RESISTANCE OF A THIN SUPERCONDUCTING THREAD AT NEAR-CRITICAL CURRENTS

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The problem of the resistance of a thin superconducting current-carrying filament in the absence of an external magnetic field is considered by taking into account the temperature fluctuations. It is shown that if the fluctuations are not taken into account, then a resistance begins to appear at the critical current density j_c but complete restoration of the current is spread out over a broad current range. Fluctuations must be taken into account for currents that are close to the critical value. In this case a resistance begins to appear at currents smaller than critical. The current dependence of the total filament resistance is found for currents close to the critical value.

INTRODUCTION

THE influence of temperature fluctuations on the "spreading" of the superconducting transition current was noted already by Pippard[1]. In recent papers[2,3] this problem is considered for a thin filament and for a film. However, Little[2] proposes that the transition occurs at the critical temperature T_c. Actually the transition occurs at $T < T_c$ but at a current $j = j_c$ which greatly changes the situation. In addition it is assumed in[1-3] that the resistance (energy dissipation) occurs only when some microscopic volume of the superconductor goes over into the normal state as a result of the fluctuations. In light of the ideas advanced by Bardeen and Stephen[4], this statement seems incorrect to us. Bardeen and Stephen considered the energy dissipation in the intermediate region near a vortex, where both a superconducting condensate and an electric field exists simultaneously. A similar situation should arise also in a thin superconducting filament.

We shall consider below the resistance of a thin superconducting film carrying a current set by an external source. There is no external magnetic field.

2. CURRENT TRANSITION

In this section we consider the current transition in a thin-superconducting filament without fluctuations. The analysis will be carried out by the method of the Ginzburg-Landau theory[5], using the idea advanced by Bardeen and Stephen[4]. We shall show that the resistance of a filament is not brought back to normal even at a current $j > j_c$ (under conditions of ideal heat transfer). In spite of the crudeness of the presented calculation it can be assumed that its results are applicable to the problem of the fluctuation resistance. A rigorous solution of the problem of the current transition of a thin filament form the superconducting state to the normal state calls, of course, for a separate analysis using the methods of the microscopic theory.

We consider a transition to the normal current state for a thin superconducting filament in which $r \ll \delta_0/\kappa$, where r is the radius of the filament, δ_0 is the depth of the penetration of the magnetic field and κ is a constant of the theory[5]. The distribution of the current over the cross section of the filament can then be regarded as uniform. Since we are using the Ginzburg-Landau theory, we assume also that $T \approx T_c$ and $r \gg \sqrt{\xi_0 l}$ where $\xi_0 = \hbar v_0/\Delta_0$ is the coherence length, v_0 the velocity of the electron on the Fermi surface, Δ_0 the energy gap at $T = 0$, and l the mean free path of the electron. The latter inequality is necessary for the applicability of the Ginzburg-Landau theory to a thin superconducting filament with allowance for gradient terms only along the filament.

If the density of the free energy for a current-carrying filament is written in the form[6]

$$F_s = F_n + \alpha\rho_s + \frac{\beta}{2}\rho_s^2 + \frac{m}{2}\rho_s v_s^2, \qquad (1)$$

then the equilibrium condition corresponding to the minimum of the free energy will be

$$\frac{\partial F_s}{\partial \rho_s} = \alpha + \beta\rho_s + \frac{m}{2}v_s^2 = 0. \qquad (2)$$

Here F_n is the density of the free energy in the normal state, ρ_s the concentration of the superconducting electrons ($\rho_s = |\Psi|^2$, where Ψ is a wave function of a theory of[5]), v_s the velocity of the superconducting electrons ($j = e\rho_s v_s$ is the density of the superconducting current, which is set by the external source), α and β the coefficients of expansion of the free energy in powers of $|\Psi|^2$, and m the electron mass.

Expressing the equilibrium value of ρ_s from (2), we obtain the dependence of the density of the superconducting current on the velocity v_s:

$$j = e\rho_s v_s = -e\frac{\alpha}{\beta}v_s - \frac{em}{2\beta}v_s^3. \qquad (3)$$

We see that the function $j(v_s)$ has a maximum, since $\alpha(T) < 0$ when $T < T_c$. This maximum current density is defined in[5] as the critical density:

$$j_c = j_{max} = \frac{e}{\beta\sqrt{m}}\left(\frac{2}{3}|\alpha|\right)^{3/2}. \qquad (4)$$

The maximum for the current is obtained at a superconducting velocity which we denote by v_d:

$$v_d = (2|\alpha|/3m)^{1/2} \qquad (5)$$

and at a superconducting-electron density ρ_c:

$$\rho_c = 2|\alpha|/3\beta. \qquad (6)$$

The states with $v_s < v_d$ (i.e., when $dj/dv_s > 0$) will be stable states with a superconducting current, and the case $v_s > v_d$ (i.e., $dj/dv_s < 0$) corresponds to unstable states. These are the well-known results of[5,6].

We now pose the following question: What processes arise in a thin superconducting filament if the external source increases the density of the current in the filament to a value larger than j_c? We note, first, that when $j = j_c$ we still have $F_s < F_n$. This can be readily verified by substituting (5) and (6) in (1). Thus, when $j = j_c$ the superconducting state is still favored. This means that when $j = j_c$ there should be no phase transition in the usual sense of this word. However, the point $j = j_c$ is singular in some sense, since the transport of an electric current $j > j_c$ with the aid of the superconducting electrons only is impossible for the very simple reason that there are not enough of them: At this stage of our exposition we shall use the idea of the mechanism of energy dissipation in the transition region near the core of an Abrikosov vortex, given in the paper of Bardeen and Stephen[4]. Let us apply this idea to our case of a thin superconducting filament.

When $j > j_c$, an electric field E is produced in the filament; the condensate is accelerated in this field (from a velocity v_d to a velocity $v_d + eE\tau/m$) within a time τ (the relaxation time of the superconducting state). After the lapse of this time, the Cooper pairs forming the condensate break up into individual electrons; the latter relax with the lattice and slow down to a velocity smaller than v_d. The electrons then are again paired and fall into the condensate (since $F_s < F_n$) and the entire process is repeated. Bearing in mind this picture and taking into account the existence of the normal component of the electron liquid, we write the following averaged equation for the current density when $j > j_c$:

$$j = e\rho_c v_d + e^2\rho_c \frac{E}{m}\tau + \sigma_n E,$$

where σ_n is the normal conductivity. Recognizing that the $j_c = e\rho_c v_d$, we have

$$j = j_c + \sigma E, \tag{7}$$

where

$$\sigma = \sigma_s + \sigma_n, \quad \sigma_s = e^2\rho_c\tau / m = c^2\tau / 6\pi\delta_0^2.$$

Thus, the experimentally observed resistivity of the filament at $j > j_c$ will be

$$R = \sigma^{-1}(1 - j_c / j). \tag{8}$$

Consequently, the restoration of the resistance of a thin filament when $j > j_c$ should occur not jumpwise at $j = j_c$ but monotonically as j becomes larger than j_c. This, of course, is valid only under the conditions of ideal heat transfer, when the filament temperature does not rise.

All the foregoing arguments are valid also for the case of a thin film. The spreading of the current transition in thin films was observed experimentally (see, for example,[7]).

3. DERIVATION OF THE DISTRIBUTION OF THE FLUCTUATIONS OF Ψ FOR A THIN SUPERCONDUCTING FILAMENT

We consider a thin current carrying superconducting filament of length 2L. Let its radius be $r \gg \sqrt{\xi_0 l}$, but $r \ll \delta_0(T)/\kappa$, where $\delta_0(T)$ is the depth of penetration of the weak magnetic field at the temperature T, and $\kappa \approx \delta_0(0)/\xi_0$. To use the Ginzburg-Landau theory, we assume that $T \approx T_c$.

Let us compare the two characteristic times, namely τ_s, which is the relaxation time of the superconducting state, and τ_T, which is the time of dissipation of the temperature fluctuation in a certain volume of the material. We have $\tau_s \sim \hbar/\Delta \sim 10^{-11}$–$10^{-12}$ sec. On the other hand, if we assume that the temperature relaxes as a result of thermal conductivity, then $\tau_T \sim \lambda^2/l_{ph}u$, where λ is the linear dimension of the volume with the fluctuation, l_{ph} is the phonon mean free path, and u is the speed of sound. If $\lambda \sim 3 \times 10^{-6}$ cm, $l_{ph} \sim 10^{-7}$ cm, and $u \sim 10^5$ cm/sec, then $\tau_T \sim 10^{-9}$ sec. Thus, conditions under which $\tau_T \gg \tau_s$ are realistic. This means that all the superconducting characteristics of a small volume subtended by the fluctuation will follow the temperature adiabatically.

We now find the law governing the distribution of the fluctuations in a thin filament. The temperature fluctuation ΔT in a small volume is determined by the minimum work that an external thermally insulated source can perform. Its density R_{min} is, according to[8],

$$\frac{R_{min}}{kT} = \frac{C_v}{2kT^2}(\Delta T)^2,$$

where C_v is the specific heat.

Let ψ be the deviation of the wave function Ψ of the Ginzburg-Landau theory from the value Ψ_0 corresponding to the equilibrium temperature. Assuming that the current in the filament is close to critical, that is, that $\Psi_0^2 \approx (2/3)|\alpha|/\beta$, and using the expression for the jump of the specific heat Δc in a second-order phase transition, $\Delta C = (d|\alpha|/dT)^2 T_c T_c/\beta$, we have

$$\frac{R_{min}}{kT} = \frac{a}{2}\psi^2, \qquad a = \frac{6|\alpha|}{kT}\frac{C_v}{\Delta C}.$$

This will take place in the case of a spatially homogeneous fluctuation ψ due to a temperature fluctuation ΔT. On the other hand, if the fluctuation is inhomogeneous, it is necessary to take into account also the additional density of the kinetic energy, and the total expression for R_{min}/kT is

$$\frac{R_{min}}{kT} = \frac{a}{2}\psi^2 + \frac{b}{2}\left(\frac{d\psi}{dx}\right)^2, \tag{9}$$

where $b = \hbar^2/mkT$. We took into account here the possible inhomogeneity of the fluctuation only along the filament (along the x axis). It is assumed that Ψ is uniformly distributed across the filament, since the filament radius is $r \ll \delta_0/\kappa$, in the spirit of the procedure used for small particles[9].

We thus have a random function $\psi(x)$. We seek the law of distribution of the random quantity ψ. The fluctuation probability is

$$w \sim e^{-\mathcal{R}_{min}/kT}, \qquad \mathcal{R}_{min} = S\int_{-L}^{L} R_{min}\,dx, \tag{10}$$

where S is the cross section area of the filament. We expand $\psi(x)$ in a Fourier series

$$\psi(x) = \sum_q \psi_q e^{iqx}. \tag{11}$$

Substituting (11) in (9) and taking into account the fact that $\psi(x)$ is real (that is, $\psi_g^* = \psi_{-g}$), we obtain

$$\frac{\mathcal{R}_{min}}{kT} = LS\sum_q (a + bq^2)|\psi_q|^2 = LS\sum_q (a + bq^2)(\varphi_q^2 + \chi_q^2), \quad (12)$$

where $\varphi_g = \mathrm{Re}\,\psi_g$ and $\chi_g = \mathrm{Im}\,\psi_g$. Substituting (12) in (10) we get $w \sim \prod_g w(\varphi_g) w(\chi_g)$, where

$$w(\varphi_q) \sim \exp\{-LS(a + bq^2)\varphi_q^2\}. \quad (13)$$

Thus, the random quantities φ_g, as well as the random quantities χ_g, are independent, and their distribution is given by (13).

We now consider the random quantity $\psi(x = 0)$. The point $x = 0$ is not distinguished physically in any way, and

$$\psi(x = 0) \equiv \psi_0 = \sum \varphi_q.$$

Thus the random quantity ψ_0 is a sum of the random quantities φ_q, the distribution of which is normal ($w_q \sim \exp[-\varphi_q^2/2D_q]$) with dispersion

$$D_q = 1/2LS(a + bq^2). \quad (14)$$

It is known from probability theory that the random quantity ψ_0 has in this case a normal distribution with dispersion $D = \Sigma D_q$. Substituting here (14) and integrating in elementary fashion, we get

$$D = 1/2S\sqrt{ab}.$$

Substituting here the expressions for a and b and using the equality [5] $|\alpha| = \hbar^2\kappa^2/2m\delta_0^2$, we get ultimately

$$D = \frac{m\delta_0 kT}{2\sqrt{3}\hbar^2 S\varkappa}\sqrt{\frac{\Delta C}{C_v}}. \quad (15)$$

We have thus found the distribution of the random quantity ψ_0:

$$w(\psi_0) = e^{-\psi_0^2/2D}/\sqrt{2\pi D}, \quad (16)$$

where D is given by (15).

4. FLUCTUATION RESISTANCE R. THE CASE $j < j_c$

We can now proceed to calculate the fluctuation resistance R. We consider first the case $j < j_c$.

The idea of the calculation can be understood with the aid of Fig. 1. It shows the dependence of the equilibrium density ρ_s on v_s at a given temperature T. It also shows the dependence of $j_s = e\rho_s v_s$ on v_s. The critical current j_c corresponds to the maximum of the curve. Assume that a current $j < j_c$ is made to flow through the filament. Corresponding to this current are the equilibrium values of the velocity v_s and of the density ρ_s^0 shown in the figure.

We now consider some point of the filament. Since the point $x = 0$ is physically indistinguishable from the others, assume that this is the point $x = 0$. As a result of the fluctuation increase of the temperature in a physically infinitesimally small volume of the filament about the point $x = 0$, the isotherm $\rho_s(v_s, T)$ will go over for such a point into $\rho_s(v_s, T + \delta T)$ (shown dashed in the figure). If the maximum of the corresponding isotherm $e\rho_s v_s$ turns out to be higher than j, this means that at the given point $x = 0$ there are still enough superconducting electrons to carry the current j, that is, the local value of the critical current at this point is still larger than j. In this case no electric field is produced here.

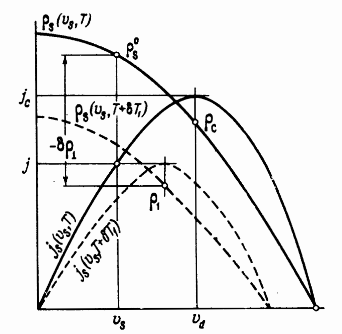

FIG. 1. Density of superconducting electrons ρ_s and of the superconducting current j_s as functions of the velocity of the condensate v_s. Solid curves - equilibrium ρ_s and j_s at the temperature T, The dashed curves correspond to a local temperature $T + \delta T_1$, where δT_1 is the threshold temperature fluctuation at which every dissipation sets in at a given current j. The threshold density fluctuation is equal to $\delta\rho_1 = \rho_1 - \rho_s^0$.

On the other, if the fluctuation temperature rise is so large that the maximum of the isotherm $e\rho_s v_s$ drops below the level j, then the situation analyzed in detail in Sec. 2 arises: the local value of the critical current turns out to be smaller than that of the current j set by the external source. In this case there occurs at the point $x = 0$ an electric field E, given by formula (7), in which j_c should be taken to mean the local value of the critical current. Averaging this value of E over the distribution (16) and dividing by j, we obtain the sought fluctuation resistance R.

Let us perform this program. Let the fluctuation be so large that a field E was produced at the point $x = 0$. Then taking (4), (6), and (7) into account we get

$$E = \frac{1}{\sigma}\left(j - \rho_s^{3/2} e\sqrt{\frac{\beta}{m}}\right), \quad (17)$$

where ρ_s is the density produced at the given point of the filament as a result of the fluctuation temperature rise. Let ρ_1 be the density at which E first appears. From (4) and (6) we can readily establish a connection between the current j_c and the density ρ_c. It is clearly seen from Fig. 1 that a similar connection exists also between the current j and the threshold density ρ_1:

$$j = \rho_1^{3/2} e\sqrt{\beta/m}. \quad (18)$$

Substituting (18) and (17), assuming that the fluctuation of ρ_s is small compared with ρ_s itself, and confining ourselves to the term linear in the fluctuation, we get

$$E = \frac{2e}{\sigma}\frac{|\alpha|}{\sqrt{m\beta}}(\psi_1 - \psi),$$

where ψ_1 is the threshold value of the fluctuation ψ, determined from the formula for ρ_1:

$$\rho_1:\ \rho_1 = \Psi_0^2 + 2\Psi_0\psi_1,\quad \rho_s = \Psi_0^2 + 2\Psi_0\psi,\quad \Psi_0^2 = \rho_s^0.$$

Averaging E over the distribution (16)

$$\bar{E} = \int_{-\infty}^{\psi_1} E w(\psi)\, d\psi,$$

we get finally

$$\bar{E} = \frac{e}{\sigma}\frac{|\alpha|}{\sqrt{m\beta}}\sqrt{2D}\left[x_1(1 + \Phi(x_1)) + \frac{1}{\sqrt{\pi}}e^{-x_1^2}\right], \quad (19)$$

where

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt,\quad x_1 = \psi_1/\sqrt{2D}.$$

It remains to connect the threshold x_1 with the difference $j_c - j$. This connection is clear from Fig. 1. Assuming that j is close to j_c, and consequently ρ_s^0 is close to $\rho_c = (2/3)|\alpha|/\beta$, we obtain first $\Delta\rho_s = \rho_s^0 - \rho_c$. To this end, we express ρ_s as a function of v_s using (3), and substitute in place of v_s the quantity $j_s/e\rho_s$. We then get for j_s

$$j_s^2\frac{m}{2\beta e^2} = -\frac{\alpha}{\beta}\rho_s^2 - \rho_s^3.$$

Expanding this expression about the point j_c in a power series in $\Delta j_s = j_c - j_s$ and $\Delta\rho$, we get

$$\Delta\rho_s = \rho_s^0 - \frac{2}{3}\frac{|\alpha|}{\beta} = \left(\frac{j_c m}{e^2|\alpha|}\right)^{1/2}\sqrt{j_c - j_s}.$$

Putting $j_s = j$ we obtain the dependence of ρ_s^0 on $j_c - j$:

$$\rho_s^0 = \frac{2}{3}\frac{|\alpha|}{\beta} + \left(\frac{j_c m}{e^2|\alpha|}\right)^{1/2}\sqrt{j_c - j}.$$

Using (18) and recognizing in addition that

$$\frac{2}{3}\frac{|\alpha|}{\beta} = \frac{j_c^{2/3}}{e^{2/3}}\left(\frac{m}{\beta}\right)^{1/3}$$

and introducing the notation

$$\varepsilon = (j_c - j)/j_c,$$

we obtain finally.

$$-\delta\rho_1 = \frac{4}{9}\frac{|\alpha|}{\beta}\varepsilon + \left(\frac{2}{3}\right)^{3/2}\frac{|\alpha|}{\beta}\sqrt{\varepsilon}.$$

Since

$$\delta\rho_1 = 2\Psi_0\psi_1, \quad \Psi_0 \approx \sqrt{\frac{2}{3}\frac{|\alpha|}{\beta}},$$

we have

$$\psi_1 = -\frac{1}{3}\sqrt{\frac{|\alpha|}{\beta}}\left(\sqrt{\varepsilon} + \sqrt{\frac{2}{3}}\varepsilon\right).$$

It now remains for us to obtain the dependence of $x_1 = \psi_1/\sqrt{2D}$ on ϵ. Then, using (15) and the expression obtained in [5] for the depth of penetration of a weak magnetic field, $\delta_0 = \sqrt{mc^2\beta/4\pi e^2|\alpha|}$, we get

$$x_1 = -\frac{3^{1/4}}{6\sqrt{\pi}}x_0\left(\sqrt{\varepsilon} + \sqrt{\frac{2}{3}}\varepsilon\right). \tag{20}$$

For convenience we have introduced here the dimensionless quantity x_0, which depends only on the dimensions and the material of the superconducting filament:

$$x_0 = \frac{\hbar c}{e}\frac{\sqrt{\varkappa S}}{\delta_0^{3/2}\sqrt{kT}}\left(\frac{C_v}{\Delta C}\right)^{1/4}. \tag{21}$$

The fluctuation resistivity of the filament $R = \bar{E}/j$ is (see (19))

$$R = \frac{1}{\sigma}\cdot 3^{3/4}\sqrt{\frac{3\pi}{2}}\frac{1}{x_0}\left[x_1(1+\Phi(x_1)) + \frac{1}{\sqrt{\pi}}e^{-x_1^2}\right]. \tag{22}$$

If $|x_1| \gg 1$, then the asymptotic expression for R is

$$R = \frac{1}{\sigma}\frac{3^{3/4}\sqrt{3}}{2\sqrt{2}}\frac{1}{x_0}\cdot\frac{e^{-x_1^2}}{x_1^2}. \tag{23}$$

It follows from (22) and (23) that the region where the fluctuations are appreciable is given by the condition

$$|x_1| \sim 1. \tag{24}$$

Recognizing that $(C_v/\Delta C)^{1/4} \sim 1$, we get from (20) and (21) the condition (24) in the form

$$\varepsilon \sim \delta_0^3 T/\varkappa S.$$

Let us present a few estimates. Let $T \sim 10°K$, $\kappa \sim 1$, $\delta_0 \sim 10^{-5}$ cm, and $S \sim 10^{-12}$ cm^2. We then have $x_0 \sim 100$ and $\epsilon \equiv (j_c - j)/j_c \sim 10^{-2}$. This is the interval of the currents near the critical current where the fluctuations are significant. If the thickness of the filament is smaller by one order of magnitude and $S \sim 10^{-14}$ cm^2, then $x_0 \sim 10$ and $\epsilon \sim 1$. In this case the fluctuations play a very important role.

If the current is equal to the critical value, then $x_1 = 0$ and the fluctuation resistance reaches a value $R \sim 3\sigma^{-1}/x_0$. If $S \sim 10^{-12}$ cm^2 we get $R \sim 0.03\ \sigma^{-1}$, and if $S \sim 10^{-14}$ cm^2 we get $R \sim 0.3\ \sigma^{-1}$.

5. TOTAL RESISTANCE R. THE CASE $j > j_c$

In this case, as follows from Sec. 2, an electric field that depends on the difference $j - j_c$ appears even in the absence of fluctuations in the filament, and its value according to (7), is

$$E = (j - j_c)/\sigma. \tag{25}$$

The density ρ_s is in this case $\rho_c = (2/3)|\alpha|/\beta$ (see (6)). The presence of the fluctuation of ρ_s causes a local change of the critical current density at the given point of the filament. Thus, if the density at the given point of the filament is now not ρ_c but ρ_s, then the local critical current equals, according to (18),

$$j_{c,\,\mathrm{loc}} = \rho_s^{3/2} e\sqrt{\beta/m}.$$

It is obvious that the local electric field will now be

$$E_{\mathrm{loc}} = \frac{1}{\sigma}\left(j - \rho_s^{3/2} e\sqrt{\frac{\beta}{m}}\right). \tag{26}$$

Recognizing that

$$\rho_s = (\Psi_c + \psi)^2 = \Psi_c^2 + 2\Psi_c\psi,$$

we get

$$\rho_s^{3/2} = \rho_c^{3/2} + 3\rho_c\psi.$$

Substituting this expression in (26), we obtain

$$E_{\mathrm{loc}} = \frac{1}{\sigma}\left(j - j_c - 3e\sqrt{\frac{\beta}{m}}\rho_c\psi\right).$$

E_{loc} will differ from zero, obviously, only so long as the fluctuation does not reach a certain threshold value ψ_2, which is determined by the fact that the local density $\rho_2 = \Psi_c^2 + 2\Psi_c\psi_2$ becomes critical for an externally set current j, that is,

$$\rho_2 = \left(\frac{j}{e}\right)^{2/3}\left(\frac{m}{\beta}\right)^{1/3}.$$

From this we get directly the difference $\delta\rho_2 = \rho_2 - \rho_c$ and a corresponding threshold ψ_2:

$$\psi_2 = \tfrac{1}{3}\sqrt{\rho_c}\varepsilon, \quad \varepsilon \equiv (j - j_c)/j_c.$$

Averaging of E_{loc} over the distribution (16) gives the average value of the electric field in the filament:

$$\bar{E} = \int_{-\infty}^{\psi_2} E_{\mathrm{loc}}\, w(\psi)\, d\psi. \tag{27}$$

Integrating (27) we obtain the following final result:

$$R = \frac{\bar{E}}{j_c} = \frac{1}{2\sigma}\varepsilon[1 + \Phi(x_2)] + \frac{1}{\sigma}\cdot 3^{3/4}\sqrt{\frac{3}{2}}\frac{1}{x_0}e^{-x_2^2}, \tag{28}$$

where $x_2 = 3^{-5/4}(2\pi)^{-1/2}\epsilon x_0$, and x_0 is given by (21).

It is clear from (28) that the region of currents where the fluctuations are significant is determined by the condition $x_2 \sim 1$, or

$$\varepsilon \sim \delta_0^{3/2}\sqrt{T_c/\varkappa S}.$$

If $T_C \sim 10^\circ K$, $\kappa \sim 1$, $\delta_0 \sim 10^{-5}$ cm and $S \approx 10^{-12}$ cm^2 then $\epsilon \sim 0.1$. When $j = j_C$, that is, when $\epsilon = 0$, formula (28) gives an expression for R which coincides with the expression that follows from (22) with $x_1 = 0$, as of course it should. On the other hand, when $x_2 \to \infty$ the total resistance $R \to \epsilon/\sigma$, corresponding to formula (25).

6. CONCLUSION

We have considered the current transition in a thin superconducting film with allowance for the fluctuation of the density of the superconducting electrons. The resistance of the filament at a current close to critical is given by formulas (22) and (28) for the cases $j < j_C$ and $j > j_C$, respectively. A qualitative picture of the transition is shown in Fig. 2.

In conclusion, we wish to examine the results obtained in Sec. 2 from a somewhat different point of view. It was assumed there that the filament radius $r \ll \delta_0/\kappa$, and satisfaction of this condition ensured the homogeneity of the distribution of the current over the cross section of the filament. There was not need anywhere else to assume that the filament radius is small. In hard superconductors of the second kind, that is, superconductors of the second kind with a large number of pinning centers for the Abrikosov vortices, the distribution of the current over the cross section will also be approximately homogeneous. If we now assume that the pinning forces are very large and the flowing current, up to the critical value, cannot be torn away from the pinning center, then the critical density will be the same as for a filament or a film, and formula (8) describes one more type of resistive state, in which the dissipation is connected not with the notion of the vortices transverse to the magnetic field (as in[4]), but with the mechanism described in Sec. 2. These questions, however, call for a separate analysis.

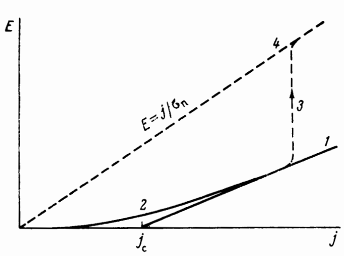

FIG. 2. Schematic representation of the current transition in a thin superconducting filament. 1 – electric field depends linearly on the current density, if the fluctuations are disregarded; 2 – transition curve with allowance for fluctuations; in the case of insufficient heat transfer, the filament temperature rises above T_C and the filament goes over into the normal state (curve 3), where Ohm's law (the straight line 4) is satififsied.

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