THERMOELECTRIC EFFECT IN SUPERCONDUCTORS

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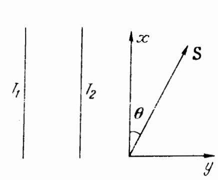
The thermoelectric effect in superconductors is studied on the basis of the microscopic theory of superconductivity. The thermoelectric current arises in an anisotropic superconductor under the action of a temperature gradient. The thermoelectric coefficients are computed. It is shown that as $T \to T_c$ the magnetic field of the thermoelectric current takes on values that are completely accessible to experimental observation.

In the present work, the thermoelectric effect in superconductors is considered on the basis of the microscopic theory. In the work of Ginzburg[1] (see also[2]), it was shown that the conclusion usually drawn as to the absence of this effect in superconducting state is not true. Actually, the normal current created by the temperature gradient in the isotropic case is completely compensated by the superconducting current. Such compensation does not generally take place when anisotropy is taken into account, which leads to a unique thermoelectric effect wherein a circulating current and a magnetic field are produced and (see below) can be measured experimentally. An expression was obtained in[1] for the thermoelectric current and the field in an anisotropic superconductor. Of course, the calculation is carried out on the basis of the London theory, generalized to the anisotropic case.

The relation for the current and field obtained in[1] contains thermoelectric coefficients that can be computed only within the framework of the microscopic theory. It is impossible to determine these coefficients from the experimental data obtained in the study of the normal metal, inasmuch as they (see below) naturally depend on the parameters of the superconducting state. At the same time, the solution of the problem of the possibility of observation of the effect is connected primarily with the determination of the coefficients. The microscopic consideration of the problem of the thermoelectric effect in superconductors, which, moreover, is based on the exact expression for the superconducting current density, is therefore of definite interest.

Let us consider an infinite superconducting plate in which there exists a temperature gradient (see the drawing). In the plane case considered, all the quantities depend only on y; the coordinate axes are chosen as shown in the drawing; s is the axis of the crystal (a uniaxial crystal is considered), and s lies in the xy plane.

We write down the Maxwell equation and also the expressions determining the superconduction and normal currents, respectively:



$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}(\mathbf{j}^{(n)} + \mathbf{j}^{(s)}), \tag{1}$$

$$j_\alpha^{(n)} = b_{\alpha\beta}\nabla_\beta T, \tag{2}$$

$$j_\alpha^{(s)}(q) = -\frac{c}{4\pi}K_{\alpha\beta}(q)A_\beta(q). \tag{3}$$

Here $b_{\alpha\beta}$ are the thermoelectric coefficients, $K_{\alpha\beta}$ are the components of the Pippard tensor, the expression for which was obtained by Pokrovskiĭ[3]. The system (1)–(3) was written in the linear approximation in ∇T. Of course, the case is considered for which $l \gg \xi_0$ (l = path length).

It is clear from these considerations that the components $j_z^{(n)}$ and $j_z^{(s)}$, together with the corresponding quantities b_{xz}, b_{zy}, K_{xz}, K_{zy}, etc., are equal to zero. From the equation of continuity it follows that in the plane case considered the component $j_y = j_y^{(n)} + j_y^{(s)}$ of the total current is also equal to zero. Only the component j_x differs from zero and, as follows from (1) and (3), equals in the case of the half space $y > 0$

$$j_x(y) = (cH/4\pi\delta)e^{-y/\delta}$$

(H is the field in the depth of the sample (see below), and $\delta = [K_{xx}(0)]^{-1/2}$). For a plate $-a \le y \le a$ (see drawing), we find

$$j_x = (cH/4\pi\delta)\operatorname{sh}(y/\delta)/\operatorname{ch}(a/\delta)$$

in agreement with[1]. Thus the anisotropy of the crystal leads to the existence of a circulating current which differs from zero in a surface layer of thickness $\sim\delta$.

We now compute the magnetic effect created by the thermoelectric current. We find from (1)–(3)

$$A_x(y) = \frac{4\pi}{c}\int\frac{j^{(n)}(q)e^{iqy}dq}{q^2 + K(q)}, \tag{4}$$

where

$$K(q) = K_{xx}(q) - K_{xy}^2(q)/K_{yy}(q), \tag{5}$$

$$j^{(n)}(q) = j_x^{(n)}(q) - \frac{K_{xy}(q)}{K_{yy}(q)}j_y^{(n)}(q). \tag{6}$$

A temperature gradient $\nabla T \lesssim 0.1$ deg/cm can be created in the sample. The normal current generated in this case changes over distances $\sim\xi_0$. Therefore, the principal role in (4) is played by $q \ll 1/\xi_0$. Furthermore, it appears that $q \ll \delta^{-1}$, with the exception of a very small range of temperatures near T_c.) The desired field is thus described by the following relation

$$H_z(y) = -\frac{4\pi}{c}\frac{d}{dy}(K^{-1}(0, T) j^{(n)}(y)). \quad (7)$$

In the isotropic case H_z vanishes as expected. Actually, the quantities $j_x^{(n)}$ and K_{xy}, which determine the value of $j^{(n)}$ according to (6), are seen to be equal to zero in this case.

Thus the problem reduces to the calculation of the normal current or, in other words, to the determination of the thermoelectric coefficients $b_{\alpha\beta}$, which characterize the anisotropic superconductor under consideration. The components of the normal current can be written in the form

$$j_x^{(n)} = (b_\parallel - b_\perp)\cos\theta\sin\theta\nabla T,$$

$$j_y^{(n)} = (b_\parallel \sin^2\theta + b_\perp\cos^2\theta)\nabla T.$$

Here $b_\parallel$ and $b_\perp$ are the principal values of the tensor $b_{\alpha\beta}$; $b_\parallel$ corresponds to $\Delta T \parallel \mathbf{S}$.

By definition,

$$\mathbf{j}^{(n)} = -e\int\frac{\partial\varepsilon}{\partial\mathbf{p}} f^{(1)}\frac{d\mathbf{p}}{(2\pi\hbar)^3},$$

where $f^{(1)}$ is the addition to the electron distribution function, the form of which is determined by the solution of the kinetic equation (see [4]):

$$-\frac{\varepsilon}{T}\frac{\partial f^{(0)}}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial p}\nabla T = I_{\text{ст}}, \quad f^{(0)} = (e^{(\varepsilon-\mu)/T} + 1)^{-1}.$$

Terms connected with the effect on the distribution function of the magnetic fields, which are proportional to $(\nabla T)^3$ in the case considered, can be neglected.

We shall consider the scattering of electronic excitations by impurities. Inasmuch as the factor exciting the electron system is the temperature gradient ∇T, the problem is analogous to the calculation of the electron thermal conductivity of a superconductor, which is considered in the isotropic case in [4,5]. We note that the consideration, which is based on a formula[6] for the thermoelectric coefficient,

$$b = (2\omega T)^{-1}\int d^4x e^{-ipx}\langle[\hat{j}(x), \hat{q}(0)]\rangle,$$

where $\hat{j}$ and $\hat{q}$ are the current density and heat flux operators, leads just as the calculation of the thermal conductivity,[7] to the same result as the method of the kinetic equation.

Carrying out the u, v transformation of Bogolyubov, we find

$$H_{int} = \sum_{p,p'} V_{p,p'}(u_p v_{p'} - v_p v_{p'})(\alpha_{p'0}^+\alpha_{p0} + \alpha_{p'1}^+\alpha_{p1}),$$

where

$$u_p^2 = \tfrac{1}{2}(1 + \xi_p/\varepsilon_p), \quad v_p^2 = \tfrac{1}{2}(1 - \xi_p/\varepsilon_p).$$

Taking into account the elastic character of the scattering, $\epsilon_p = \epsilon'_p$, and assuming, moreover, that $\Delta(p) \approx \Delta = \text{const}$, which is entirely permissible in view of the smallness of the anisotropy of the gap in comparison with the anisotropy of the Fermi surface, we find (see also [8])

$$-\frac{\varepsilon}{T}\frac{\partial f^{(0)}}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial p}\nabla T = -\frac{2\pi}{\hbar}\frac{|\xi|}{\varepsilon}\int |V_{pp'}|^2(f_p^{(1)} - f_{p'}^{(1)})\frac{d\sigma}{v_F}, \quad (8)$$

where $d\sigma$ is the element of area of the Fermi surface (see [3]).

The solution of (8) can be written in the form (see, for example,[9])

$$f^{(1)} = -\frac{\partial f^{(0)}}{\partial\varepsilon}\frac{\varepsilon}{T}(\Lambda\nabla T), \quad (9)$$

where $\mathbf{\Lambda} = \tau(\mathbf{p})\mathbf{v}$ is the vector free path length and τ, $\mathbf{v}$ are the relaxation time and the Fermi velocity of normal electrons. The electron path lengths in the normal and superconducting states are identical, as is well known.

We now compute the normal current, with account of (9). Carrying out the integration over $d\epsilon$, we find

$$\mathbf{j}^{(n)} = \frac{2e}{T}\mathcal{F}\left(\frac{\Delta}{kT}\right)\int\frac{d\sigma}{v_F}\mathbf{v}(\mathbf{\Lambda}\nabla T), \quad (10)$$

where

$$\mathcal{F}(x) = x(e^x + 1)^{-1} - \ln(1 + e^{-x}). \quad (11)$$

We determine the thermoelectric coefficients $b_\parallel$ and $b_\perp$ from (10):

$$b_{\parallel,\perp} = \frac{2ek}{(2\pi\hbar)^3}\mathcal{F}\left(\frac{\Delta}{kT}\right)\int\frac{d\sigma}{v_F}v_{\parallel,\perp}^2\tau(\mathbf{n}). \quad (12)$$

It is significant that the thermoelectric coefficients depend on the parameters of the superconducting state.

From (7), (5), and (6) we obtain

$$H_z(y) = -\frac{4\pi}{c}(\nabla T)^2\frac{dR}{dT}, \quad (13)$$

where

$$K(0)R = (b_\parallel - b_\perp)\cos\theta\sin\theta - \frac{K_{xy}}{K_{yy}}(b_\parallel\sin^2\theta + b_\perp\cos^2\theta),$$

and $b_\parallel$ and $_\perp$ are determined from (12). Expressing the components of the Pippard tensor entering in this relation in terms of its principal value, we obtain the following final formula:

$$H_z(y) = \frac{4\pi kc}{e}(\nabla T)^2(\varphi_\parallel - \varphi_\perp)\frac{d}{dT}\left(\mathcal{F}\left(\frac{\Delta}{kT}\right)\frac{\delta_L^2(T)}{\delta_L^2(0)}\right)\sin 2\theta,$$

$$\varphi_{\parallel,\perp} = \int\frac{d\sigma}{v_F}v_{\parallel,\perp}^2\tau(n) \Big/ \int\frac{d\sigma}{v_F}v_{\parallel,\perp}^2. \quad (14)$$

It is significant that the thermoelectric field is expressed in terms of the universal function T/T_c and quantities characterizing the normal metal. In the temperature range near the critical, Eq. (14) takes the form

$$H = -\pi\frac{ck}{e}(\nabla T)^2(\varphi_\parallel - \varphi_\perp)\frac{2\ln(2\sin 2\theta)}{T_c}\left(1 - \frac{T}{T_c}\right)^{-2}. \quad (15)$$

It is seen that the effect increases as $T \to T_c$, with the exception of a very small interval ΔT near T_c, where the circulating current vanishes (see below). From (14) (as also from (7)) it is seen that the effect vanishes in the transition to the isotropic state.

We now estimate the possible value of the field as $T \to T_c$:

$$H \approx \frac{ck}{e}(\nabla T)^2\frac{\pi\Lambda}{v_F}\frac{1}{T_c}\left(1 - \frac{T}{T_c}\right)^{-2}. \quad (16)$$

We assume $\nabla T \sim 0.1$ deg/cm and $\Lambda \sim 0.1$ cm (see, e.g.,[10]). For $T/T_c \sim 0.99$, which is consistent with the assumed value of ∇T for pure superconductors, field $H \sim 10^{-3}$ G are possible. The value of $T/T_c \sim 0.9$ corresponds to $H \sim 10^{-5}$ G. The values of H are quite accessible to experimental observation. From (16) it is seen that the sample investigated should be sufficiently pure.

We thus see that the anisotropy of the crystal leads to the appearance in the superconductor of thermoelectric current under the action of a temperature gradient.

The appearance of this current can be detected experimentally. The performance of the corresponding experiments is therefore of interest.

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