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STATISTICAL THEORY OF INELASTIC PROCESS IN A PLASMA II. PROCESSES DUE TO A TRANSVERSE ELECTROMAGNETIC FIELD

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A set of kinetic equations for the electron, ion, and atom distribution functions of a partially ionized plasma is presented, in which all processes due to interaction between the particles and a transverse electromagnetic field are taken into account. The derivation of the kinetic equations is similar to that described in [1] for a Coulomb plasma. By employing the second-moment approximation for the initial microscopic equations it has been possible to obtain kinetic equations in which polarization of the plasma is taken into account. As a result, the kinetic equations do not only take into account the usual processes of photoionization, photorecombination, emission, absorption etc., but also take into account all so-called superluminal ("anomalous") effects. These include, besides the Cerenkov effect, the anomalous Doppler effect and anomalous bremsstrahlung considered previously by a number of authors in the prescribed-medium approximation. Effects of spontaneous and stimulated ionization of atoms, involving emission of quanta and of stimulated recombination involving absorption etc., are also considered. A classification is presented for processes taken into account by the collision integral. Equations for the population of atoms for a Maxwellian velocity distribution of plasma particles and a set of equations for the concentration are considered. Expressions are obtained for the rates of the ordinary and "anomalous" processes. In particular, expressions are obtained for the photoionization and photorecombination coefficients and for excitation and absorption in a discrete spectrum (Einstein coefficients), by taking into account thermal motion and polarization of the medium. Expressions for the field spectral function and the dielectric constant of a partially ionized plasma are considered. Recombination instability arising in a plasma in the presence of an excess of nonequilibrium free charged particles is studied.

INTRODUCTION

THE present investigation is a continuation of earlier work [1] (henceforth cited as I) on the statistical theory of inelastic processes in a Coulomb plasma. It is devoted to a statistical description of the processes occurring in a non-equilibrium partially-ionized plasma as a result of interaction between the atoms and the free charged particles via a transverse electromagnetic field.

The paper consists of four sections. In Sec. 1 we consider the system of kinetic equations for the distribution functions of the electrons, ions, and atoms. As in I, the kinetic equations are obtained from a system of microscopic equations for four density operator matrices, describing the free and bound states of the plasma particles. In Sec. 2 we investigate the polarization properties of a partially ionized plasma. We consider, in particular, the conditions for the occurrence of recombination instability in the plasma.

In Secs. 3 and 4 we investigate the kinetic equations that describe the distribution of the atoms among the levels under the condition of a Maxwellian particle momentum distribution. We obtain expressions for the coefficients of photoionization and photorecombination of a non-equilibrium plasma.

It follows from the kinetic equations obtained in this paper that besides "normal" processes there can exist in a plasma "anomalous" processes, namely processes of spontaneous and induced ionization with emission of photons (or plasmons). For the transitions in the discrete spectrum, we obtain expressions for the Einstein coefficients with allowance for the thermal motion and the polarization of the medium. We describe the "anomalous" effects that occur in the case of transitions in the discrete spectrum as a result of the energy of thermal motion of the atoms, namely the spontaneous transitions from the lower levels to the higher ones with emission of photons, and the transitions to lower levels with absorption of quanta.

Some of the "anomalous" effects indicated here, particularly the anomalous Doppler effect, were considered earlier by I. M. Frank, V. L. Ginzburg, V. M. Faĭn, and V. Ya. Éĭdman. The corresponding literature and the results are given in Ginzburg's review [2]. It follows from our work that all the "anomalous" effects are automatically included in the kinetic equations if the latter are derived with allowance for the polarization of the medium. The probabilities of all the processes are then determined.

Non-equilibrium processes in a partially ionized plasma are usually investigated either on the basis of the balance equations or on the basis of the generalized Boltzmann kinetic equations [3-8]. The derivation of the kinetic equations for a partially ionized plasma from the microscopic equations is of interest from several points of view. It is possible to obtain in this way the most general expression for the collision integrals with allowance for the effects of polarization and all types of inelastic processes, both in Boltzmann form and in the more general electrodynamic form. Such an approach makes it possible to go also much farther, namely to obtain more general kinetic equations in the presence of average fields and spatial inhomogeneity, and by the same token to take consistent account of the influence of these factors on the form of the distribution functions.

We confine ourselves in this paper to an approximation in which the average fields and the spatial inhomogeneities do not exert a noticeable influence on the form of the collision integrals, i.e., we consider the so-called approximation of a spatially homogeneous plasma. In addition, we consider here only a system of equations for the distribution functions of the electrons, ions, and atoms. Under certain conditions it should be supplemented by the equation for the photons (or plasmons). This question will be considered separately.

1. SYSTEM OF KINETIC EQUATIONS. CLASSIFICATION OF ELEMENTARY PROCESSES

Just as in I, we consider here a partially ionized plasma consisting of three components: electrons, ions, and atoms. We start out again with the equation for the operator density matrix ρ_{ab} of pairs of charged particles, both bound (atoms) and free. The equation for ρ_{ab} differs from I(1.2) only in that the pair interaction U_{ab} is realized via a transverse field, i.e., now

$$\hat{U}_{ab}(\mathbf{R},\mathbf{r})=\frac{ie_a\hbar}{m_a c}\mathbf{A}\left(\mathbf{R}+\frac{m_b}{m_a+m_b}\mathbf{r}\right)\left(\frac{m_a}{m_a+m_b}\frac{\partial}{\partial\mathbf{R}}+\frac{\partial}{\partial\mathbf{r}}\right)+\frac{ie_b\hbar}{m_b c}\mathbf{A}\left(\mathbf{R}-\frac{m_a}{m_a+m_b}\mathbf{r}\right)\left(\frac{m_b}{m_a+m_b}\frac{\partial}{\partial\mathbf{R}}-\frac{\partial}{\partial\mathbf{r}}\right). \quad (1.1)$$

The system of equations for the density matrix $\rho_{\alpha\beta}(\mathbf{P}', \mathbf{P}'', t)$ (α = n, p'; β = m, p'') coincides with I(1.8), the only difference being that now the matrix element of the function (1.1) is given by the expression

$$U_{\alpha\beta}(\mathbf{P}',\mathbf{P}'',\omega)=-\frac{\hbar^3}{V}\int \mathbf{P}_{\alpha\beta}\mathbf{A}(\omega,\mathbf{k})\delta(\hbar\mathbf{k}-(\mathbf{P}'-\mathbf{P}''))\,d\mathbf{k}, \quad (1.2)$$

and

$$\mathbf{P}_{\alpha\beta}=\int\Big[\frac{e_a}{c}\exp\Big\{i\frac{m_b}{m_a+m_b}\mathbf{kr}\Big\}\Big(\frac{\mathbf{P}'+\mathbf{P}''}{2(m_a+m_b)}+\frac{\hat{\mathbf{p}}_\alpha^*+\hat{\mathbf{p}}_\beta}{2m_a}\Big)+\frac{e_b}{c}\exp\Big\{-i\frac{m_a}{m_a+m_b}\mathbf{kr}\Big\}\Big(\frac{\mathbf{P}'+\mathbf{P}''}{2(m_a+m_b)}-\frac{\hat{\mathbf{p}}_\alpha^*+\hat{\mathbf{p}}_\beta}{2m_b}\Big)\Big]\psi_\alpha^*(\mathbf{r})\psi_\beta(\mathbf{r})\,d\mathbf{r}. \quad (1.3)$$

The operators $\hat{\mathbf{p}}_\alpha$ and $\hat{\mathbf{p}}_\beta$ act respectively on ψ_α and ψ_b, which are the eigenfunctions of the isolated atoms.

For free charged particles we get from (1.3)

$$|\mathbf{P}_{\mathbf{p}'\mathbf{p}''}|_{ij}^2=\frac{(2\pi\hbar)^3}{V}\Big[\frac{e_a^2}{c^2}\delta(\mathbf{p}_b'-\mathbf{p}_b'')\Big(\frac{\mathbf{v}_a'+\mathbf{v}_a''}{2}\Big)_{ij}^2+\frac{e_b^2}{c^2}\delta(\mathbf{p}_a'-\mathbf{p}_a'')\Big(\frac{\mathbf{v}_b'+\mathbf{v}_b''}{2}\Big)_{ij}^2\Big] \quad (1.4)$$

(V—volume of the system, $v_{a,t}$—particle velocity). For the discrete spectrum in the dipole approximation we get from (1.3)

$$|\mathbf{P}_{nm}|_{ij}^2=\frac{e^2}{c^2\mu^2}|\mathbf{p}_{nm}|_{ij}^2. \quad (1.5)$$

Similar expressions are obtained in the dipole approximation for transitions from the discrete spectrum to the continuous one, and vice-versa.

The kinetic equations for the distribution functions of the electrons, ions, and atoms will be represented in the form

$$\frac{\partial f_a(\mathbf{p}_a',t)}{\partial t}=(J_a)_1+(J_a)_2,\qquad \frac{\partial f_n(\mathbf{P}',t)}{\partial t}=(J_n)_1+(J_n)_2; \quad (1.6)$$

$f_n(\mathbf{P}', t)$ is the distribution function of the atoms in the state n. The normalization of the distribution functions is the same as in I. The first parts of the collision integrals in (1.6) have the properties

$$\int (J_a)_1\,d\mathbf{p}_a'=0,\qquad \sum_n\int (J_n)_1\,d\mathbf{P}'=0. \quad (1.7)$$

This means that they make no contribution to the continuity equations.

The expressions for the collision integrals are obtained in the same approximation as in I. We write them out in an electrodynamic form, which is more convenient for further use

$$(J_a)_1=\frac{e_a^2}{2(2\pi)^3\hbar}\int d\omega\,d\mathbf{k}\,d\mathbf{p}_a''\frac{[\mathbf{kv}']^2}{k^2\omega^2}\delta(\hbar\mathbf{k}-(\mathbf{p}_a'-\mathbf{p}_a''))\times\delta(\hbar\omega-(E_{\mathbf{p}_a'}-E_{\mathbf{p}_a''}))\{(\delta\mathbf{E}\,\delta\mathbf{E})_{\omega\mathbf{k}}^{\perp}(f(\mathbf{p}_a'')-f(\mathbf{p}_a'))-R(\omega,\mathbf{k})(f(\mathbf{p}_a'')+f(\mathbf{p}_a'))\}, \quad (1.8)^*$$

$$(J_a)_2=\frac{1}{2(2\pi)^3\hbar}\frac{V}{(2\pi\hbar)^3}\sum_m\int d\mathbf{p}_b'\,d\omega\,d\mathbf{k}\,d\mathbf{P}''\frac{c^2|\mathbf{P}_{\mathbf{p}'m}^{\perp}|^2}{\omega^2}\times\delta(\hbar\mathbf{k}-(\mathbf{p}_a'+\mathbf{p}_b'-\mathbf{P}''))\,\delta(\hbar\omega-(E_{\mathbf{p}_a'}+E_{\mathbf{p}_b'}-E_m-E_{\mathbf{P}''}))\times\{(\delta\mathbf{E}\delta\mathbf{E})_{\omega\mathbf{k}}^{\perp}[f_m(\mathbf{P}'')-Nf(\mathbf{p}_a')f(\mathbf{p}_b')]-R(\omega,\mathbf{k})[f_m(\mathbf{P}'')+Nf(\mathbf{p}_a')f(\mathbf{p}_b')]\}, \quad (1.9)$$

$$(J_n)_1=\frac{1}{2(2\pi)^3\hbar}\sum_m\int d\omega\,d\mathbf{k}\,d\mathbf{P}''\frac{c^2|\mathbf{P}_{nm}^{\perp}|^2}{\omega^2}\delta(\hbar\mathbf{k}-(\mathbf{P}'-\mathbf{P}''))\times\delta(\hbar\omega-(E_n+E_{\mathbf{P}'}-E_m-E_{\mathbf{P}''}))\{(\delta\mathbf{E}\delta\mathbf{E})_{\omega\mathbf{k}}^{\perp}[f_m(\mathbf{P}'')-f_n(\mathbf{P}')]-R(\omega,\mathbf{k})[f_m(\mathbf{P}'')+f_n(\mathbf{P}')]\}, \quad (1.10)$$

$$(J_n)_2=\frac{1}{2(2\pi)^3\hbar}\frac{V}{(2\pi\hbar)^3}\int d\omega\,d\mathbf{k}\,d\mathbf{p}_a''\,d\mathbf{p}_b''\frac{c^2}{\omega^2}|\mathbf{P}_{n\mathbf{p}''}^{\perp}|^2\times\delta(\hbar\omega-(E_n+E_{\mathbf{P}'}-E_{\mathbf{p}_a''}-E_{\mathbf{p}_b''}))\,\delta(\hbar\mathbf{k}-(\mathbf{P}'-\mathbf{p}_a''-\mathbf{p}_b''))\{(\delta\mathbf{E}\delta\mathbf{E})_{\omega\mathbf{k}}^{\perp}\times[Nf(\mathbf{p}_a'')f(\mathbf{p}_b'')-f_n(\mathbf{P}')]-R(\omega,\mathbf{k})[Nf(\mathbf{p}_a'')f(\mathbf{p}_b'')+f_n(\mathbf{P}')]\}. \quad (1.11)$$

we have used here the notation

$$R(\omega,\mathbf{k})=\frac{8\pi\hbar\varepsilon''^{\perp}(\omega,\mathbf{k})\,\omega^4}{|\omega^2\varepsilon^{\perp}(\omega,\mathbf{k})-c^2k^2|^2}, \quad (1.12)$$

$$E_{\mathbf{p}_a}=\frac{p_a^2}{2m_a},\quad E_{\mathbf{P}}=\frac{P^2}{2M},\quad \mathbf{p}=\frac{m_b\mathbf{p}_a-m_a\mathbf{p}_b}{m_a+m_b},\quad \mathbf{P}=\mathbf{p}_a+\mathbf{p}_b, \quad (1.13)$$

$\mathbf{P}^{\perp}$ is the transverse component of the vector (1.3), $(\delta\mathbf{E}\,\delta\mathbf{E})_{\omega\mathbf{k}}^{\perp}$ is the spectral function of the transverse electromagnetic field, $\epsilon^{\perp}(\omega, \mathbf{k})$ is the transverse dielectric constant, and $\epsilon^{\shortparallel}$ is the imaginary part of the dielectric constant. The expressions for these functions will be considered in the next section.

The collision integrals (1.8)–(1.11) have the usual properties. They ensure conservation of the total number of particles, the total angular momentum, and total energy of the plasma. The collision integrals vanish if the distribution over the momenta and the energy levels are Maxwell-Boltzmann distributions, if ionization equilibrium obtains, and if the spectral function of the field is determined by the well-known equilibrium expression (see, for example, the book of Silin and Rukhadze [9], p. 69, and the book by the author [10], p. 183).

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*$[\mathbf{kv}'] \equiv \mathbf{h}\times\mathbf{v}'$.

Just as in the case of a Coulomb plasma, each of the functions $(\delta E\,\delta E)^{\perp}_{\omega k}$ and $\epsilon^{\perp}(\omega, k)$ consists of four parts, therefore on going over to the Boltzmann (collision) form we obtain eight collision integrals in the right side of each of the equations of (1.7).

The "collision" processes take place when ω and k are not connected by the equation $\omega^2\epsilon^{\perp} - c^2k^2 = 0$, i.e., outside the transparency region. Inside the transparency region, the spectral function cannot be expressed in the general case in terms of the distribution functions of free particles and it is impossible to go over to the "collision" form of the collision integrals. In this case the change of the state of the atoms and of the free particles is due to the interaction with the photons. We present here a classification of the corresponding processes, for when polarization is taken into account processes which are usually not considered become possible.

We start with the integral (1.8). This integral describes processes for which the conservation laws are satisfied:

$$\pm\hbar\omega = \frac{p_a'^2}{2m_a} - \frac{p_a''^2}{2m_a}, \quad \pm\hbar\mathbf{k} = \mathbf{p}_a' - \mathbf{p}_a'', \quad \omega > 0. \tag{1.14}$$

In the transparency region, neglecting the spatial dispersion, we have $|\mathbf{k}| = \omega\sqrt{\epsilon'}/c$. When $\sqrt{\epsilon'} > 1$, the conservation laws (1.14) admit of change in the state of the free particles as a result of emission and absorption of quanta (Cerenkov radiation and absorption). In order to take into account the bremsstrahlung and absorption processes it is necessary to use in the derivation of the expression for the collision integral $(J_a)_1$ a more accurate expression for the matrix element $|\mathbf{P}_{p'p''}|^2_{ij}$ than expression (1.4).

The collision integral (1.9) takes into account processes that are allowed by the conservation laws

$$\pm\hbar\omega = \frac{p_a'^2}{2m_a} + \frac{p_b'^2}{2m_b} - E_m - E_{\mathbf{P}''},$$

$$\pm\hbar\mathbf{k} = \mathbf{p}_a' + \mathbf{p}_b' - \mathbf{P}'', \quad \omega > 0. \tag{1.15}$$

From (1.15) with the plus sign we get the processes of photoionization and photorecombination (recombination accompanied by radiation). Let us consider the processes that are possible with the minus sign. Using the variables (1.13) and the condition $|\mathbf{k}| = \omega\sqrt{\epsilon'}/c$, we find from (1.15) that such processes are possible if

$$V''\cos\theta > c/\sqrt{\varepsilon'}. \tag{1.16}$$

This is either decay of the atom, accompanied by radiation, or a recombination accompanied by photon absorption. The decay of the atom is at the expense of the kinetic energy of the atom. When $\sqrt{\epsilon'} \gtrsim 1$, it is necessary to use the corresponding relativistic formulas.

The collision integral (1.10) describes processes that are admitted by the conservation laws

$$\pm\hbar\omega = (E_n + E_{\mathbf{P}'} - E_m - E_{\mathbf{P}''}),$$

$$\pm\hbar\mathbf{k} = \mathbf{P}' - \mathbf{P}'', \quad \omega > 0, \quad E_n > E_m. \tag{1.17}$$

In the case of a plus sign, the conservation laws allow the excitation of the atom upon absorption of a photon and a transition to the lower states with emission of a photon. In the case of a minus sign, we get from (1.17) the possibility of processes of excitation with emission of photons (anomalous Doppler effect[2]) and of transitions to the lower states with absorption. These processes are also possible only under condition (1.16).

Expressions for the probabilities of the processes (1.15) and (1.16) will be obtained in Sec. 3.

The "collision" processes due to the interaction of particles via a transverse field for a nonrelativistic plasma are much less probable than the corresponding processes for a Coulomb plasma. Because of this, we shall consider below only processes in which the change of the state of the particles is accompanied by emission or absorption of quanta of the electromagnetic field. The corresponding processes with participation of plasmons can be considered in similar fashion on the basis of the kinetic equations of I.

The collision integrals (1.8)–(1.11) can be simplified by describing the motion of the free particles and the centers of the atoms classically. For example, for an isotropic plasma in the classical approximation the expression (1.8) takes the form

$$(J_a)_1 = \frac{\partial}{\partial p_{ai}'} D_{ij}{}^a \frac{\partial f_a}{\partial p_{aj}'} + \frac{\partial}{\partial p_{ai}'} A_i{}^a f_a. \tag{1.18}$$

the coefficients of "diffusion" and "friction" are determined by the expressions

$$D_{ij}{}^a = \frac{e_a^2}{16\pi^3}\int d\omega\, d\mathbf{k}\,(\delta E_i \delta E_j)_{\omega\mathbf{k}}\,\delta(\omega - \mathbf{k}\mathbf{v}_a'),$$

$$A_i{}^a = \frac{e_a^2}{2\pi^2}\int d\omega\, d\mathbf{k}\,\frac{\omega^3\varepsilon''^{\perp}[\mathbf{k}[\mathbf{v}_a'\mathbf{k}]]_i}{|\omega^2\varepsilon^{\perp} - c^2k^2|^2 k^2}\,\delta(\omega - \mathbf{k}\mathbf{v}_a').$$

Equation (1.8) coincides in form with the corresponding expression for a fully ionized plasma (see the appendix to the article of Silin[11] and formulas (13.42, 46) of the book[10]).

2. SPECTRAL FUNCTION OF THE FIELD AND DIELECTRIC CONSTANT. IONIZATION AND RECOMBINATION INSTABILITIES OF A PLASMA

The spectral function of the transverse field can be represented in the form of a sum of four parts:

$$(\delta\mathbf{E}\delta\mathbf{E})_{\omega\mathbf{k}}{}^{\perp} = (\delta\mathbf{E}\delta\mathbf{E})_{\omega\mathbf{k}}{}^{ff} + (\quad)^{fb} + (\quad)^{bf} + (\quad)^{bb}. \tag{2.1}$$

The indices f and b denote the free and bound states. The first term in (2.1) is of the form

$$(\delta\mathbf{E}\delta\mathbf{E})^{ff}_{\omega\mathbf{k}} = \frac{V}{(2\pi\hbar)^3}\sum_a \frac{(4\pi)^2\pi n e^2\omega^2}{k^2}$$
$$\times \int d\mathbf{p}\,\frac{[\mathbf{k}\mathbf{v}]^2\,\delta(\omega - \mathbf{k}\mathbf{v})[f_a(\mathbf{p} + {}^1/_2\hbar\mathbf{k}) + f_a(\mathbf{p} - {}^1/_2\hbar\mathbf{k})]}{|\omega^2\varepsilon^{\perp} - c^2k^2|^2}. \tag{2.2}$$

when $\hbar = 0$, Eq. (2.2) coincides in form with the well-known expression for the spectral function of the field of a fully ionized plasma (see, for example,[10], p. 181). The second and third terms in (2.1) are given by the expression

$$(\delta\mathbf{E}\delta\mathbf{E})^{bf}_{\omega\mathbf{k}} = (\delta\mathbf{E}\delta\mathbf{E})^{fb}_{-\omega,-\mathbf{k}} = 4\pi^2\pi n\hbar\,\frac{V^2}{(2\pi\hbar)^6}$$
$$\times \sum_{m_1}\int d\mathbf{p}_{1a}'\,d\mathbf{p}_{1b}'\,d\mathbf{P}_1''\,\frac{c^2\omega^2\,|\mathbf{P}^{\perp}_{\mathbf{p}_1' m_1}|^2}{|\omega^2\varepsilon^{\perp} - c^2k^2|^2}\,\delta(\hbar\mathbf{k} - (\mathbf{p}_{1a}' + \mathbf{p}_{1b}' - \mathbf{P}_1''))$$
$$\times\,\delta(\hbar\omega - (E_{\mathbf{p}_{1a}'} + E_{\mathbf{p}_{1b}'} - E_{m_1} - E_{\mathbf{P}_1''}))\,[Nf(\mathbf{p}_{1a}')f(\mathbf{p}_{1b}') + f_{m_1}(\mathbf{P}_1'')]. \tag{2.3}$$

The last function in (2.1) is given by

$$(\delta E\delta E)^{bb}_{\omega k} = (4\pi)^2 \pi n\hbar \frac{V}{(2\pi\hbar)^3} \sum_{n_1 m_1} \int dP_1' dP_1'' [f_{n_1}(P_1') + f_{m_1}(P_1'')]$$
$$\times \frac{c^2\omega^2 |P^{\perp}_{n_1 m_1}|^2}{|\omega^2\varepsilon^{\perp} - c^2k^2|^2} \delta(\hbar k - (P_1' - P_1''))\, \delta(\hbar\omega - (E_{n_1} + E_{P_1'} - E_{m_1} - E_{P_1''})). \quad (2.4)$$

The subdivision of the spectral function into four parts in (2.1) is to some degree arbitrary, since each part contains the total dielectric constant, which takes into account all the polarization processes, viz., those due to the motion of the free particles transitions in the discrete spectrum, and transitions from the bound state into free states and vice-versa.

The dielectric constant can in turn be represented in the form

$$\varepsilon(\omega, k) = 1 + 4\pi(\alpha_{ff} + \alpha_{bf} + \alpha_{fb} + \alpha_{bb}). \quad (2.5)$$

Here

$$\alpha_{ff} = -\sum_a \frac{e_a^2 n_a}{m_a\omega^2}$$
$$+ \frac{V}{(2\pi\hbar)^3} \sum_a \frac{e_a^2 n_a}{2\hbar\omega^2 k^2} \int \frac{[kv]^2 (f_a(P + {}^1\!/_2\hbar k) - f_a(P - {}^1\!/_2\hbar k))}{\omega + i\Delta - kv} dp. \quad (2.6)$$

This expression differs from the corresponding expression for a fully ionized plasma (see formula (26.14) of the book of Silin and Rukhadze [9]) only in that the concentration of the charged particles n_a are variable in this case.

The expression for α_{bb} is of the form

$$\alpha_{bb} = -\sum_a \frac{e_a^2 n_{ab}}{m_a\omega^2} + \frac{n}{2} \frac{V}{(2\pi\hbar)^3} \sum_{n_1 m_1} \int dP_1' dP_1'' \frac{c^2}{\omega^2} |P^{\perp}_{n_1 m_1}|^2$$
$$\times \frac{\delta(\hbar k - (P_1' - P_1''))(f_{n_1}(P_1') - f_{m_1}(P_1''))}{\hbar(\omega + i\Delta) - (E_{n_1} + E_{P_1'} - E_{m_1} - E_{P_1''})}. \quad (2.7)$$

This expression goes over in the dipole approximation into the well-known expression when the degree of ionization is zero.

The polarizabilities α_{bf} and α_{fb}, which are determined by the transitions from the discrete spectrum into the continuous spectrum and vice-versa, are equal to

$$\alpha_{bf}(\omega, k) = \alpha_{fb}^*(-\omega, -k) = -\frac{n}{2} \frac{V^2}{(2\pi\hbar)^6} \sum_m \int dp_{1a} dp_{1b} dP_1 |P^{\perp}_{p_1, m_1}|^2 \frac{c^2}{\omega^2}$$
$$\times \frac{\delta(\hbar k - (p_{1a} + p_{1b} - P_1)) [Nf(p_{1a}) f(p_{1b}) - f_{m_1}(P_1)]}{\hbar(\omega + i\Delta) - (E_{p_{1a}} + E_{p_{1b}} - E_{m_1} - E_{P_1})}. \quad (2.8)$$

The imaginary parts of the polarizabilities determine the rates of absorption or emission of the electromagnetic field in a partially ionized plasma. The expression for α''_{ff} determines the decrement (or increment) of the Landau damping. It differs from the usual expression only in that now the densities of the charged particles are variable. Let us consider in greater detail the expression for α''_{bf}.

From (2.8) for $\omega > 0$ we get in the dipole approximation and as $M \to \infty$

$$\alpha_{bf}'' = -\frac{\pi V e^2}{2(2\pi\hbar)^3} \sum_m \int dp |r^{\perp}_{pm}|^2 \delta(\hbar\omega - |E_m| - E_p)$$
$$\times [Nf(p) n_b - n_{ab}\rho_m]. \quad (2.9)$$

Here ρ_m is the distribution over the levels, $\Sigma\rho_m = 1$. We assume that the distribution of the levels is in equilibrium, and the electron velocity distribution over the velocities is of the form

$$\frac{nV}{(2\pi\hbar)^3} f(p) = \frac{n_a}{(2\pi m\varkappa T)^{3/2}} \exp\left\{-\frac{p^2}{2m\varkappa T}\right\}$$
$$+ \frac{n_1}{(2\pi m\varkappa T_1)^{3/2}} \exp\left\{-\frac{(p - mu)^2}{2m\varkappa T_1}\right\} \equiv n_a f_0 + n_1 f_1, \quad (2.10)$$

i.e., a beam of electrons has been introduced into a plasma with a Maxwellian distribution. From (2.9) and (2.10) it follows that

$$\alpha_{bf}'' = \frac{e^2 V}{2}\left[\frac{n_{ab}}{Z}\left(\frac{m\varkappa T}{2\pi\hbar^2}\right)^{3/2} e^{\hbar\omega/\varkappa T} - n_a n_b\right] \sum_m \int dp |r^{\perp}_{pm}|^2 f_0$$
$$\times \delta(\hbar\omega - |E_m| - E_p) - \frac{e^2 V}{2} n_1 n_b \sum_m \int dp |r^{\perp}_{pm}|^2 \delta(\hbar\omega - |E_m| - E_p) f_1. \quad (2.11)$$

The first term in this expression determines α''_{bf} for an equilibrium distribution with respect to the velocities and the levels. When $\hbar\omega > 0$, the photoionization process is more effective than the photorecombination process. As a result, absorption takes place. The second term is non-equilibrium. It decreases the absorption. Let us determine the conditions under which the absorption becomes negative.

We consider by way of an example transitions from the ground level only. We represent the expression for the function α''_{bf} in the form

$$\alpha_{bf}'' = (\alpha_{bf}'')_0 + (\alpha_{bf}'')_1. \quad (2.12)$$

The first part is determined by the equilibrium part of the distribution function, and the second is determined by the beam, i.e., by the second term of (2.11). We integrate with respect to p. If the Saha formula holds, then the following expression is true:

$$\frac{|\alpha_{bf}''|_1}{(\alpha_{bf}'')_0} = \frac{n_1}{n_a}\sqrt{\frac{T}{T_1}} \frac{\varkappa T}{\sqrt{2mE}u}\left[\exp\left\{-\frac{(\sqrt{2mE} - mu)^2}{2m\varkappa T_1}\right\}\right.$$
$$\left. - \exp\left\{-\frac{(\sqrt{2mE} + mu)^2}{2m\varkappa T_1}\right\}\right] e^{-I/\varkappa T} \cdot (1 - e^{-\hbar\omega/\varkappa T})^{-1}. \quad (2.13)$$

If this ratio is larger than unity, i.e., $\alpha''_{bf} < 0$, then recombination instability occurs as a result of the excess recombination of the beam electrons at the ground level. It follows from (2.13) that such an instability is possible for a weakly ionized gas with sufficiently low ionization potential I. We can similarly obtain from (2.11) the conditions for the recombination instability at higher levels.

An expression for the matrix element in (2.11) for transitions from the ground level can be obtained by using the expression for the effective cross section of ionization from the ground level (see the book of Landau and Lifshitz [12], p. 667). It is of the form

$$|r^{\perp}_{0p}|^2 = \frac{2}{3} \frac{2\pi^2 2^8 a^5}{Vp^*} \exp\left(-\frac{4}{p^*}\operatorname{arctg} p^*\right)$$
$$\times (1 + p^*)^{-5}\left[1 - \exp\left(-\frac{2\pi}{p^*}\right)\right]^{-1}, \quad p^* = \frac{pa}{\hbar}. \quad (2.14)$$

When $p^* \gg 1$ (Born approximation) we have $|r^{\perp}_{0p}|^2 \approx 2^9\pi a^5/3Vp^{*4}$.

We now consider an expression for the function α''_{fb}. In the dipole approximation we get from (2.8)

$$\alpha_{fb}'' = \frac{\pi n}{2}\frac{V^2}{(2\pi\hbar)^6}\sum_m\int dp_a\,dp_b\,d\mathbf{P}\,|\mathbf{r}_{pm}^{\perp}|^2\,\delta(\hbar\mathbf{k}-(\mathbf{P}-\mathbf{p}_a-\mathbf{p}_b))$$
$$\times\,\delta(\hbar\omega-(E_m+E_\mathbf{P}-E_{\mathbf{p}_a}-E_{\mathbf{p}_b}))\,[Nf(\mathbf{p}_a)f(\mathbf{p}_b)-f_m(\mathbf{P})],\quad \omega>0. \tag{2.15}$$

This expression differs from zero if the conservation laws (1.15) (with a minus sign) are satisfied.

When $\omega > 0$ the polarizability α_{fb}'' is due to two processes: spontaneous decay (ionization) with emission, and recombination with absorption. In the equilibrium state, the recombination process prevails. In order to cause instability as a result of radiation in spontaneous decay of the atoms, it is necessary to disturb the equilibrium by producing an excess of neutral particles. Naturally, in order to produce instability as a result of any nonequilibrium process it is necessary to reverse the sign of the imaginary part of the total polarizability.

We note that, unlike (2.9), it is impossible to go over to the approximation of immobile atoms in expression (2.15), since the processes causing the polarization are impossible in this approximation.

Let us consider, finally, the expression for the function α_{bb}''. From (2.7) we obtain in the dipole approximation

$$\alpha_{bb}'' = \frac{\pi n}{2}\frac{V}{(2\pi\hbar)^3}\sum_{nm}\int d\mathbf{P}'\,d\mathbf{P}''\,|\mathbf{r}_{nm}^{\perp}|^2\,\delta(\hbar\mathbf{k}-(\mathbf{P}'-\mathbf{P}''))$$
$$\times\,\delta(\hbar\omega-(E_n+E_{\mathbf{P}'}-E_m-E_{\mathbf{P}''}))\,[f_m(\mathbf{P}'')-f_n(\mathbf{P}')]. \tag{2.16}$$

The processes that determine this function are possible if the conservation laws (1.17) are satisfied. The processes with the plus sign in (1.17) correspond to terms with $E_n > E_m$ in (2.16). These are ordinary processes of resonance absorption and emission. In the equilibrium state, excitation with absorption of quanta prevails, as the result of which the corresponding part of the function α_{bb}'' is positive.

Processes with the minus sign in (1.17) correspond to terms with $E_n < E_m$ in (2.16). These are processes of excitation with emission of quanta, as well as transitions to the lower states with absorption. In the equilibrium state, the absorption processes prevail, and therefore $\alpha_{bb}'' > 0$. The probabilities of all these processes will be determined in the next section.

The possibility of using a plasma as an active laser medium has been under extensive discussion of late. Thus, for example, Gudzenko and Shelepin[13] obtained corresponding estimates for a hydrogen plasma. They have shown that in such a medium it is possible to obtain amplification at a number of frequencies of the discrete spectrum. More detailed calculations and the corresponding literature are given in[14].

3. KINETIC EQUATION DESCRIBING THE ATOM-LEVEL DISTRIBUTION IN THE CASE OF A MAXWELLIAN VELOCITY DISTRIBUTION

Let us multiply the second equation of (1.6) by $nV/(2\pi\hbar)^3$ and integrate with respect to $\mathbf{P}'$. We put

$$(n_{ab})_m = n\frac{V}{(2\pi\hbar)^3}\int f_m\,d\mathbf{P}\equiv n_{ab}\rho_m,\qquad \sum_m \rho_m = 1. \tag{3.1}$$

The equation for the function $(n_{ab})_n$ will be represented in the form

$$\frac{\partial (n_{ab})_n}{\partial t} = (S_n)_1 + (S_n)_2. \tag{3.2}$$

The integrals in the right side are determined in the dipole approximation by expressions that follow from (1.10) and (1.11), with (1.5) taken into account. The integral $(S_n)_1$ describes the variation of the function $(n_{ab})_n$ due to transitions in the discrete spectrum, and $(S_n)_2$ describes the result of transitions from the discrete to the continuous spectrum and vice-versa. In the expressions for these integrals, account is taken of the contribution made for all possible values of ω and $\mathbf{k}$. Let us consider here the transparency region, when ω and $\mathbf{k}$ are connected by the equation $\omega^2\epsilon^{\perp} - c^2k^2 = 0$, in which $\epsilon' = \epsilon'(\omega)$ is the real part of the dielectric constant. We disregard spatial dispersion.

The expression for $(S_n)_1$ in the transparency region can be represented in the form

$$(S_n)_1 = \sum_{m,\mp}\left[B_{nm}\tilde{\rho}^{(\mp)}_{\omega_{nm}}\left((n_{ab})_m-(n_{ab})_n\right)\mp A^{(\mp)}\frac{(n_{ab})_n}{(n_{ab})_m}\right]. \tag{3.3}$$

Here B_{nn} is the usual Einstein coefficient, and

$$\tilde{\rho}^{(\mp)}_{\omega_{nm}} = Mc\int_0^\infty \rho_\omega^{(T)} f\left(Mc\frac{\omega\mp\omega_{nm}}{\omega\sqrt{\varepsilon'}}\mp\frac{1}{2}\frac{\hbar\omega\sqrt{\varepsilon'}}{c}\right)\frac{d\omega}{\omega\sqrt{\varepsilon'}} \tag{3.4}$$

is the effective spectral density, averaged over the Doppler line, $\rho_\omega^{(T)}$ is a spectral function, which coincides in the equilibrium state with the temperature part of the Planck formula, and f is the one dimensional Maxwellian distribution

$$A^{(\mp)}_{nm} = \frac{4e^2|\mathbf{r}_{nm}|^2}{3\hbar c^3}Mc\int_0^\infty \omega^3\sqrt{\varepsilon'}f\left(Mc\frac{\omega\mp\omega_{nm}}{\omega\sqrt{\varepsilon'}}\mp\frac{1}{2}\frac{\hbar\omega\sqrt{\varepsilon'}}{c}\right)\frac{d\omega}{\omega\sqrt{\varepsilon'}} \tag{3.5}$$

is the Einstein coefficient with allowance for the polarization and thermal motion (Doppler broadening).

The terms with the minus sign in (3.3)--(3.5) are determined by "normal" processes, and the terms with the plus sign by "anomalous" processes. Under this condition we can see from (3.4) and (3.5) that the "anomalous" effects, unlike the "normal" effect, are nonresonant. Formulas (3.4) and (3.5) are valid under the condition that the Doppler width exceeds the width due to the collisions.

The expression for $(S_n)_2$ can be represented in the form

$$(S_n)_2 = (b_n^{(i)} + b_n^{(sp)})\,n_a n_b - (a_a^{(i)} + a_n^{(sp)})\,(n_{ab})_n. \tag{3.6}$$

Here $b^{(i)}$ and $b^{(sp)}$ are the coefficients of the induced and spontaneous recombinations at the level n; $a_m^{(i)}$ and $a_n^{(sp)}$ are the coefficients of induced and spontaneous ionizations from the level n. These coefficients are determined by the expressions

$$b_n^{(i)} = \sum_{-,+} b_n^{(\mp)} = \frac{e^2V}{2(2\pi)^3\hbar}\sum_{-,+}\int_0^\infty d\omega\int dk\,dp\,d\mathbf{P}\,d\mathbf{P}'\,|\mathbf{r}_{np}^{\perp}|^2$$
$$\times\,\delta(\hbar\mathbf{k}\mp(\mathbf{P}'-\mathbf{P}))\,\delta(\hbar\omega\mp(E_n+E_{\mathbf{P}'}-E_p-E_\mathbf{P}))\,f(p)\,f(\mathbf{P})\,(\delta E\delta E)^{(T)}_{\omega\mathbf{k}}; \tag{3.7}$$

$(\delta E\,\delta E)^{(T)}_{\omega k}$ is the part of the spectral function which coincides in the equilibrium state with the temperature part of this function. The coefficient

$$b_n^{(i)} \to a_n^{(i)}\ \text{as}\ (\delta E\delta E)^{(T)}_{\omega\mathbf{k}} \to \left(\frac{\mu\varkappa T}{2\pi\hbar^2}\right)^{3/2}\frac{e^{\mp\hbar\omega/\varkappa T}}{Z}(\delta E\delta E)^{(T)}_{\omega\mathbf{k}}; \tag{3.8}$$
$$b_n^{(+)} \to b_n^{(sp)},\ a_n^{(-)} \to a_n^{(sp)}\ \text{as}\ (\delta E\delta E)^{(T)}_{\omega\mathbf{k}} \to 2R(\omega,\mathbf{k}).$$

For the transparency region, the expression for $(S_n)_2$ takes the form

$$(S_n)_2 = \frac{4\pi^2 V e^2}{3\hbar} \sum_{-,+} \int_0^\infty d\omega \int dP_x\, d\mathbf{p}\, |r_{n\mathbf{p}}^{\perp}|^2 \delta\left(\hbar\omega\left(1 - \frac{v_x\sqrt{\varepsilon'}}{c}\right) \mp \left(\frac{\hbar^2\omega^2\varepsilon'}{2Mc^2} + E_n - E_{\mathbf{p}}\right)\right) \left\{\rho_\omega^{(T)}\left[n_a n_b f(\mathbf{p}) f(P_x) - \frac{(n_{ab})_n f(P_x + \hbar\omega\sqrt{\varepsilon'}/c)}{(2\pi\hbar)^3}\right] \mp \frac{\hbar\omega^3\sqrt{\varepsilon'}}{\pi^2 c^3}\left[\begin{matrix}(n_{ab})_n f(P_x + \hbar\omega\sqrt{\varepsilon'}/c)/(2\pi\hbar)^3 \\ (n_a n_b f(\mathbf{p}) f(P_x)\end{matrix}\right]\right\} \quad (3.9)$$

This expression can also be represented in the form (3.6). The form of the coefficient in this case is clear from a comparison of expressions (3.6) and (3.9).

Thus, in expressions (3.6) and (3.9), besides the "normal" processes, account is taken of three "anomalous" processes: stimulated and spontaneous ionization with emission of a photon, and stimulated recombination with absorption of a photon. From the foregoing formulas it is clear that all the "anomalous" effects are superluminal, i.e., they are possible at heavy-particle velocities exceeding the velocity of light in the medium.

All the "anomalous" effects considered here are possible also in the case of a Coulomb plasma, when the motion of the atoms exceeds the phase velocity of the longitudinal waves in the plasma. The probabilities of these processes can be obtained with the aid of the kinetic equations given in I. The difference lies in the fact that the spectral function of the transverse field and the dielectric constant must be replaced by the corresponding functions for the longitudinal field. Naturally, the "anomalous" effects are more probable for a Coulomb plasma, since the phase velocities of the plasmons (especially the acoustic ones) are much smaller than those of the photons in the plasma.

4. EQUATIONS FOR THE CONCENTRATIONS OF THE CHARGED PARTICLES IN ATOMS

In order to obtain equations for the concentrations, we shall use the kinetic equation (1.6). Taking into account the properties (1.7), we obtain the following system of equations:

$$\frac{dn_a}{dt} = (a^{(i)} + a^{(sp)}) n_{ab} - (b^{(i)} + b^{(sp)}) n_a n_b, \quad \frac{dn_{ab}}{dt} = -\frac{dn_a}{dt}. \quad (4.1)$$

The recombination and ionization coefficient are connected with the coefficients $b_n^{(i)}$ (3.7) and $a_n^{(i)}$ (3.8) by the relations

$$b = \sum_n b_n, \quad a = \sum_n a_n. \quad (4.2)$$

Let us consider the expression for the coefficients of the "normal" photoionization and photorecombination. In the approximation in which the heavy particles have infinite mass, we have

$$b^{(i)} = \frac{V}{2\hbar} \frac{1}{(2\pi m\varkappa T)^{3/2}} \sum_n \int d\omega\, d\mathbf{p}\, |\mathbf{r}_{\mathbf{p}n}^{\perp}|^2 \delta\left(\hbar\omega - |E_n| - \frac{p^2}{2m}\right) \times (\delta\mathbf{E}\delta\mathbf{E})_\omega^{(T)} e^{-p^2/2m\varkappa T}. \quad (4.3)$$

For recombination at the ground level, using expression (2.14) for the matrix element, we get

$$b^{(i)} = \frac{2^8 e^{-4}}{3\hbar}\left(\frac{2\pi\hbar^2}{m\varkappa T}\right)^{3/2} a^3 \int_{\hbar\omega > I} \exp\left\{-\frac{\hbar\omega - I}{\varkappa T}\right\} (\delta\mathbf{E}\delta\mathbf{E})_\omega^{(T)} d\omega. \quad (4.4)$$

From this, using the first expression of (3.8), we get an expression for the photoionization from the ground level:

$$a^{(i)} = \frac{2^8 e^{-4}}{3\hbar} a^3 \int_{\hbar\omega > I} (\delta\mathbf{E}\delta\mathbf{E})_\omega^{(T)} d\omega. \quad (4.5)$$

For the equilibrium state with $I \gg \kappa T$ we have

$$a^{(i)} = \frac{2^{10} e^{-4}}{3} a^3 \frac{I^3 \sqrt{\varepsilon'(I/\hbar)}}{\hbar^3 c^3} \frac{\varkappa T}{\hbar} e^{-I/\varkappa T}. \quad (4.6)$$

This expression differs only by a numerical factor from expression (6.96) obtained by a quasiclassical method in the book of Zel'dovich and Raĭzer[4].

The method presented here and in I for describing the nonequilibrium states of a plasma with allowance for inelastic processes can be used with slight modifications also for other cases, for example, to describe nonequilibrium processes in semiconductors, inelastic processes in neutral gases, in the statistical theory of chemical reactions, etc.

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