THE MOTION OF A PLASMA IN THE FIELD OF AN ELECTROMAGNETIC WAVE

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We obtain equations that describe the motion of a cold plasma under the action of the pressure of a high-frequency electromagnetic wave propagating along a constant, uniform magnetic field. We study in detail the case of short wave lengths when the geometric optics approximation is applicable. We show that it is in that case possible that electrodynamical waves arise in the plasma; we also solve the problem of the self-similar spreading of a plasma through which an electromagnetic wave passes.

INTRODUCTION

WHEN considering the propagation of electromagnetic waves in a plasma, one usually assumes that the wave does not affect the motion of the plasma.[1] Such an assumption is justified if the kinetic pressure of the plasma particles is larger than the pressure of the electromagnetic wave. However, in investigations of the radiative acceleration of a plasma often such field intensities are produced that this condition is violated. In that case it is important to take the pressure of the electromagnetic wave into account.[1)]

In the present paper we use the hydrodynamic equations for the electrons and ions and the Maxwell equations for the field to obtain equations which describe the motion of a cold plasma under the influence of a high-frequency electromagnetic wave propagating along a constant, uniform magnetic field. Using these equations we consider the simplest case when in a transparent plasma an electromagnetic wave propagates with a wavelength much shorter than the characteristic distance over which the plasma density changes. We show that in that case there exists a well-defined relation between the electromagnetic pressure and the plasma density (which is similar to an equation of state). We consider small perturbations in the plasma (electrodynamic waves) and also the problem of a self-similar spreading of the plasma under the action of the pressure of an electromagnetic wave passing through the plasma (stationary dilatation wave).

1. PLASMA EQUATIONS OF MOTION

We use the hydrodynamic equations for the electrons and ions and the Maxwell equations for the field as the basis for our description of the motion of a plasma in the field of an electromagnetic wave. We shall assume that all hydrodynamic variables and all fields depend on one spatial variable z and on the time t. The electromagnetic wave also propagates along the z axis which is parallel to a constant, uniform magnetic field B_0. The set of equations for electrons (a = e) and ions (a = i) and for the field has in the usual notation the form

$$\frac{\partial v_x^{(a)}}{\partial t}+v_z^{(a)}\frac{\partial v_x^{(a)}}{\partial z}=\frac{e_a}{m_a}E_x+\frac{e_a}{m_ac}(v_y^{(a)}B_0-v_z^{(a)}B_y), \quad (1.1)$$

$$\frac{\partial v_y^{(a)}}{\partial t}+v_z^{(a)}\frac{\partial v_y^{(a)}}{\partial z}=\frac{e_a}{m_a}E_y+\frac{e_a}{m_ac}(v_z^{(a)}B_x-v_x^{(a)}B_0), \quad (1.2)$$

$$\frac{\partial v_z^{(a)}}{\partial t}+v_z^{(a)}\frac{\partial v_z^{(a)}}{\partial z}=\frac{e_a}{m_a}E_z+\frac{e_a}{m_ac}(v_x^{(a)}B_y-v_y^{(a)}B_x), \quad (1.3)$$

$$\frac{\partial n^{(a)}}{\partial t}+\frac{\partial}{\partial z}(n^{(a)}v_z^{(a)})=0, \quad (1.4)$$

$$\frac{\partial E_x}{\partial z}=-\frac{1}{c}\frac{\partial B_y}{\partial t}, \quad (1.5)$$

$$\frac{\partial E_y}{\partial z}=\frac{1}{c}\frac{\partial B_x}{\partial t}, \quad (1.6)$$

$$\frac{\partial B_x}{\partial z}=\frac{1}{c}\frac{\partial E_y}{\partial t}+\frac{4\pi}{c}\sum_a e_an^{(a)}v_y^{(a)}, \quad (1.7)$$

$$-\frac{\partial B_y}{\partial z}=\frac{1}{c}\frac{\partial E_x}{\partial t}+\frac{4\pi}{c}\sum_a e_an^{(a)}v_x^{(a)}, \quad (1.8)$$

$$\frac{\partial E_z}{\partial z}=-4\pi\sum_a e_an^{(a)}v_z. \quad (1.9)$$

We neglect fast-changing motions of the ions and electrons in the magnetic field and in the wave field. In that case Eq. (1.3) for the longitudinal velocity of the ions takes the form

$$\frac{\partial v_z^{(i)}}{\partial t}+v_z^{(i)}\frac{\partial v_z^{(i)}}{\partial z}=\frac{e_i}{m_i}E_z. \quad (1.10)$$

Assuming that the phase velocity of the wave is much larger than the velocity of the electron longitudinal motion $v_z^{(e)}$, we can write Eqs. (1.1) and (1.2) for the transverse electron velocities in the form

$$\frac{\partial v_+^{(e)}}{\partial t}=\frac{e}{m}E_++i\Omega v_+^{(e)}, \qquad \frac{\partial v_-^{(e)}}{\partial t}=\frac{e}{m}E_--i\Omega v_-^{(e)},$$

where $v_\pm^{(e)}=v_x^{(e)}\pm iv_y^{(e)}$, $E_\pm=E_x\pm iE_y$, $\Omega=eB_0/mc$ is the cyclotron frequency of the electrons. The solution of these equations has the form

$$v_\pm^{(e)}(z,t)=\frac{e}{m}\int^t dt'E_\pm(z,t')\exp(\pm i\Omega(t-t')). \quad (1.11)$$

In order to describe the slow longitudinal motion of the electrons we average in Eq. (1.3) over a time large compared to the period over which the field of the wave changes. Using Eqs. (1.5), (1.6), and (1.11), we write

[1)]We must note that the problem of the plasma equilibrium configuration in the field of a high-frequency standing wave has been considered in a number of papers (e.g.,[2,3]).

$$\frac{\partial v_z^{(e)}}{\partial t}+v_z^{(e)}\frac{\partial v_z^{(e)}}{\partial z}=\frac{e}{m}E_z-\frac{e^2}{2m^2}\left\langle\int^t dt'\int^t dt''\left\{\cos\Omega(t-t')\left(E_x(t')\frac{\partial E_x(t'')}{\partial z}+E_y(t')\frac{\partial E_y(t'')}{\partial z}\right)+\sin\Omega(t-t')\left(E_x(t')\frac{\partial E_y(t'')}{\partial z}+E_y(t')\frac{\partial E_x(t'')}{\partial z}\right)\right\}\right\rangle \tag{1.12}$$

where the brackets $\langle\ldots\rangle$ indicate averaging over the time.

The fields in Eq. (1.12) satisfy, according to (1.7) to (1.9), the equations

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_\pm=\frac{4\pi e^2}{mc^2}\frac{\partial}{\partial t}\left\{n^{(e)}\int^t dt'\,E_\pm(t')\exp(\pm i\Omega(t-t'))\right\}, \tag{1.13}$$

$$\partial E_z/\partial z=4\pi(en^{(e)}+e_in^{(i)}). \tag{1.14}$$

The set of Eqs. (1.4), (1.10), (1.12) to (1.14) describes the slow motion of the plasma under the action of the pressure of the electromagnetic wave. These equations can be simplified considerably if the condition for the quasi-neutrality of the plasma, $en^{(e)}+e_in^{(i)}=0$, $v_z^{(e)}=v_z^{(i)}\equiv v(z,t)$, is satisfied during the motion. Assuming for the sake of simplicity that $|e_i|=|e|$ and using the notation $n(z,t)=n^{(e)}=n^{(i)}$, we get then

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial z}=-\frac{e^2}{2mm_i}\left\langle\int^t dt'\int^t dt''\left\{\cos\Omega(t-t')\left(E_x(t')\frac{\partial E_x(t'')}{\partial z}+E_y(t')\frac{\partial E_y(t'')}{\partial z}\right)+\sin\Omega(t-t')\left(E_x(t')\frac{\partial E_y(t'')}{\partial z}+E_y(t')\frac{\partial E_x(t'')}{\partial z}\right)\right\}\right\rangle, \tag{1.15}$$

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_\pm=\frac{4\pi e^2}{mc^2}\frac{\partial}{\partial t}\left\{n\int^t dt'E_\pm(t')\exp(\pm i\Omega(t-t'))\right\}, \tag{1.16}$$

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial z}(nv)=0. \tag{1.17}$$

The expression on the right-hand side of Eq. (1.15) describes the force acting upon the plasma in the field of a high-frequency wave. The dependence of the field of the wave on the plasma density is determined by Eq. (1.16). The set of Eqs. (1.15) to (1.17) determines thus at the same time the changes in time and space of the field in the plasma and of the density and velocity of the plasma.

We note that when there is no magnetic field it is convenient to use a somewhat different set of equations. In that case Eqs. (1.1) and (1.2) have the form

$$\frac{\partial v_{x,y}^{(e)}}{\partial t}=\frac{e}{m}E_{x,y}$$

and Eqs. (1.5) and (1.6) enable us to obtain simple expressions for the magnetic field of the wave

$$B_y=-\frac{mc}{e}\frac{\partial v_x^{(e)}}{\partial z},\qquad B_x=\frac{mc}{e}\frac{\partial v_y^{(e)}}{\partial z}. \tag{1.18}$$

Equation (1.3) for the longitudinal velocity of the electrons then transforms to

$$\frac{\partial v_z^{(e)}}{\partial t}+v_z^{(e)}\frac{\partial v_z^{(e)}}{\partial z}=\frac{e}{m}E_z-\frac{1}{2}\frac{\partial}{\partial z}\langle v_x^{(e)2}+v_y^{(e)2}\rangle, \tag{1.19}$$

and we get the equation to determine $v_{x,y}^{(e)}$ from Eqs. (1.7) and (1.8)

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)v_{x,y}^{(e)}=\frac{4\pi}{mc^2}e^2n^{(e)}v_{x,y}^{(e)}. \tag{1.20}$$

2. GEOMETRIC OPTICS APPROXIMATION

We solve Eq. (1.15) assuming that the phase of the electromagnetic wave changes fast not only in time, but also in space. In other words, we consider sufficiently short wavelengths for which the geometric optics approximation is valid. We write the field of the wave in the form

$$E_\pm(z,t)=E_\pm^{(0)}(z,t)e^{i\varphi_\pm(z,t)}, \tag{2.1}$$

where $E_\pm^{(0)}$ and $\varphi_\pm$ are real functions while the latter changes in space and time faster than the first one. Restricting ourselves to the largest terms we get from Eq. (1.16) for the field E_+

$$(E_+^{(0)''}-\varphi_+'^2E_+^{(0)})-\frac{1}{c^2}(\ddot{E}_+^{(0)}-\dot{\varphi}_+^2E_+^{(0)})=-\frac{\omega_{L_e}^2}{c^2}E_+^{(0)}\frac{\dot{\varphi}_+}{\Omega-\dot{\varphi}_+}+\frac{(\omega_{L_e}^2)^{\cdot}}{c^2}\frac{1}{\Omega-\dot{\varphi}_+}\left(\frac{\dot{E}_+^{(0)}}{\Omega-\dot{\varphi}_+}+\frac{E_+^{(0)}\ddot{\varphi}_+}{(\Omega-\dot{\varphi}_+)^2}\right), \tag{2.2}$$

$$2E_+^{(0)'}\varphi_+'+\varphi_+''E_+^{(0)}-\frac{1}{c^2}(2\dot{E}_+^{(0)}\dot{\varphi}_++\ddot{\varphi}_+E_+^{(0)})=\frac{(\omega_{L_e}^2)^{\cdot}}{c^2}\frac{E_+^{(0)}}{\Omega-\dot{\varphi}_+}+\frac{\Omega\omega_{L_e}^2}{c^2}\frac{1}{\Omega-\dot{\varphi}_+}\left(\frac{E_+^{(0)}}{\Omega-\dot{\varphi}_+}\right)^{\cdot}, \tag{2.3}$$

where $\omega_{L_e}^2=4\pi e^2n/m$, the prime indicates differentiation with respect to z and the dot differentiation with respect to the time. The equations for E_- have a similar form.

We shall assume that the following inequalities are satisfied:

$$|k_\pm^2|\gg|E_\pm^{(0)''}/E_\pm^{(0)}|,\quad \omega^2\gg|\ddot{E}_\pm^{(0)}/E_\pm^{(0)}|,\quad |\dot{n}/n|\gg|(\dot{E}_\pm^{(0)}/\omega\pm\Omega)/E_\pm^{(0)}\omega|, \tag{2.4}$$

where $k_\pm=\varphi_\pm'$, $\dot{\varphi}_+=\dot{\varphi}_-=-\omega$. Equation (2.2) and the analogous equation for E_- take the form

$$c^2k_\pm^2=\omega^2(1-\nu_\pm n(z,t)), \tag{2.5}$$

where $\nu_\pm=4\pi e^2/m\omega(\omega\pm\Omega)$.

When we assume that the velocity of the plasma motion is much smaller than the velocity of light, we get from Eq. (2.3) and the analogous equation for E_-[2)]

$$E_\pm^{(0)2}(z,t)k_\pm(z,t)=\text{const}=C_\pm^2. \tag{2.6}$$

Equations (2.5) and (2.6) enable us to write down an expression for the field of the wave in the form

$$E_\pm(z,t)=\frac{C_\pm}{\sqrt{k_\pm(z,t)}}\exp\{-i\omega t+ik_\pm(z,t)z\}. \tag{2.7}$$

It is clear that the amplitude of the wave increases with increasing plasma density (apart from the case of the extraordinary wave for $|\Omega|>\omega$ when the wave amplitude decreases with increasing density).

Using Eq. (2.7) we find E_x and E_y and from Eq. (1.15) we get[3)]

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial z}=-C_\pm^2\frac{c\nu_\pm}{16\pi m_i\omega}\frac{\partial}{\partial z}\frac{1}{\sqrt{1-n\nu_\pm}}. \tag{2.8}$$

[2)]The constants $C_\pm$ may in general depend on the time. However, in all problems considered by us there is a region where the properties of the medium are constant (either a vacuum or a plasma with a constant density). Since Eq. (2.6) is valid also in these regions, the constants $C_\pm$ are in those cases independent of the time.

[3)]To obtain Eq. (2.8) we assumed that the wave frequency is constant. Taking the time-dependence of the frequency into account leads to corrections of the order of the ratio of the plasma velocity to the light velocity, which we neglect.

Equations (2.8) and (1.17) together form a set of equations describing the dynamics of the plasma in the field of a short-wavelength high-frequency electromagnetic wave propagating along the magnetic field. It is clear that $n \neq |\nu_\pm|^{-1}$, for only in that case can the inequalities (2.4) be satisfied.

We can write Eq. (2.8) also in the form

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial z}=-\frac{1}{n}\frac{\partial p}{\partial z},$$

where the pressure is connected with particle number density through the relation

$$p=C_\pm^2\frac{c}{16\pi m_i\omega}\frac{2-n\nu_\pm}{\sqrt{1-n\nu_\pm}}. \quad (2.9)$$

Equation (2.9) is similar to an equation of state and in the low density limit when $n|\nu_\pm| \ll 1$ it transforms to

$$p=C_\pm^2\frac{c}{8\pi\omega m_i}\left[1+\frac{1}{8}(n\nu_\pm)^2\right]. \quad (2.10)$$

3. ELECTRODYNAMIC WAVES

As the simplest example we use Eqs. (1.17) and (2.8) to consider low-frequency, long-wavelength oscillations which can propagate in a plasma through which a high-frequency electromagnetic wave passes. We shall assume that $n(z, t) = n_0 + \delta n(z, t)$ where n_0 is the unperturbed particle number density in the plasma ($n_0 \gg \delta n$) and $v(z, t) = \delta v(z, t)$. The linearized set of equations for δn and δv has the form

$$\frac{\partial \delta n}{\partial t}+n_0\frac{\partial \delta v}{\partial z}=0, \quad (3.1)$$

$$\frac{\partial \delta v}{\partial t}+C_\pm^2\frac{c}{32\pi m_i\omega}\nu_\pm^2\frac{1}{(1-n_0\nu_\pm)^{3/2}}\frac{\partial \delta n}{\partial z}=0. \quad (3.2)$$

Assuming that $\delta n, \delta v \sim \exp(-i\omega_0 t + i\varkappa z)$ we get a dispersion law for the electrodynamic waves

$$\omega_0^2=\varkappa^2 s_0^2\equiv\varkappa^2 C_\pm^2\frac{c}{32\pi m_i\omega}\nu_\pm\frac{n_0\nu_\pm}{(1-n_0\nu_\pm)^{3/2}}. \quad (3.3)$$

It is convenient to express the constants $C_\pm$ in terms of the wave amplitude in the unperturbed plasma. From Eqs. (2.6) and (2.5) for the phase velocity of the wave s_0 we get

$$s_0=\frac{1}{2\sqrt{2}}\omega_{L_i}\left(\frac{eE_\pm^{(0)}}{m\omega|\omega\pm\Omega|}\right)\frac{1}{(1-n_0\nu_\pm)^{3/4}},\ \text{where}\ \omega_{L_i}=\sqrt{4\pi e^2 n_0/m_i}. \quad (3.4)$$

In the limit as $n_0|\nu_\pm| \ll 1$ and in the absence of a magnetic field we get from Eq. (3.4) the results from Volkov's paper[4] for low-frequency, long-wavelength waves.

4. SELF-SIMILAR MOTION

We consider the problem of the self-similar spreading of a rarefied plasma, through which an electromagnetic wave passes, in a vacuum. We shall assume that the quantities n and v depend only on the combination $\xi = z/t$ of the variables and we write Eqs. (1.17) and (2.8) in the form

$$(v-\xi)\frac{dn}{d\xi}+n\frac{dv}{d\xi}=0, \quad (4.1)$$

$$(v-\xi)\frac{dv}{d\xi}+C_\pm^2\frac{c}{32\pi\omega m_i}\nu_\pm^2\frac{1}{(1-n\nu_\pm)^{3/2}}\frac{dn}{d\xi}=0. \quad (4.2)$$

From this it follows that

$$v=\xi-s(n), \quad (4.3)$$

where

$$s^2(n)=C_\pm^2\frac{c}{32\pi\omega m_i}\nu_\pm^2\frac{n}{(1-n\nu_\pm)^{3/2}}. \quad (4.4)$$

The sign of s in (4.3) is chosen in such a way that the leading edge of the dilatation wave moves in the direction of negative z-values (see[5]).

It is convenient to express s in terms of its values in the region where the plasma is not perturbed (in our case for $z \to \infty$)

$$s^2(n)=s_0^2\frac{n}{n_0}\frac{(1-n_0\nu_\pm)^{3/2}}{(1-n\nu_\pm)^{3/2}}, \quad (4.5)$$

where

$$s^2(n_0)=s_0^2=C_\pm^2\frac{c}{32\pi m_i\omega}\nu_\pm^2\frac{n_0}{(1-n_0\nu_\pm)^{3/2}}. \quad (4.6)$$

As we should expect, Eq. (4.6) is the same as the square of the electrodynamic wave velocity (3.4). If we express the constants $C_\pm$ in terms of the wave amplitude in vacuo through Eq. (2.6) we get for s_0

$$s_0=\frac{1}{2\sqrt{2}}\left(\frac{eE_\pm^{(0)}}{m\omega}\right)\frac{\omega_{L_i}}{|\omega\pm\Omega|}\left[1-\frac{\omega_{L_e}^2}{\omega(\omega\pm\Omega)}\right]^{-3/4}. \quad (4.7)$$

Using Eq. (4.3) we get from Eq. (4.1)

$$v=\int\frac{s(n)}{n}dn.$$

Using Eq. (4.5) and the condition that $v = 0$ when $n = n_0$ we get when $0 < n\nu_\pm \le n_0\nu_\pm < 1$

$$v=2\sqrt{2}s_0\frac{(1-n_0\nu_\pm)^{3/4}}{(n_0\nu_\pm)^{1/2}}\left[F\left(\arccos\sqrt[4]{1-n\nu_\pm},\frac{1}{\sqrt{2}}\right)-F\left(\arccos\sqrt[4]{1-n_0\nu_\pm},\frac{1}{\sqrt{2}}\right)\right], \quad (4.8)$$

where $F(\alpha, \varphi)$ is an elliptical integral of the first kind. However, if $n\nu_\pm < 0$, which is possible only for the extraordinary wave (ν_-) in a strong magnetic field ($|\Omega| > \omega$), we have

$$v=2\sqrt{2}s_0\frac{(1-n_0|\nu_-|)^{3/4}}{(n_0|\nu_-|)^{1/2}}\left[F\left(\arccos(1+n|\nu_-|)^{-1/4},\frac{1}{\sqrt{2}}\right)-F\left(\arccos(1+n_0|\nu_-|)^{-1/4},\frac{1}{\sqrt{2}}\right)\right]. \quad (4.9)$$

Equations (4.8), (4.9), and (4.3) enable us to obtain a connection between the particle number density and the variable ξ

$$\xi=s_0\frac{(1+n_0|\nu_\pm|)^{3/4}}{(n_0\nu_\pm)^{1/2}}\left\{\frac{\sqrt{n\nu_\pm}}{(1-n\nu_\pm)^{3/4}}+2\sqrt{2}F\left(\arccos\sqrt[4]{1-n\nu_\pm},\frac{1}{\sqrt{2}}\right)-2\sqrt{2}F\left(\arccos\sqrt[4]{1-n_0\nu_\pm},\frac{1}{\sqrt{2}}\right)\right\},\quad 0<n_0\nu_\pm<1; \quad (4.10)$$

$$\xi=s_0\frac{(1+n_0|\nu_-|)^{3/4}}{\sqrt{n_0|\nu_-|}}\left\{\frac{\sqrt{n|\nu_-|}}{(1+n|\nu_-|)^{3/4}}+2\sqrt{2}F\left(\arccos(1+n|\nu_-|)^{-1/4},\frac{1}{\sqrt{2}}\right)-2\sqrt{2}F\left(\arccos(1+n_0|\nu_-|)^{-1/4},\frac{1}{\sqrt{2}}\right)\right\},\quad n\nu_-<0. \quad (4.11)$$

Determining $n(\xi)$ from Eqs. (4.10) and (4.11) we can use Eqs. (4.8) and (4.9) to find the function $v(\xi)$.

The simplest expressions for $n(\xi)$ and $v(\xi)$ occur in the limit of a rarefied plasma $n_0|\nu_\pm| \ll 1$. We get from Eqs. (4.8) to (4.11)

$$\frac{n}{n_0} = \frac{1}{9}\left(\frac{\xi}{s_0} + 2\right)^2, \tag{4.12}$$

$$v = {}^2/_3(\xi - s_0). \tag{4.13}$$

We are interested in the value of ξ for which the density vanishes, since it follows from Eq. (4.3) that this value of ξ is the same as the velocity of the leading edge, which has the maximum velocity. We get from Eqs. (4.10) and (4.11) when n = 0

$$v_{max} = \xi_0 = -2\sqrt{2}\, s_0 \begin{cases} \dfrac{(1 - n_0\nu_\pm)^{3/4}}{\sqrt{n_0\nu_\pm}} F\left(\arccos(1 - n_0\nu_\pm)^{1/4}, \dfrac{1}{\sqrt{2}}\right), & (4.14) \\ \qquad 0 < n_0\nu_\pm < 1, \\ \dfrac{(1 + n_0|\nu_-|)^{3/4}}{\sqrt{n_0|\nu_-|}} F\left(\arccos(1 + n_0|\nu_-|)^{-1/4}, \dfrac{1}{\sqrt{2}}\right), \\ \qquad n\nu_- < 0. & (4.15) \end{cases}$$

If $n_0|\nu_\pm| \ll 1$ we have

$$v_{max} = -2s_0 = -\frac{1}{\sqrt{2}}\left(\frac{eE_\pm^{(0)}}{m\omega}\right)\frac{\omega_{Li}}{|\omega \pm \Omega|}. \tag{4.16}$$

It follows in the general case from Eqs. (4.14) and (4.15) that v_{max} may exceed the value (4.16) by approximately a factor two (the maximum value of the elliptical integral is 1.85).

We must note that the magnetic field affects the velocity profile and the density in the dilatation wave, but does not affect the character of the self-similar motion itself. Independent of the polarization of the incoming wave there is a spreading of the plasma and not a compression, both in a strong magnetic field ($|\Omega| \gg \omega$) and in a weak magnetic field ($|\Omega| \ll \omega$).

CONCLUSIONS

Allowance for the pressure of a high-frequency electromagnetic wave which passes through a plasma leads thus to a number of specific effects. In the short wave case considered by us a new branch of oscillations—electrodynamic waves—turns out to be possible. Moreover, in the same case, a stationary dilatation wave appears when an electromagnetic wave passes through the boundary of a transparent plasma. The longitudinal magnetic field then changes the wave profile and the velocity with which the discontinuities move, but it does not affect the character of the self-similar spreading of the plasma itself.

In the present paper we have assumed the plasma to be cold. It is clear that taking into account in the equations of motion of the plasma the thermal pressure does not present difficulties of principle. We indicate, however, the conditions when our considerations are valid and the thermal pressure can be neglected. In the case, considered by us, of short wavelengths and under the condition $n|\nu_\pm| \ll 1$, we get by equating the thermal pressure and the pressure of the electromagnetic field the condition for the validity of our considerations

$$\frac{1}{4}\frac{eE_\pm^{(0)}}{m\omega}\frac{\omega_{Le}}{|\omega \pm \Omega|} > v_{T_e},$$

where v_{T_e} is the electron thermal velocity.

In conclusion I express my gratitude to V. P. Silin for his interest in the present paper.

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