EFFECT OF A HIGH-FREQUENCY MAGNETIC FIELD ON PLASMA INSTABILITY

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The effect of a high-frequency magnetic field $H_\sim$ on the oscillations of a plasma confined by a fixed magnetic field H_0 is investigated. It is shown that even with relatively weak high-frequency fields $H_\sim/H_0 \sim (mT_i/MT_e)^{1/2}$ it is possible to suppress loss-cone, drift and cyclotron instabilities. When $H_\sim/H_0 \sim \rho_i/a$ (ρ_i is the mean ion Larmor radius and a is the plasma radius) the growth rate for the drift and drift-temperature instabilities can be reduced.

THERE exists a broad class of plasma instabilities, for plasmas confined by a magnetic field, in which the waves that are excited are characterized by values of k_z which are small $k_z \ll k$, where k_z is the projection of the wave vector **k** on the magnetic field H_0. For the most part, these instabilities are among the class of those which are presently taken to be the most dangerous: in particular, the various kinds of drift instabilities, the instabilities associated with anisotropic distribution functions (loss-cone and drift loss-cone instabilities) and instabilities at the cyclotron frequency.

A number of authors[1,2] have investigated the effect of a high-frequency field on the drift instabilities. Mikhaĭlovskiĭ and Siderov[1] took account of the additional electron drift produced by the pressure gradient of the RF magnetic field. Faĭnberg and Shapiro[2] have shown that as a consequence of oscillations of electrons along magnetic lines of force under the effect of the RF electric field the temperature of the electrons is effectively increased. As a result the oscillation frequency is increased and the instability region is reduced in terms of the parameter $\eta = \partial \ln T/\partial \ln n_0$.

1. PLASMA STABILIZATION BY AN RF MAGNETIC FIELD

In the present work we direct attention to a completely different method of stabilization proposed by the authors in a short note.[3] First we shall consider the physical picture behind the method. The instabilities in question lead to perturbations of equilibrium values of the density, potential etc., which appear in the form of flutes that are highly elongated along the magnetic lines of force. When the primary magnetic field $H_z \equiv H_0$ is supplemented by a transverse RF magnetic field $\tilde{H}_y$, where $|\tilde{H}_y| \ll H_0$, which varies at a frequency Ω, much higher than the oscillation frequency ω, at certain instants of time the instantaneous magnetic line of force $H_0 + \tilde{H}$ can connect neighboring flutes of the perturbation. Consider the electrons or ions moving with thermal velocities along the lines of force across the flutes; if they succeed in moving across the fixed magnetic field by a distance of the order of the transverse wavelength $\lambda \sim k_\perp^{-1}$ (the transverse particle velocity is $v_T H_y/H_0$) they can then reduce the electric field associated with the perturbations and the instability can be quenched (cf. Fig. 1). The conditions under which the RF magnetic field will have an appreciable effect on the instability can then be written in the form

$$\frac{k_\perp v_T}{\Omega}\frac{H_y}{H_0} > 1.$$

We have omitted the subscript e or i on v_T since the electron motion is important in certain instabilities and the ion motion in others.

2. CALCULATION OF THE CORRECTIONS TO THE DISTRIBUTION FUNCTION

We consider a plasma which is inhomogeneous in the x direction and located in a strong magnetic field H_0 in the z direction. Assume that there is an RF field with a component $H_y = H_1 \cos \Omega_1 t$ ($H_1 \ll H_0$, $\Omega_1 \ll \omega_{He}$). We wish to find the correction to the equilibrium distribution function for the electrons $f_0 = f_0(\epsilon, x)$ in the frequency region $\omega \ll \omega_{He}$. It is convenient to use the drift kinetic equation for this purpose.[4] In the case of electrostatic oscillations the equation for the correction in the linear approximation is

$$\frac{\partial \delta f}{\partial t} + (\mathbf{h}\nabla)\delta f + \frac{c}{H_0}([\mathbf{Eh}]\nabla)f_0 - \frac{e}{m}(\mathbf{Eh})\frac{\partial f_0}{\partial v_\parallel} = 0, \qquad (1)^*$$
$$\mathbf{E} = -\nabla\Phi.$$

Using the fact that f_0, H_0 and **h** are independent of y and z we make a Fourier transformation of Eq. (1) in terms of y and z:

$$\frac{\partial f_k}{\partial t} + i(\mathbf{hk})f_k - i\frac{c}{H_0}k_y\frac{\partial f_0}{\partial x}\Phi_k - \frac{e}{m}(\mathbf{hk})\frac{\partial f_0}{\partial v_\parallel}\Phi_k = 0. \qquad (2)$$

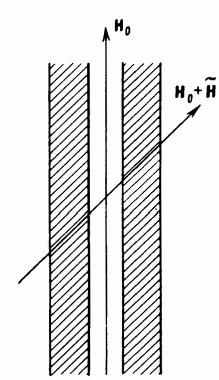

FIG. 1

*$[\mathbf{Eh}] \equiv \mathbf{E} \times \mathbf{h}$, $(\mathbf{Eh}) \equiv \mathbf{e} \cdot \mathbf{h}$.

Substituting in Eq. (2) $h = e_y(H_1/H_0)\cos\Omega_1 t + e_z$ and integrating Eq. (2) by the method of variation of constants with the initial condition Φ_k, f_k at $t = -\infty$, we have

$$f_k = \int_{-\infty}^{\infty} dt' \left\{ i\frac{c}{H_0}k_y\frac{\partial f_0}{\partial x} + \frac{e}{m}i\left(k_z + k_y\frac{H_1}{H_0}\cos\Omega_1 t\right)\frac{\partial f_0}{\partial v_\parallel}\right\} \times \exp\left[ik_z v_\parallel(t'-t) + iv_\parallel\frac{k_yH_1}{\Omega_1 H_0}\sin\Omega_1 t' - iv_\parallel\frac{k_yH_1}{\Omega_1 H_0}\sin\Omega_1 t\right]\Phi_k(t'). \quad (3)$$

Assuming that $\Phi_k(t) \sim \exp(-i\omega t)$ where ω has a small imaginary part $i\nu$ $(\nu > 0)$ and using the fact that

$$\exp\left(iv_\parallel\frac{k_yH_1}{\Omega_1H_0}\sin\Omega_1 t\right) = \sum_{l=-\infty}^{\infty} J_l\left(\frac{k_yH_1}{\Omega_1H_0}v_\parallel\right)\exp(il\Omega_1 t),$$

we can now write f_k in the form

$$f_k = -\Phi_{k\omega}\frac{e}{m}\frac{\partial f_0}{v_\parallel\partial v_\parallel}\exp(-i\omega t) + \Phi_{k\omega}\exp(-i\omega t) \times\left(\frac{c}{H_0}k_y\frac{\partial f_0}{\partial x} - \omega\frac{e}{m}\frac{\partial f_0}{v_\parallel\partial v_\parallel}\right) \times\left[\sum_{l=-\infty}^{\infty}\frac{J_l^2(k_yv_\parallel H_1/\Omega_1H_0)}{k_zv_\parallel-\omega+l\Omega_1} + \sum_{\substack{l,n\\ l\neq n}}\frac{J_l(k_yv_\parallel H_1/\Omega_1H_0)}{k_zv_\parallel-\omega+l\Omega_1}J_n\left(\frac{k_yv_\parallel}{\Omega_1}\frac{H_1}{H_0}\right) \times\exp\{i\Omega_1 t(l-n)\}\right]. \quad (4)$$

It is evident from the expression for f_k that there are terms of two types: one of these depends on time in the form $\exp(-i\omega t)$ while the others are rapidly oscillating functions, being proportional to $\exp(i\Omega_1 t)$. Averaging over the fast oscillations (this is equivalent to an expansion in ω/Ω_1) and neglecting small terms of order ω/Ω_1, with $f_{k\omega} = \int \exp(-i\omega t)f_k(t)dt$ we find

$$f_{k\omega} = \Phi_{k\omega}\left\{-\frac{e}{m}\frac{\partial f_0}{v_\parallel\partial v_\parallel} + \left[\frac{c}{H_0}k_y\frac{\partial f_0}{\partial x} - \frac{e}{m}\omega\frac{\partial f_0}{v_\parallel\partial v_\parallel}\right] \times(k_zv_\parallel-\omega)^{-1}J_0^2\left(\frac{k_yv_\parallel}{\Omega_1}\frac{H_1}{H_0}\right)\right\}. \quad (5)$$

3. LOSS-CONE AND CYCLOTRON INSTABILITIES

We now use Eq. (5) in order to examine the effect of the RF field on instabilities associated with anisotropies in the distribution function.

a. We shall first treat an important form of instability in a homogeneous plasma $\partial f_0/\partial x = 0$ confined in a mirror, the so-called velocity-space instability first investigated by Post and Rosenbluth.[5] The dispersion relation for this instability is

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} = \frac{\omega_{pe}^2}{\omega^2}\frac{k_z^2}{k^2} + \frac{\omega_{pi}^2}{k^2\bar{v}_{Ti}^2}F\left(\frac{\omega}{k\bar{v}_{Ti}}\right), \quad (6)$$

$\omega_{pi} \gg \omega_{Hi}$, $k_z \ll k_\perp$ and $\bar{v}_{Ti}$ is the ion velocity

$$F(y) = -2\int_{-\infty}^{\infty} dx(\partial\psi/\partial x)^2(1-x/y^2)^{-1/2},$$

$$\psi(x) = \text{const}\cdot\int_{-\infty}^{\infty} f_{0i}(v_\perp^2, v_\parallel)dv_\parallel;$$

$$\text{const} = \left[\int_{-\infty}^{\infty}\psi(x)dx\right]^{-1}, \quad x = v_\perp^2/\bar{v}_{Ti}^2,$$

where f_{0i} is the distribution function for the ions confined in the mirror. The first term on the right side of the dispersion relation arises from perturbations in the electron motion along the magnetic field. In order to understand how this is modified in the presence of the RF magnetic field, we direct attention to the fact that it derives from the expression

$$-2\frac{\omega_{pe}^2}{k^2v_{Te}^2}\left(1 - \omega\int\frac{f_{0e}}{\omega - k_zv_\parallel}dv_\parallel\right) \quad (7)$$

where $\omega \gg k_zv_{Te}$. In the presence of the RF magnetic field, Eq. (7) becomes the following when the relation in (5) is introduced:

$$-2\frac{\omega_{pe}^2}{k^2v_{Te}^2}\left(1 - \omega\int\frac{f_{0e}J_0^2(\mu v_\parallel)}{\omega - k_zv_\parallel}dv_\parallel\right), \quad (8)$$

The dispersion relation (6) now becomes

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} + 2\left(1 - \omega\int\frac{f_{0e}J_0^2(\mu v_\parallel)}{\omega - k_zv_\parallel}dv_\parallel\right)\frac{\omega_{pe}^2}{k^2v_{Te}^2} - \frac{\omega_{pi}^2}{k^2\bar{v}_{Ti}^2}F\left(\frac{\omega}{k\bar{v}_{Ti}}\right) = 0. \quad (9)$$

If $\mu = k_yH_1/\Omega_1H_0 = 0$ $(H_1 = 0)$ then Eq. (9) transforms into Eq. (6) when $\omega \gg k_zv_{Te}$. It follows from[5] that perturbations with phase velocity ω/k_z smaller than the electron thermal velocity are stable. In this case the second term in the curved brackets in Eq. (9) is smaller than unity when $H_1 = 0$. In the presence of the RF field, with a sufficiently high value of μ the following inequality is satisfied

$$\omega\int\frac{f_{0e}J_0^2(\mu v_\parallel)}{\omega - k_zv_\parallel}dv_\parallel < 1 \quad (9a)$$

for any value of ω/k_zv_{Te}. Actually, the maximum value for the left side of the inequality is reached when $\mu = 0$ and can be found using tables of the Cramp function. This value is approximately 1.5. The square of the Bessel function that appears in the numerator acts to reduce the integral, this effect becoming stronger the larger the value of μ. The inequality in (9a) is satisfied when $\mu v_{Te} \sim 1$. In this case the first term on the right side of the dispersion relation (9) changes sign, going from positive to negative. Let us see what effect this has on the stability of the plasma. An investigation of (9) by means of the Nyquist method shows that the plasma is stable if (9a) is satisfied. This result can be understood as follows. In the presence of the RF magnetic field the term $\omega_{pe}^2k_z^2/\omega^2k^2$ in the dispersion relation changes, roughly speaking, to $2\omega_{pe}^2/k^2v_{Te}^2$ and the oscillations will in general, disappear. We note that a sufficient condition for stabilization can be written in the form

$$\mu v_{Te} \gtrsim 1 \text{ or } (k_yv_{Te}/\Omega_1)H_1/H_0 \gtrsim 1 \quad (10)$$

In the absence of the RF field the phase velocity of the unstable waves ω/k is always smaller than the thermal velocity of the ions. If we take the frequency to be the minimum allowable value for the appearance of an instability with a given value of the wave vector $\Omega_1 \approx \omega_k$, we obtain the following condition on the amplitude of the RF field:

$$H_1/H_0 > v_{Ti}/v_{Te}. \quad (11)$$

Fields of this kind can be produced by exciting a helicon wave in the plasma.[6]

b. We now consider the case in which the plasma density is a function of x and $k_z = 0$, the so-called drift-velocity-space instability. For reasons of simplicity we use the ion distribution function for a mirror device given by Mikhaĭlovskiĭ:[7]

$$f_{0i} = M(T_1 + T)T^{-2}\exp(-Mv_\perp^2/2T)[1-\exp(-Mv_\perp^2/2T_1)]$$

[T has the meaning of a temperature while the quantity $2TT_1/M(T_1+T)^{1/2} \equiv v_c$ corresponds to the velocity scale below which $\partial f_{0i}/\partial\epsilon_\perp > 0$ ($\epsilon_\perp = v_\perp^2/2$, $T_1 \lesssim T$) that is to say, this is the characteristic velocity of the loss cone].

When $H_1 = 0$ the dispersion relation is the following[7] [$\omega_{Hi} \ll \omega \ll \omega_{He}$, $\omega \ll k_\perp(T_1/M)^{1/2}$]:

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - k_y\frac{\partial \ln n_0}{\partial x}\frac{\omega_{pi}^2}{\omega_{Hi}\omega k^2} - i\pi^{1/2}\frac{\omega}{|k|v_c}\frac{\omega_{pi}^2}{k^2}\frac{M(T+T_1)}{T^2}\left[1-\left(\frac{T_1}{T_1+T}\right)^{1/2}\right] = 0. \quad (12)$$

In the presence of the RF field the correction to the electron distribution function is given by Eq. (5). In this case, Eq. (12) is replaced by

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} + 2\frac{\omega_{pe}^2}{k^2v_{Te}^2}(1-A) - Ak_y\frac{\partial \ln n_0}{\partial x}\frac{\omega_{pe}^2}{\omega_{He}\omega k^2} - i\pi^{1/2}\frac{\omega}{|k|v_c}\frac{\omega_{pi}^2}{k^2}\frac{M(T+T_1)}{T^2}\left[1-\left(\frac{T_1}{T+T_1}\right)^{1/2}\right] = 0;$$

$$A = \int_{-\infty}^{\infty} J_0^2(\mu v_\parallel)f_{0e}\,dv_\parallel \leqslant 1. \quad (13)$$

Equation (13) can conveniently be written in the following form:

$$2(1-A)\frac{T}{T_e} + k^2(\rho_e^2 + r_{Di}^2) + A\frac{\omega^*}{\omega} - ib\frac{\omega}{|k|v_c} = 0,$$

$$\omega^* = -k_y\frac{cT}{eH_0}\frac{\partial \ln n_0}{\partial x}, \quad b = \pi^{1/2}\frac{T+T_1}{T}\left[1-\left(\frac{T_1}{T+T_1}\right)^{1/2}\right],$$

$$\rho_e^2 = mTc^2/e^2H_0^2, \quad r_{Di}^2 = T/4\pi n_0e^2.$$

Formally this equation always has an unstable solution. When $1 - A \gg \sqrt{\rho_i/a}$ the growth rate is

$$\gamma = \frac{2bA^2\omega^{*2}}{|k|v_c[2(1-A)T/T_e + k^2(\rho_e^2 + r_{Di}^2)]^3}. \quad (14)$$

In order for the dispersion relation (13) to apply the inequality $\gamma > \omega_{Hi}$ must be satisfied (cf. Mikhaĭlovskiĭ[7]). Hence, this instability can arise if

$$\frac{k}{[2(1-A)T/T_e + k^2(\rho_e^2 + r_{Di}^2)]^3} > \frac{a^2v_c}{2\rho_i^3bA^2v_{Ti}},$$

or, minimizing the left side with respect to k,

$$a \lesssim 3^{-1}\rho_iAM^{1/4}/(1-A)m^{1/4}. \quad (15)$$

(the last inequality is obtained under the assumption that $b \sim 1$, $v_c \sim v_{Ti}$).

c. We now consider the case in which the plasma density is independent of x and excitation occurs at harmonics of the ion-cyclotron frequency. In the presence of the RF field the dispersion relation for this case is[8]

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} = -2\pi\frac{\omega_{pi}^2}{k^2}\omega\int_0^\infty dv_\perp^2\int_{-\infty}^{\infty} dv_\parallel\frac{\partial f_0}{\partial v_\perp^2}\sum_{n=-\infty}^{\infty}\frac{J_n^2(k_\perp v_\perp/\omega_{Hi})}{\omega + n\omega_{Hi}}. \quad (16)$$

We note that the most dangerous distribution, from the point of view of confinement, is a delta-function distribution in the transverse velocity. In this limit the loss-cone instability becomes an instability that has been studied by Dory, Guest, and Harris.[8] Assume that

$$f_{0i} = (2\pi)^{3/2}M^{1/2}T^{-1/2}\exp(-Mv_\parallel^2/2T)v_\perp^{-1}\delta(v_\perp - v_\perp^0).$$

Then Eq. (16) assumes the form

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} = \frac{\omega_{pi}^2}{\omega_{Hi}^2}\sum_{n=-\infty}\frac{\omega}{\omega + n\omega_{Hi}}\frac{1}{p}\frac{\partial}{\partial p}J_n^2(p) = 0, \quad (17)$$

$$p = k_\perp v_\perp^0/\omega_{Hi}.$$

In the presence of an RF magnetic field that oscillates at a frequency $\Omega_1 \gg \omega_{Hi}$, Eq. (17) becomes

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} + 2\frac{\omega_{pe}^2}{k^2v_{Te}^2}(1-A) - \frac{\omega_{pi}^2}{\omega_{Hi}^2}\sum_{n=-\infty}\frac{\omega}{\omega + n\omega_{Hi}}\frac{1}{p}\frac{\partial}{\partial p}J_n^2(p) = 0. \quad (18)$$

Consider the frequency region near the first harmonic. Retaining the terms with n = 0, 1, −1 in the summation and using the recursion relations for the Bessel functions we obtain an expression for the frequency

$$\omega^2 = \frac{B\omega_{Hi}^2/\omega_{pi}^2 + 2J_0J_1/p}{B\omega_{Hi}^2/\omega_{pi}^2 + 2J_1J_2/p}\omega_{Hi}^2, \quad B = 1 + \frac{\omega_{pe}^2}{\omega_{He}^2} + 2\frac{\omega_{pe}^2}{k^2v_{Te}^2}(1-A). \quad (19)$$

Thus if $\omega_{pi}^2/\omega_{Hi}^2 \gg 1$ while $\omega_{pe}^2/\omega_{He}^2 \ll 1$, in the absence of the RF field (A = 1) an instability always occurs. Formally the effect of the RF magnetic field is such that the quantity B is increased greatly for $A \lesssim 1$. It is known that an instability of this type disappears if $\omega_{Hi}^2\omega_{pe}^2/\omega_{pi}^2k^2v_{Te}^2$ is larger than $2J_0J_1/p$, that is to say when $T_i \gtrsim T_e$. In the present case the stabilization condition $A < 1$) ($k_yv_{Te}H_1/H_0\Omega_1 > 1$) is reduced to the following relation ($k_{y\,min} \sim \rho_i^{-1}$, $\Omega_1 \gtrsim \omega_{Hi}$):

$$H_1/H_0 > v_{Ti}/v_{Te}. \quad (20)$$

4. DRIFT INSTABILITIES

Drift instabilities represent a broad class of instabilities that arise because of spatial inhomogeneity in a plasma confined by a magnetic field. At the present time the most dangerous of these is believed to be the so-called drift-temperature instability, first described in[9]. We now wish to investigate the effect of an RF field on the various drift-instabilities, starting with the drift-temperature instability.

a. The dispersion relation for the drift-temperature instability can be obtained most simply by using the neutrality condition, taking the electron perturbation in the density to be of the form $n_e/n_0 = e\Phi/T_e$ and the ion-perturbation to be given by Eq. (5). As a result we find

$$1 + \frac{T_i}{T_e} - \int_{-\infty}^{\infty}\frac{\omega^*(1 - \eta/2 + \eta v_\parallel^2/v_{Ti}^2)}{\omega - k_zv_\parallel}\frac{f_{0i}}{n_0}J_0^2(\mu v_\parallel)dv_\parallel = 0;$$

$$\omega_i^* = -k_y\frac{cT_i}{eH_0}\frac{\partial \ln n_0}{\partial x}, \quad \eta = \frac{\partial \ln T_i}{\partial \ln n_0}, \quad T_i \sim T_e. \quad (21)$$

We now determine the stability boundary from Eq. (21). We will assume that ω is real (at the stability limit Im $\omega = 0$) and equate the real and imaginary parts that arise from the residue in Eq. (21) to zero separately. After some simple calculations we find

$$\omega = \omega_i^*(\eta/2 - 1)A(2-A)^{-1},$$

$$k_z^2 = \omega_i^{*2}v_{Ti}^{-2}(\eta/2 - 1)\eta A^2 2^{-1}(2-A)^{-1}. \quad (22)$$

From the requirement $k_z^2 > 0$ we find that the instability condition is $\eta > 2$. In other words, the RF field has no effect on the stability boundary. Now let us see how the growth rate and frequency are effected by the RF field. In the limit of large values of $\ln \mu v_{Ti} \gg 1$ it is possible to estimate the integral in Eq. (21) in the sense of the principal value, making use of the fact that the basic contribution to the integral comes from values $\omega/k_z > v > \mu^{-1}$ ($\omega/k_z \gtrsim v_{Ti}$). The expressions for the frequency and growth rate are then

$$\omega = \omega_i^* A (2-A)^{-1}(\eta/2-1),$$

$$\gamma = -\pi^{1/2}\frac{\omega^2}{|k_z|v_{Ti}}\frac{J_0^2(\omega\mu/k_z)}{A} \times \exp\left(-\frac{\omega^2}{k_z^2 v_{Ti}^2}\right)\left[1+\frac{\omega^2}{k_z^2 v_{Ti}^2}\eta(1-\eta/2)\right]. \quad (23)$$

b. Another important drift-instability is due to the excitation of drift waves in the plasma by resonance electrons. One of these instabilities was first investigated in[9] and is frequently called the universal instability. The dispersion relation which describes the unstable drift waves with wavelengths much larger than the ion-Larmor radius is obtained from the neutrality condition

$$\sum_\alpha \int e_\alpha f_\alpha dv_\parallel = 0$$

[α = e, i refers to the electron and ions while f_α is determined by Eq. (5)]. This is of the form

$$\sum_\alpha \frac{1}{T}\left(1-\int_{-\infty}^{\infty}\frac{\omega_\alpha^*(1-\eta_\alpha/2+\eta_\alpha v_\parallel^2/v_{T\alpha}^2)+\omega}{\omega-k_z v_\parallel}J_0^2(\mu v_\parallel)f_{0\alpha}dv_\parallel\right)=0,$$

$$\omega_\alpha^* = -k_y\frac{cT_\alpha}{eH_0}\frac{\partial \ln n_0}{\partial x}, \quad \mu = \frac{H_1}{H_0}\frac{k_y}{\Omega_1}, \quad \eta_\alpha = \frac{\partial \ln T_\alpha}{\partial \ln n_0},$$

$$f_{0\alpha} = \pi^{-1/2}v_{T\alpha}^{-1}n_0(x)\exp(-m_\alpha v_\parallel^2/2T_\alpha(x)). \quad (24)$$

In addition to the solutions characteristic of the drift temperature instability that has been considered above this equation also has solutions in the range of phase velocities $v_{Te} \gg \omega/k_z \gg v_{Ti}$; $\eta_i = 0$:

$$\omega = -\frac{\omega_e^* A_i}{1+(1-A_i)T_e/T_i}, \quad A_\alpha = \int_{-\infty}^{\infty} f_{0\alpha}J_0^2(\mu v_\parallel)dv_\parallel. \quad (25)$$

These are the drift waves in the presence of the RF magnetic field. These waves can be excited by resonance electrons. The growth rate γ, which formally arises from the electron residue, is

$$\gamma = \pi^{1/2}\frac{\omega_e^{*2}}{|k_z|v_{Te}}J_0^2\left(\mu\frac{\omega}{k_z}\right)A_i\frac{1+(1-A_i)T_e/T_i-\eta/2}{[1+(1-A_i)T_e/T_i]^3}. \quad (26)$$

It turns out that as the amplitude of the RF field increases the region of instability for long wave ($k\rho_i \ll 1$) drift waves increases in parameter space: $\eta_e < 2[1+(1-A_i)T_e/T_i]$ but the growth rate of the instability can be reduced appreciably. For example, whereas in the absence of the RF field the maximum growth rate $\gamma \sim \omega_k$ for waves characterized by $\omega/k_z \lesssim v_{Te}$, in the presence of the RF magnetic field with relatively small intensity $H_1 \sim H_0\rho_i/a$ and the minimum allowed frequency $\Omega_1 \sim \omega_e^*$ the growth rate for a given value of k_y can be reduced by approximately the factor $(M/M)^{1/2}$.

c. Finally we consider the effect of the RF magnetic field on the so-called drift-cyclotron instability of an inhomogeneous plasma, first studied by Mikhaĭlovskiĭ and Timofeev.[10] We shall limit our analysis to the case of the flute mode $k_z = 0$, which has the highest growth rate in the absence of the RF field. The dispersion equation for this instability for shortwave perturbations $k\rho_i \gg 1$, is of the form ($\nabla T = 0$)

$$\frac{1}{T_i}\left[1+k^2r_{Di}^2-\frac{\omega-\omega_i^*}{\omega-n\omega_{Hi}}\frac{1}{(2\pi)^{1/2}k\rho_i}\right] = -\frac{1}{T_e}\left[1-A_e\frac{\omega-\omega_e^*}{\omega}\right]. \quad (27)$$

The left and right sides of this equation contain terms that describe the perturbation in the density due to the ions and electrons respectively. The effect of the RF field on the ion motion can be neglected since we assume that $\Omega_1 \gg \omega_{Hi}$. This equation has unstable solutions

$$\omega = n\omega_{Hi}+\gamma, \quad \gamma^2 = \frac{n^2\omega_{Hi}^2}{(2\pi)^{1/2}k\rho_i}A_e^{-1}(1-n\omega_{Hi}/\omega_i^*), \quad (28)$$

if

$$\omega_i^* = n\omega_{Hi}A_e^{-1}[1+k^2r_{Di}^2+T_iT_e^{-1}(1-A_e)]. \quad (29)$$

This equation should be regarded as an equation for values of k_y for the unstable perturbations. It does not have a solution in the allowable region of values for k_y, assuming $k_y\rho_e \ll 1$, if

$$\frac{H_1}{H_0} > \left(\frac{m}{M}\right)^{1/2}\frac{\rho_i}{a}\frac{\Omega_1}{n\omega_{Hi}} \quad (30)$$

Consequently if this condition is satisfied the drift-cyclotron instability does not appear.

d. The flute instability of the plasma is described by the following dispersion equation when $c < v_A$:

$$2\frac{\omega_{pe}^2}{k^2v_{Te}^2}\left[1-\frac{\omega_e^*+\omega}{\omega}A_e\right] = -\frac{\omega_{pi}^2k_y\,\partial\ln n_0/\partial x}{k^2\omega_{Hi}(\omega-\omega_g)}, \quad (31)$$

$$\omega_g = k_y g/\omega_{Hi}.$$

The effect of curvature of the primary magnetic field can be simulated, in the usual way, by the introduction of a gravitational force $g \sim T_iM^{-1}\partial \ln H_0/\partial x$ directed perpendicularly to the plasma boundary (usually $|\omega_g/\omega^*| \approx |\partial \ln H_0/\partial \ln n_0| \ll 1$). It is assumed that $H_1/H_0 < \Omega_1/k_y v_{Ti}$ so that the motion of the ions along the instantaneous direction of the lines of force can be neglected. The stability condition follows from Eq. (31):

$$1-A_e > 4|\omega_g/\omega_i^*|k^2\rho_i^2. \quad (32)$$

Expanding the Bessel function under the assumption that $\mu v_{Te} \ll 1$, we can rewrite it in the form

$$\frac{H_1^2}{H_0^2} > 4\left|\frac{\omega_g}{\omega_i^*}\right|\frac{\Omega_1^2}{\omega_{Hi}^2}\frac{m}{M}.$$

If we substitute the minimum allowed frequency Ω_1 in this inequality, this frequency being approximately equal to the growth rate of the flute instability $\gamma^2 \sim |g\partial \ln n_0/\partial x|$, we can obtain an estimate of the minimum RF field that will have an effect on the flute instability:

$$\frac{H_1}{H_0} > \left|\frac{\partial \ln H_0}{\partial x}\right|\rho_e. \quad (33)$$

For a real system with finite length there is an additional necessary condition for stabilization

$$H_1/H_0 > 2\pi a/mL, \quad (34)$$

where a and L are the radius and length of the plasma while m is the azimuthal mode number for the perturbation. This condition is such that a line of force at some given time intersects two neighboring flutes of the instability. A sufficient condition for stabilization then is represented by the two inequalities, which are difficult to satisfy.

We note that the inequality in (34) is also relavent to other kinds of instabilities characterized by $k_z = 0$ (3b, 3c, 3d). Analyzing the examples considered above we note that the RF magnetic field has essentially two effects. First, as a consequence of the fact that charged particles can easily move across the fixed magnetic field because of the time varying direction of the magnetic lines of force, there is an important change in the dispersion properties of the plasma [the real part of the dielectric constant $\epsilon(\omega, k)$]. Thus, in the example considered in 3a, in general the oscillations disappear (we recall that in an isothermal plasma in the absence of magnetic field there are no low-frequency oscillations). In other examples (3b, 4a, 4b) the oscillation frequency is reduced. In the case of the loss-cone (3a), and drift-velocity space (3b) and cyclotron (3c, 3d and 4c) instabilities the RF field does not effect the ions, which produce the instability, but acts on the electrons and only the change in the dispersion properties can cause the disappearance of instability.

In the case of the drift-temperature instability (3a) a sufficiently large magnetic field suppresses the instabilities for which $\omega/k_z \gg v_{Ti}$, but those characterized by $\omega/k_z \sim v_{Ti}$ remain, although the growth rates are reduced. The nature of the instability is such as to make it a kinetic instability, its growth rate being determined by the residue [cf. Eq. (23)] that is to say, the instability is due to resonant ions. Similarly, it is not possible to achieve complete stabilization (by an RF magnetic field) of the instabilities associated with drift waves in an inhomogeneous plasma excited by resonance electrons (4b). A resonant particle moves, in one period of the RF magnetic field $2\pi/\Omega_1$, through many spatial periods of the perturbation and experiences an averaging effect of the electric field of the perturbation; however, the field does not vanish completely, but is only reduced by the factor $\pi^{-1/2}k_z(\mu\omega)^{-1} = \pi^{-1/2}H_1k_y\omega/H_0\Omega_1k_z$. This is also the situation, as in the case of kinetic instabilities with short wavelengths $k\rho_i \gg 1$ in a magnetic field (without the RF field) where the effect of the resonant particles is reduced by a factor of $\pi^{-1/2}(k\rho_i)^{-1}$. Starting from these considerations one can draw the following conclusions: in order to achieve a sizable reduction in the growth rate of the drift instabilities it would be necessary to use an RF magnetic field which represents a mixture of two or more frequencies which are not related by integers. In this case there will be an additional averaging effect of the electric field associated with the perturbation on the resonance particle.

5. EFFECT OF AN RF MAGNETIC FIELD WHICH IS A SUPERPOSITION OF FIELDS CHARACTERIZED BY DIFFERENT FREQUENCIES

We now assume that the RF magnetic field is the following function of time: $H_y = H_1 \cos \Omega_1 t + H_2 \cos \Omega_2 t$, $\Omega_1 - \Omega_2 \gg \omega$, $\Omega_1, \Omega_2 \gg \omega$. In this case the unit vector **h** along the instantaneous magnetic field is given by

$$\mathbf{h} = \mathbf{e}_y\left[\frac{H_1}{H_0}\cos\Omega_1 t + \frac{H_2}{H_0}\cos\Omega_2 t\right] + \mathbf{e}_z.$$

Substituting the expression for **h** in Eq. (2) and integrating this equation in the same way as in Sec. 1, we find that Eq. (4) is replaced by

$$f_k(t) = -\Phi_{k\omega}\frac{e}{m}\frac{\partial f_0}{v_\parallel \partial v_\parallel}\exp(-i\omega t) + \Phi_{k\omega}\exp(-i\omega t)\cdot \times\left(\frac{c}{H_0}k_y\frac{\partial f_0}{\partial x} - \omega\frac{e}{m}\frac{\partial f_0}{v_\parallel \partial v_\parallel}\right)\left[\sum_{l,p}\frac{J_l^2(\mu_1 v_\parallel)J_p^2(\mu_2 v_\parallel)}{k_z v_\parallel - \omega + l\Omega_1 + p\Omega_2} + \sum_{\substack{l\neq n\\ p\neq q}}\frac{J_l(\mu_1 v_\parallel)J_n(\mu_1 v_\parallel)J_p(\mu_2 v_\parallel)J_q(\mu_2 v_\parallel)}{k_z v_\parallel - \omega + l\Omega_1 + p\Omega_2}\exp\{i\Omega_1(l-n)t + i\Omega_2(p-q)t\}\right]. \quad (35)$$

It is evident from Eq. (35) that there are high-frequency corrections $\sim \exp\{i(\Omega_1 - \Omega_2)t\}$. If $\Omega_1 - \Omega_2 \gg \omega$ and the frequency Ω_1 is not a multiple of the frequency Ω_2, averaging over the fast oscillations (this is equivalent to an expansion in $\omega/(\Omega_1 - \Omega_2)$) and neglecting terms of order $\omega/(\Omega_1 - \Omega_2)$, we find that $f_{k\omega} = \int \exp(+i\omega t) f_k(t) dt$ becomes

$$f_{k\omega} = \Phi_{k\omega}\left[-\frac{e}{m}\frac{\partial f_0}{v_\parallel \partial v_\parallel} + \left[\frac{c}{H_0}k_y\frac{\partial f_0}{\partial x} - \omega\frac{e}{m}\frac{\partial f_0}{v_\parallel \partial v_\parallel}\right] \times (k_z v_\parallel - \omega)^{-1} J_0^2(\mu_1 v_\parallel) J_0^2(\mu_2 v_\parallel)\right]. \quad (36)$$

If Eq. (36) is now substituted, in place of Eq. (5), in all of the expressions derived above, it is evident that the effect of the RF magnetic field is enhanced. For example, the growth rate for the drift-temperature stability is reduced by a factor A, the growth rate for the drift velocity space by a factor of A^4 etc. If we follow the conclusions of Eq. (36) it becomes evident that if one takes n frequencies (differing from each other by amounts that are larger than the characteristic frequency of the instability to be suppressed) the effect of the RF field is enhanced by a factor A^n $(A < 1)$. This means that even those instabilities that are not finally stabilized will have growth rates that are essentially equal to zero in the RF field.

6. DISCUSSION OF THE RESULTS

The present calculations show that in the majority of cases by means of an RF field it is possible to partially or completely stabilize a broad class of plasma instabilities in a magnetic field, these instabilities being dangerous from the point of view of magnetic confinement of a plasma. There is no doubt that the RF field can have an effect on related instabilities, which have not been considered here because of space limitations. As far as the effect on the velocity space (3a), drift-velocity space (3b) and cyclotron instabilities (3c, 3d, 4c), which are especially dangerous for mirror systems, it is found convenient to use a configuration in which the variable magnetic field is obtained through the use of a helicon wave; in other cases one can use the fast magneto-acoustic wave (cf. for example[6]). The frequency of the alternating field Ω must be much greater than the cyclotron frequency of the ions in order to suppress cyclotron instabilities and larger than $\min(\omega_{pi}, \omega_{i,e})$ to have an effect on the velocity-space instabilities. In this frequency region for Ω the expression for the refractive index of the plasma becomes

$$N^2 = \frac{\omega_{pe}^2}{\omega_{He}\Omega\cos(\widehat{kH_0})}, \quad \cos(\widehat{kH_0}) \gg (m/M)^{1/2}.$$

Generally speaking the magnetic field of the helicon has three components and is elliptically polarized. The ions do not really participate in these oscillations and this is extremely important because as far as the effect on the instability it is necessary that the line of force of the magnetic field rotate with respect to the perturbations in ion density.

In order to have an effect on velocity space and drift-velocity space instabilities, these being characterized by frequency spectra which theoretically occupy the region from ω_{Hi} to $\min(\omega_{pi}, \omega_{i,e})$, it is necessary to use a frequency $\Omega \gtrsim \min(\omega_{pi}, \omega_{i,e})$. However, the effect of the RF field is a maximum for perturbations characterized by a frequency ω close to the frequency Ω (stabilization of perturbations at these frequencies requires $H_1/H_0 > v_{Ti}/v_{Te}$). Hence, stabilization of the entire spectrum of stabilities would require a higher amplitude for the RF magnetic field. From this point of view the most desirable case is a rarefied plasma.

In order to obtain an effect on drift instabilities one can use a magnetic field produced by an alternating current that flows through the plasma or a magnetic field produced by current-carrying conductors located alongside the plasma. However, this alternating field only penetrates into the plasma by an amount equal to the skin depth. It would appear that to obtain a volume effect on the drift instabilities one could use the Alfvèn wave in a plasma, in particular, a symmetric mode with a field H_φ and E_r. Unfortunately, the alternating magnetic field of the low-frequency oscillations $\Omega \ll \omega_{Hi}$ is not suitable for this purpose. This stabilizing effect occurs only when the magnetic line of force rotates with respect to the perturbations and because of the fact that the magnetic field is frozen into the plasma the perturbations will be displaced along with the magnetic line of force. Formally this can be shown using the example of an Alfvèn wave if, in the drift kinetic equation

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial z} + v_\parallel \frac{H^\varphi}{H_0}\frac{\partial f}{r\partial\varphi} - \frac{cE_r}{H_0}\frac{\partial f}{r\partial\varphi} + \frac{e}{m}(\mathbf{Eh})\frac{\partial f_0}{\partial v_\parallel} = 0$$

we convert to the variables ξ and φ associated with the moving line of force:

$$\xi = \int \sqrt{1 + \frac{H_\varphi^2}{H_0^2}}\, dz \simeq z,$$

$$\psi = -\int \frac{H_\varphi}{rH_0} dz + \varphi, \quad t' = t, \quad E_r = \frac{i}{k_z c}\frac{\partial H_\varphi}{\partial t}$$

As a result we obtain an equation that does not contain the field of the Alfvèn wave explicitly:

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial \xi} + \frac{e}{m}(\mathbf{Eh})\frac{\partial f}{\partial v_\parallel} = 0.$$

It is evident that a perturbation taken in the form $\exp(ik_\xi \xi + im\psi)$ in the presence of a radial temperature gradient and a radial density gradient will be unstable. Thus, an RF magnetic field will have an effect on drift stabilities only in the region of the skin depth, where the lines of force are not frozen in the plasma. The nature of the change in the frequency of the drift wave due to the effect of the RF magnetic field of frequency Ω is shown in Fig. 2.

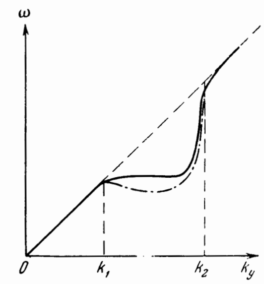

FIG. 2

Here, the dashed curve shows the function $\omega(k_y)$ in the absence of the RF field. The value of the wave vector $k_1 = \Omega H_0/H_1 v_{Ti}$ is determined by the condition $1 - A_i \sim 1$. The value

$$k_2 = \Omega e H_0 (cT\partial \ln n_0 / \partial x)^{-1}$$

is determined by the condition $\Omega \sim \omega$. The lower dot-dashed curve shows the qualitative behavior of $\omega(k_y)$ when the magnetic field is the superposition of two irrational frequencies. This same dependence is obtained for the frequency and growth rate of the drift-temperature instability.

The investigation carried out above can be considered from another point of view, that is, the mutual effect of two or more plasma oscillations. In most work devoted to the nonlinear theory of the instabilities considered above, the nonlinear interaction is treated in the random-phase approximation and the characteristic time for the nonlinear transfer of wave energy between modes is assumed to be on different time scales. Our results, using the terminology adopted here, has been obtained under a fixed-phase approximation, and show that in addition to decay and other interactions, which lead to the transfer of energy, oscillations of finite amplitude have an important effect on the real and imaginary parts of dielectric constant of the plasma. For instance, the possibility cannot be excluded that as a consequence of the nonlinear transfer of energy between different waves and modes in a plasma, the instability due to the presence of a loss-cone can excite helicons in the frequency region from ω_{Hi} to ω_{pi} with an amplitude $H_1 \sim H_0 v_{Ti}/v_{Te}$ and cause a saturation of the velocity-space instability. And since the helicon wave can decay into a plasma wave only in a very weak way, the stable state may exist for a long period of time.

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162