \tag{8}$$

where ω_0/Ω is an integer and $\Omega T = 2\pi$. The right side of (5) can be represented in the form

$$[K(t)r\eta_0]^{1/2}e^{i\omega_0 t}\sum_m A_m(t)\Big(1+\frac{\rho}{4}e^{-i\Omega t}+\frac{\rho}{4}e^{i\Omega t}\Big)\exp\{i\varphi_m(t)+i\psi_m\}. \tag{9}$$

Substituting (8) and (9) in (5) and equating coefficients of the corresponding harmonics, we can readily obtain a system of finite-difference equations for the amplitudes and phases. To this end, however, it is necessary to clarify the character of the time variation of the gain K(t). In the region of the nonlinear development of the generation, K is constant and is determined by the initial value. With increasing pulse energy, a small decrease of the gain takes place during each pass. For example, for neodymium glass, the saturation energy $\mathscr{E}_S$ is several J/cm², and the maximum pulse energy in a mode-locking laser is 10^{-3} J/cm².[2,3,5] Consequently, the amplitudes of the Fourier components of the function K(t) at the frequencies $\pm\Omega$ do not exceed 10^{-3}–10^{-4}. An approximation is therefore possible, in which the changes of the gain within times on the order of T are neglected and only the slow variations of the gain within times $\tau_S \gg T$ are taken into account. Physically this means that we neglect the distortions of the pulse waveform (such as the preferred amplification of the leading front) on passing through the active medium.[11–14] 4)

Thus, substituting (8) and (9) in (5) we get

$$\begin{aligned} A_m(t+T)\exp\{i\varphi_m(t+T)\} \\ =(\eta_0 rK)^{1/2}[A_m(t)\exp\{i\varphi_m(t)+i\psi_m\} \\ +{}^1/_4\rho A_{m-1}(t)\exp\{i\varphi_{m-1}(t)+i\psi_{m-1}+i\Omega t\} \\ +{}^1/_4\rho A_{m+1}(t)\exp\{i\varphi_{m+1}(t)+i\psi_{m+1}-i\Omega t\}], \end{aligned} \quad (10)$$

where $m = 0, \pm 1, \pm 2, \ldots$.

In considering ultrashort light pulses, when the number of locked modes is large, the amplitude and phase differences between neighboring modes can be regarded as sufficiently small:

$$|A_{m\pm1}-A_m|\ll A_m, \quad |\varphi_{m\pm1}-\varphi_m|\ll\varphi_m. \quad (11)$$

Then Eq. (10) reduces to two equations for the amplitudes and phases:

$$A_m(t+T)=(\eta_0 rK)^{1/2}[A_m(t)+{}^1/_4\rho A_{m-1}(t)+{}^1/_4\rho A_{m+1}(t)], \quad (12)$$

$$\varphi_m(t+T)=\varphi_m(t)+\psi_m. \quad (13)$$

The physical meaning of Eqs. (12) and (13) is perfectly clear. The amplitude equation describes the coupling of the neighboring modes as a result of loss modulation, and the phase equation describes the phase shift due to the dispersion of the medium.

The finite-difference amplitude equation (12) is best transformed into a differential equation. The condition (11) allows us to go over to differentiation with respect to m. If the effective gain per pass $\kappa = \eta_0 rK$ differs little from unity, we can also go over to differentiation with respect to time. As a result we obtain for the mode amplitudes a partial differential equation of the parabolic type:

$$T\frac{\partial A_m(t)}{\partial t}=[\sqrt{\varkappa(1+\rho)}-1]A_m(t)+\frac{\rho}{4}\sqrt{\varkappa}\frac{\partial^2 A_m(t)}{\partial m^2}. \quad (14)$$

At a constant gain K(t), the solution of (14) is

$$A_m(t)=A_0\frac{\exp\{(\sqrt{\varkappa(1+\rho)}-1)t/T\}}{\sqrt{\pi\rho\varkappa^{1/2}t/T}}\exp\left\{-\frac{m^2}{\rho\varkappa^{1/2}t/T}\right\}, \quad (15)$$

where approximately $A_0 \sim A_0(0)$. It is meaningless to determine A_0 more accurately, since (15) has been derived under the assumption that $t/T \gg 1$.

The phase equation (13) has a solution

$$\varphi_m(t)=\frac{t}{T}\psi_m, \quad (16)$$

where we put $\varphi_m(0) = 0$. The phase shift of the field in the m-th mode $\psi_m = \omega_m c^{-1} Ln(\omega_m)$ in the region $m\Omega \ll \omega_0$ is determined, accurate to terms of second order in $m\Omega/\omega_0$, by the expression

$$\psi=\omega_0\frac{L}{c}n_0+m\Omega\frac{L}{c}\left(n_0+\omega_0\frac{\partial n}{\partial\omega}\right)+m^2\Omega^2\frac{L}{c}\left(\frac{\partial n}{\partial\omega}+\frac{\omega_0}{2}\frac{\partial^2 n}{\partial\omega^2}\right) \quad (17)$$

or

$$\psi_m=2\pi m(1+m\Omega\mathscr{L}(n)), \quad (18)$$

where

$$\mathscr{L}(n)=\left(\frac{\partial n}{\partial\omega}+\frac{\omega_0}{2}\frac{\partial^2 n}{\partial\omega^2}\right)\Big/\left(n_0+\omega_0\frac{\partial n}{\partial\omega}\right) \quad (19)$$

and we have discarded the phase shift $\omega_0 L n_0/c$ which is the same for all modes. The final solution of the phase equation (16) is

$$\varphi_m(t)=m\Omega(1+m\Omega\mathscr{L}(n))t. \quad (20)$$

4. EVOLUTION OF THE PULSE DURATION

Substituting the expressions for the amplitudes and phases of the modes into the initial expansion (1), we can find the field generated at any instant of time and, in particular, consider the evolution of the pulse duration.

Let us consider first the region of linear development of the generation when the effective gain per pass κ is constant and equal to the initial value. In this case the instantaneous amplitudes of the modes are determined by (15). Substituting the expressions for the amplitudes and phases (15) and (20) into the initial expansion (1), we get

$$\begin{aligned} E(t)=\mathrm{Re}\,e^{i\omega_0 t}A_0\frac{\exp\{[\sqrt{\varkappa(1+\rho)}-1]t/T\}}{\sqrt{\pi\rho\varkappa^{1/2}t/T}}\sum_m\exp\Big\{im\Omega t \\ +\left[i\Omega^2\mathscr{L}(n)t-\frac{T}{\rho\varkappa^{1/2}t}\right]m^2\Big\}. \end{aligned} \quad (21)$$

The sum in (21) is a quasiperiodic function with maximum at the point $t_k = kT$, where k is an integer. Each maximum corresponds to a laser emission pulse. In the absence of dispersion ($\mathscr{L}(n) = 0$) the width of the pulses gradually decreases, since the number of modes in the sum, determined by the factor $\exp\{-Tm^2/\rho\kappa^{1/2}t\}$, increases with time like $m_{ph} \approx \sqrt{\rho\kappa^{1/2}t/T}$. In the presence of dispersion, dephasing of the different modes takes place and accumulates in time. The dephasing tends to broaden the pulses and, starting with a certain instant, the contraction of the pulses gives way to broadening. For a quantitative answer it is necessary to calculate the sum in (21). In our approximation, where the number of modes is large, the summation in (21) can be replaced by integration. The resultant integral can be calculated exactly:

$$\begin{aligned} \int_{-\infty}^{\infty}\exp\{i\gamma m+m^2(i\beta-\alpha)\}dm=\left[\frac{\pi}{2(\alpha^2+\beta^2)}\right]^{1/2}\exp\left\{-\frac{\gamma^2}{4(\alpha-i\beta)}\right\} \\ \times[(\sqrt{\alpha^2+\beta^2}+\alpha)^{1/2}+i(\sqrt{\alpha^2+\beta^2}-\alpha)^{1/2}]. \end{aligned} \quad (22)$$

Then the expression for the field intensity $I(t) = c\langle E^2(t)\rangle/8\pi$ is given by

$$\begin{aligned} I(t)=\frac{I_0\exp\{[(1+\rho)\varkappa-1]t/T\}}{(\rho\varkappa^{1/2}t/T)[(T/\rho\varkappa^{1/2}t)^2+(\Omega^2\mathscr{L}t)^2]^{1/2}} \\ \times\exp\left\{-\frac{\Omega^2 t^2}{2(\rho\varkappa^{1/2}t/T)[(T/\rho\varkappa^{1/2}t)^2+(\Omega^2\mathscr{L}t)^2]}\right\} \end{aligned} \quad (23)$$

where I_0 is the initial intensity of the central mode.

When the sum in (21) is replaced by an integral, the periodicity of the maxima is lost. It can, however, be

4)This effect can appear in the case, not considered here, when the nonlinear development of the generation does not continue long enough, or even goes over into the stationary regime. This is possible, for example, at sufficiently high pump levels, if the gain of the medium is restored after the passage of each pulse. In this case the nonlinear distortions of the pulse waveform can accumulate during many passes.

readily reconstructed. If we introduce the number of passes through the resonator after the start of generation, $k = [t/T]$, where the square brackets denote the integer part of the number, then the intensity of the radiation in the time interval $(k - \frac{1}{2})T \le t \le (k + \frac{1}{2})T$, i.e., the intensity of the k-th pulse after the start of generation, $I_k(t)$, is given by

$$I_k(t) = \frac{I_0 \exp\{[\varkappa(1+\rho)-1]t/T\}}{[1+(\rho\varkappa^{1/2}\mathcal{L}\Omega^2t^2T^{-1})^2]^{1/4}} \cdot \exp\left\{-\frac{\Omega^2(t-kT)^2}{2(\rho\varkappa^{1/2}t/T)[(T/\rho\varkappa^{1/2}t)^2+(\Omega^2\mathcal{L}t)^2]}\right\}, \quad (24)$$

where $(k - \frac{1}{2})T \le t < (k + \frac{1}{2})T$.

Expression (24) pertains to the case of linear development of the generation. It can be extended, however, to include the case when the gain κ begins to depend on the time as a result of the saturation of the gain under the influence of the exponentially increasing average radiation power

$$I_{av}(t) = \frac{1}{T}\int_t^{t+T} I(t')\,dt'.$$

Formally this complicates the solution, since the coefficients in the right side of (14) begin to depend on the time. But it is physically clear that a change of 5–10% in the gain κ at the derivative $\partial^2 A_m/\partial m^2$ has little effect on the rate of increase of the number of locked modes. At the same time, a similar change of κ greatly changes the coefficient $\sqrt{\kappa(1+\rho)} - 1$ in the term with $A_m(t)$, and consequently greatly influences the change of the intensity. Taking into account the change of $\kappa(t)$ only in $A_m(t)$, we can easily solve (14). The intensity of the k-th pulse is then given by the more general expression

$$I_k(t) = I_0 \exp\left\{\frac{1}{T}\int_0^t \alpha(t')\,dt'\right\}[1+(\rho\varkappa^{1/2}\mathcal{L}\Omega^2t^2T^{-1})^2]^{-1/4} \times\exp\left\{-\frac{\Omega^2(t-kT)^2}{2(\rho\varkappa^{1/2}t/T)[(T/\rho\varkappa^{1/2}t)^2+(\Omega^2\mathcal{L}t)^2]}\right\}, \quad (25)$$

where

$$\alpha(t) = \ln[\varkappa(1+\rho)] = \sigma_0 N(t)l - \ln\frac{1}{\eta_0 r(1+\rho)}.$$

The first two factors of expression (25) describe the evolution of the peak power (envelope) of the pulses. The third exponential factor describes the form of the pulses. Let us consider first the evolution of the pulse duration. The duration of the k-th pulse at the 1/e level is equal to

$$\tau_k = \frac{2}{\Omega}\left\{2\rho\varkappa^{1/2}k\left[\left(\frac{1}{\rho\varkappa^{1/2}k}\right)^2+(2\pi\Omega\mathcal{L}k)^2\right]\right\}^{1/2}. \quad (26)$$

It follows from (26) that the minimum of the pulse duration is reached at the instant of time

$$t_{min} = 2\pi(2\pi\sqrt{3}\rho\varkappa^{1/2}\Omega^3\mathcal{L})^{-1/2} \quad (27)$$

after a number of passes $k_{min} = t_{min}/T$ given by the expression

$$k_{min} = (2\pi\sqrt{3}\rho\varkappa^{1/2}\Omega\mathcal{L})^{-1/2}. \quad (28)$$

The minimum pulse duration τ_{min} is determined by the expression (26) with $k = k_{min}$:

$$\tau_{min} = \frac{2}{\pi}T\left(\frac{8\pi\sqrt{3}}{9}\frac{\Omega\mathcal{L}}{\rho\varkappa^{1/2}}\right)^{1/4} \quad (29)$$

We note that the minimum duration obtained in the present paper is somewhat higher than the minimum duration obtained in [7] for the stationary generation regime. For example, at $T = 2.5\times10^{-9}$ sec, $\rho = 0.1$, $\mathcal{L}(n) \approx 10^{-18}$ and $\kappa \approx 1$, expression (29) yields $\tau_{min} \approx 3\times10^{-11}$ sec, and from expression (18) of [7] we get 0.8×10^{-11} sec. This discrepancy can be naturally attributed to the fact that mode capture takes place in the stationary regime even in the presence of a certain mode dephasing. Formally this is the result of the nonlinearity of the oscillation equations, which is neglected in the present case when the nonstationary regime is considered. The mode locking effect in the presence of a certain dephasing (detuning) ensures also establishment of a stationary duration.

During the time $t \ll t_{min}$, when the role of the dispersion is negligibly small, the light pulses are compressed in accordance with the law $\tau_k = T/\pi(\rho\kappa^{1/2}k/2)^{1/2}$. When $\tau_k \ll T$, the compression process becomes very slow. Physically this is perfectly understandable, since the modulation of the transmission during the time T cannot noticeably influence the wave form of the pulses with duration $\tau_k \ll T$. This is valid, of course, for an arbitrary law of periodic modulation of the transmission $\eta(t)$. Indeed, the waveforms of the k-th and (k + 1)-st pulses are connected by the relation

$$I_{k+1}(t) = \eta(t)I_k(t). \quad (30)$$

Assuming that the change in the waveform per pass is small, we go over to the differential equation

$$dI_k(t)/dk = [\eta(t) - 1]I_k(t), \quad (31)$$

the solution of which is $I_k(t) = I_0(t)\exp\{[\eta(t) - 1]k\}$. When $\tau_k \ll T$, the waveform of the pulse is given by

$$I_k(t) \sim \exp\left\{\frac{1}{2}\frac{d^2\eta}{dt^2}kt^2\right\}, \quad (32)$$

and the duration is determined by the expression

$$\tau_k = \left(-\frac{k}{8}\frac{d^2\eta}{dt^2}\right)^{-1/2}. \quad (33)$$

In the particular case of sinusoidal modulation, expression (33) coincides with (26) in the absence of dispersion.

We note that inasmuch as the character of the pulse compression remains unchanged under an arbitrary law of transmission modulation $\eta(t)$,[5)] all the results obtained above for the case of sinusoidal modulation $\eta(t) = \eta_0(1 + \rho\cos\Omega t)$ remain valid also for an arbitrary modulation law $\eta(t)$, provided $\eta_0\rho\Omega^2$ is replaced by $|d^2\eta/dt^2|$ at the point of the maximum of the function $\eta(t)$.

In the region $t \gg t_{min}$, the pulses broaden in accordance with $\tau_k \approx 4\pi\mathcal{L}k\sqrt{\rho\kappa^{1/2}k}$. To obtain light pulses of minimum duration, it is necessary to synchronize the

[5)] There is only the physically insignificant requirement that the function $\eta(t)$ be continuous and doubly differentiable.

instant when maximum average power is reached and the instant of smallest duration.

The average radiation power $I_{av}(t)$ is given by Eq. (25), which is averaged over the time interval $(t - T/2, t + T/2)$:

$$I_{av}(t) = \frac{I_0}{(2\pi\rho\varkappa^{1/2}t/T)^{1/2}} \exp\left\{\frac{\sigma_0 l}{T}\int_0^t N(t')\,dt' - \frac{t}{T}\ln\frac{1}{\eta_0 r(1+\rho)}\right\}. \quad (34)$$

The change of the average power is determined by simultaneously solving Eqs. (6) and (34). We shall not go into a detailed analysis of this question, since it is physically clear that the change of $I_{av}(t)$ is analogous to the evolution of the power in a laser with instantaneous Q switching.[15, 16] During the first and most prolonged phase, an exponential increase takes place in the average power, from the level of the spontaneous noise to a level almost sufficient to saturate the gain, but no noticeable saturation takes place. The duration of the first phase of development of the generation is called the delay time τ_d.[15] During the second phase, saturation of the gain of the medium takes place (emission by the majority of the active particles), and the maximum average power is reached. It is clear that to obtain powerful pulses with minimum duration it is necessary that the delay time τ_d be approximately equal to the time necessary to reach the minimum duration t_{min}. Since t_{min}, determined by (27), is relatively large ($\sim 10^{-5}$ sec), the initial gain per pass $(\kappa - 1)$ should be quite small ($\sim 10^{-2}$). In this case the number of generated pulses in the train is unavoidably large, since its order of magnitude is several times $1/(\kappa - 1)$. When the initial gain is increased, the number of pulses decreases and the pulse energy increases accordingly, but the duration also increases, owing to the decrease in the generation development time.

These two contradictory conditions can be circumvented by additionally increasing the Q at the instant when the minimum duration is reached. In this case the number of pulses in the train greatly decreases, and accordingly the power increases appreciably, and the duration remains practically minimal.

5. COMPARISON WITH EXPERIMENT

In the experiments of [2, 3], the characteristic generation development time t_{gen} (distance between spikes in [2, 3], delay time after Q switching in [2]) is much shorter than the time necessary to reach minimum duration t_{min}, when the significant role is assumed by dispersion. On the other hand, in the region $t_{gen} \ll t_{min}$ the number of locked modes is determined, in accordance with (26), by the expression

$$m = (1/4\pi\rho\sqrt{\varkappa}\Omega t_{gen})^{1/2}. \quad (35)$$

In the spike mode, apparently, $t_{gen} \approx 10^{-6}$ sec. Then, when $\rho\sqrt{\kappa} \approx 1$ and $\Omega = 3 \times 10^8$ rad/sec we have $m \approx 15$, which agrees with the experimental value $m \approx 20$ in [2, 3]. In the regime of simultaneous Q switching,[2] we apparently have $t_{gen} \approx 10^{-7}$ sec. Then $m \approx 5$, which also agrees with the experimental value $m \approx 4$ in [2]. Thus, the light pulse durations in the experiments of [2, 3] were limited by the generation development time.

A more detailed comparison of the present theory with experiment can be carried out in the laser described in [4], by observing the transient process of gradual pulse shortening. With the aid of this laser it is also possible to verify the dependence of the limiting duration on the dispersion in the stationary regime, as described by the theory.[7]

6. CONCLUSION

We considered in this paper the dynamics of the generation of ultrashort light pulses by the mode locking method in a laser with instantaneous Q switching. The developed approach is applicable also to the case of a laser with self-capture of the modes by means of a saturable filter, which has gained widely in popularity following the work by De Maria et al.[5] The results of such an analysis are briefly reported in [10], and a detailed consideration of the dynamics of the generation of ultrashort pulses will be presented for this case later. It is appropriate to note here several qualitatively new features that characterize a laser with self-capture of modes by means of a saturable solution. For example, the shortening of pulses in self-capture of modes occurs more rapidly than in the case of mode locking by external modulation. This follows from the expression for the pulse evolution, which is analogous to (31), but it employs a transmission $\eta(t)$ that depends on the pulse waveform $I_k(t)$. To reach a minimum duration it is necessary to have a very small excess of the initial gain above threshold. In the case of self-capture the minimum duration depends strongly on the time of spontaneous collapse of the filter T_1, and after the minimum duration is reached the evolution is determined to a considerable degree by the dispersion of the medium. In addition, at sufficiently small values of T_1 ($< 10^{-12}$ sec), a regime of induced "collapse" ("self-collapse") of a filter is possible, of the type of a 360° pulse, arising as the result of coherent interaction between the light pulse and the particles of the solution, if the pulse duration is comparable with the transverse relaxation time T_2 of the filter particles.

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