RESONANCE EFFECTS IN THE FIELD OF AN INTENSE LASER RAY. II

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Resonant Compton scattering of photons by electrons in the field of coherent light is considered. The dependence of the scattered photon frequency on the electron and incident photon 4-momenta and also on the intensity of the light beam is derived. The resonance nature of the differential cross section for Compton scattering is discussed. The lifetime of electron states with a definite quasi-energy is computed[11] in order to evaluate the magnitude of the cross section at resonance points. A numerical estimate shows that the resonant Compton scattering total cross section for laser radiation intensities attainable at present may exceed the ordinary Compton-effect cross section by several orders of magnitude. The magnitude of the integral cross section for resonance Moller scattering is estimated.

1. INTRODUCTION

IN the author's preceding paper[1] (henceforth cited as I) it was shown, using the effect of scattering of an electron by an electron, that the probabilities of the processes occurring in the field of a plane monochromatic light wave, which are processes of second order of perturbation theory with respect to the interaction with the quantized electromagnetic field, have a resonant character: at certain ratios of the 4-momenta of the particles taking part in the scattering process, the differential probability of such a process becomes infinite. The resonant behavior of the probabilities is connected with the discrete nature of the virtual energy spectrum of the electron in the field of the plane wave. It is of interest to investigate other second-order processes in the field of a laser beam, particularly Compton scattering.

Compton scattering of a laser beam was considered from various points of view.[2-5] The frequency of the scattered photons, as shown by the investigations, depends on the intensity of the laser beam.[2-4] In addition, at sufficiently high intensity of the coherent light beam, processes in which several laser photons are absorbed simultaneously become important. In the cited investigations they considered a process of first-order in the interaction with the quantized electromagnetic field. In the present paper we consider a second-order process—scattering of a photon by an electron moving in the field of a laser beam.

In the derivation of the equations for the matrix elements of the processes under consideration, it is possible to use a quasiclassical approximation, in which the laser beam is described classically as an external field of definite intensity, and the electrons and photons which do not belong to the laser beam are described quantum-mechanically. The amplitude of the probability of the process is determined as the S-matrix element

$$\langle \Phi_{p_f}^{(m)} | S | \Phi_{p_i}^{(n)} \rangle. \tag{A}$$

Here $|\Phi_{p_i}^{(n)}\rangle \equiv |\mathbf{k}_i^{(1)}\lambda_i^{(1)}; \ldots; \mathbf{k}_i^{(n)}\lambda_i^{(n)}; \mathbf{p}_i\sigma_i\rangle$ is the state of the field with one free electron with momentum $\mathbf{p}_i$ and spin σ_i and n photons with momenta and polarizations $\mathbf{k}_i^{(s)}$ and $\lambda_i^{(s)}$ (s = 1, 2, 3, ..., n) respectively. The interaction Hamiltonian H_{int} with the aid of which the S-matrix is determined describes the interaction of the electrons with either a quantized or an external electromagnetic field. The reduced S-matrix element corresponds to a process in which the scattering of the electron is accompanied by absorption of n photons and emission of m photons. If we now confine ourselves in the expansion of the S-matrix contained in expression (A), in the perturbation-theory series in H_{int}, to the required number of orders in the interaction with the quantized electromagnetic field (this number of orders is determined by the sum of the number of real photons and by double the number of virtual photons taking part in the process) and sum over all orders of the interaction with the external field (see, for example, [6]), we then obtain the following result: all the wave and Green's functions of the free electrons in the matrix element are replaced respectively by the wave and Green's functions of the electrons in a classical external field.

Besides the quasiclassical treatment, it is also possible to use a full quantum-mechanical description of the processes occurring in the field of a laser beam. This is done by introducing coherent quantum states[7] for the description of the laser field. The coherent states, which contain both information on the average number of photons in the beam and phase information, are apparently the most suitable approximation of the field of a laser beam. As shown by Kibble[8], both approaches based on the use of coherent states, the semiclassical and the quantum-mechanical one, are perfectly equivalent if radiative corrections are neglected (cf. also [9]).

In Sec. 2 we derive an expression for the probability of scattering of a photon of frequency ω_i by an electron in the field of a plane monochromatic wave of frequency ω. We prove the gauge invariance of the obtained expression. The formula for the frequency of the scattered quantum in different limiting cases leads to the usual formula for the Compton effect and the formula obtained by Brown and Kibble[2] and by Gol'dman.[4] A resonant behavior of the scattering probability is noted and a formula is derived for the resonant frequency of the photon.

The probability at the resonance points can be calculated by the same method as in the case of resonant transitions between the stationary states of a discrete spectrum, for example, in the effect of scattering of a photon by a bound electron,[10] the only difference being that we consider a nonstationary state with definite quasi-energy in lieu of the stationary state of the electron.[11] When account is taken of the interaction between an electron situated in the field of a laser beam and a quantized electromagnetic field, the electron undergoes transitions from a state with a given quasi-energy into a state with a different value of the quasi-energy. The lifetime of the state of an electron with a given quasi-energy becomes finite. Formally, the finite lifetime can be taken into account by ascribing complex values to the quasi-energy. The imaginary part of the quasi-energy (the radiative width of the state) is calculated in Sec. 3.

In Sec. 4 we present a numerical estimate for the cross section of the resonant Compton scattering. As shown by calculation, for modern lasers the cross section of resonant scattering can exceed by several orders of magnitude the cross section of the ordinary Compton scattering.

In I, to eliminate the infinities at the resonant points in the cross section of the Moller scattering, the Green's function of the free photon was replaced by the Green's function of the photon in an external field. For a correct analysis of the resonances it is necessary, besides making such a substitution, to take into account also the "spreading" of the states of the electron with definite quasi-energy under the influence of the quantized electromagnetic field. In Sec. 5 we present an expression for the integral cross section of the Moller scattering with allowance for the radiative width of the electrons with a definite quasi-energy, as calculated in Sec. 3.

2. RESONANT COMPTON SCATTERING

The probability amplitude for the scattering of a photon with 4-momentum k_i by an electron with 4-momentum p_i in a classical external field A is determined by the expression[1)] (k_f and p_f are the 4-momenta of the photon and of the electron in the final state)

$$M_{i\to f} = -ie^2 \int d^4z_1\, d^4z_2\, \bar{\psi}_{p_f}(z_1|A)\{\hat{A}^*_{k_f}(z_1)G(z_1,z_2|A)\hat{A}_{k_i}(z_2) + \hat{A}_{k_i}(z_1)G(z_1,z_2|A)\hat{A}^*_{k_f}(z_2)\}\psi_{p_i}(z_2|A), \tag{1}$$

$$A_{k_\alpha}(z) = \frac{e_\alpha}{\sqrt{2\omega_\alpha}} e^{-ik_\alpha z}, \tag{2}$$

where $A_{k\alpha}(z)$ is the wave function of the photon with 4-momentum $k_\alpha = (\omega_\alpha, \mathbf{k}_\alpha)$, and e_α is the photon polarization vector. Just as in I, we take as the external field the field of a plane monochromatic wave:

$$A(z) = a\cos k'z, \quad k' = \omega n, \; n = (1, \mathbf{n}). \tag{3}$$

The wave function ψ_p of the electron is best represented in the form of a Fourier expansion:

[1)]The notation is the same as in I. Only new symbols are defined in the text.

$$\psi_p(z|A) = \frac{1}{\sqrt{V}} \sum_{r=-\infty}^{\infty} \left[L_r^{(0)}(\varkappa_p) - \frac{e}{2}\frac{\hat{a}\hat{n}}{np} L_r^{(1)}(\varkappa_p) \right] u_p \times \exp[-i(pz + 2k'z\varkappa_{2p} + rk'z)], \tag{4a}$$

$$L_r^{(j)}(\varkappa_p) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} d\varphi (\cos\varphi)^j \exp i(\varkappa_{1p}\sin\varphi + \varkappa_{2p}\sin 2\varphi - r\varphi). \tag{4b}$$

Substitution of (4a) and (2) into the probability amplitude (1) and subsequent integration lead to the following result (the expression for the Green's function is given in the Appendix of I):

$$M_{i\to f} = -\frac{ie^2(2\pi)^4}{2\sqrt{\omega_i\omega_f}} \sum_{r',s',r,s} \sum \delta^4[p_f + k_f - p_i - k_i + 2k'(\varkappa_{2f} - \varkappa_{2i}) - sk'] \times \left[\bar{u}_f \left[L_{r'}^{(0)}(f) + \frac{e}{2}\frac{\hat{a}\hat{n}}{np_f} L_{r'}^{(1)}(f)\right]\left\{\frac{\hat{e}_f B_{\mu\nu}(p)\hat{e}_i}{D_r(k_i)}\Big|_{p=p_i+k_i+2k'\varkappa_{2i}+s'k'} + \frac{\hat{e}_i B_{\mu\nu}(p)\hat{e}_f}{D_r(-k_f)}\Big|_{p=p_i-k_f+2k'\varkappa_{2i}+s'k'}\right\}\left[L_{s'}^{(0)}(i) - \frac{e}{2}\frac{\hat{a}\hat{n}}{np_i}L_{s'}^{(1)}(i)\right]u_i\right], \tag{5}$$

$$L_r^{(i)}(\alpha) \equiv L_r^{(i)}(\varkappa_{p_\alpha}), \quad (\alpha = i, f), \quad \mu = r + s', \quad \nu = r + r' + s,$$
$$D_r(k_i) = (p_i + k_i - r\omega n)^2 + 4\varkappa_{2i}\omega nk_i - m^2 + i\varepsilon, \quad \varepsilon \to +0. \tag{6}$$

The functions $B_\mu(p)$ can be transformed into

$$B_{\mu\nu}(p)|_{p=\bar{p}+2\varkappa_{2i}k'+s'k'} = [\hat{\bar{p}} + 2\omega\hat{n}(\varkappa_{2i} - \varkappa_{2\bar{p}}) - r\omega\hat{n} + m]B_{\mu\nu}^{00}(\bar{p}) + (\varkappa_{1\bar{p}}\omega\hat{n} - e\hat{a})B_{\mu\nu}^{10}(\bar{p}) - \frac{e}{2n\bar{p}}(-\hat{a}\hat{n}\hat{\bar{p}} + m\hat{n}\hat{a})[B_{\mu\nu}^{10}(\bar{p}) - B_{\mu\nu}^{01}(\bar{p})] + 4\varkappa_{2\bar{p}}\,\omega\hat{n}B_{\mu\nu}^{11}(\bar{p}), \quad \bar{p} = p_i + k_i, \quad B_{\mu\nu}^{ij}(p) \equiv L_\mu^{(i)}(\varkappa_p)L_\nu^{(j)}(\varkappa_p). \tag{7}$$

The summation over r′ and s′ in the foregoing expression can be readily carried out with the aid of the formula

$$\sum_{r'=-\infty}^{+\infty} L_{r'}^{(n)}(\varkappa_{p_1})L_{r'+r}^{(m)}(\varkappa_{p_2}) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} d\varphi(\cos\varphi)^{n+m}\exp i[(\varkappa_{1p_1} - \varkappa_{1p_2})\sin\varphi + (\varkappa_{2p_1} - \varkappa_{2p_2})\sin 2\varphi + r\varphi] \equiv L_{-r}^{(n+m)}(\varkappa_{p_1} - \varkappa_{p_2}). \tag{8}$$

With the aid of (7) and (8) it is easy to prove the invariance of the matrix element (5) against the transformation $a \to a' = a + \lambda n$, where λ is an arbitrary constant.

We now demonstrate the invariance of the probability amplitude $M_{i\to f}$ against the gauge transformation

$$e_f \to e_f' = e_f + \lambda_1 k_f, \quad e_i \to e_i' = e_i + \lambda_2 k_i, \tag{9}$$

where λ_α are arbitrary constants. When $\hat{e}_f = \hat{k}_f$, part of the matrix element (5), corresponding to the first term in the curly bracket, can be reduced by means of very cumbersome manipulations to the expression

$$\sum_{r',s',r}\left(\bar{u}_f\left[L_{r'}^{(0)}(f) + \frac{e}{2}\frac{\hat{a}\hat{n}}{np_f}L_{r'}^{(1)}(f)\right]\frac{\hat{k}_f B_{\mu\nu}(p)\hat{e}_i}{D_r(k_i)}\Big|_{p=p_i+k_i+2\varkappa_{2i}k'+s'k'} \times \left[L_{s'}^{(0)}(i) - \frac{e}{2}\frac{\hat{a}\hat{n}}{np_i}L_{s'}^{(1)}(i)\right]u_i\right) = \sum_{r'=-\infty}^{+\infty}\left(\bar{u}_f\left[L_{r'}^{(0)}(f) + \frac{e}{2}\frac{\hat{a}\hat{n}}{np_f}L_{r'}^{(1)}(f)\right]\hat{e}_i\left[L_{r'+s}^{(1)}(i) - \frac{e}{2}\frac{\hat{a}\hat{n}}{np_i}L_{r'+s}^{(1)}(i)\right]u_i\right) \tag{10}$$

The second part of the matrix element can be transformed, after making the substitution $\hat{e}_f \to \hat{k}_f$, into an expression that differs from (10) only in sign.

The differential scattering probability, averaged over the initial polarization states of the electrons and photons and summed over the final states, can be written in the form

$$dw_{i\to f}=\frac{1}{4}\sum_{\sigma,\lambda}\frac{|M_{i\to f}|^2}{VT}\frac{d\mathbf{p}_f}{(2\pi)^3}\frac{d\mathbf{k}_f}{(2\pi)^3}=\frac{1}{4}\frac{1}{4\varepsilon_i\varepsilon_f}\frac{e^4}{4\omega_i\omega_f}\frac{d\mathbf{p}_f d\mathbf{k}_f}{(2\pi)^2}$$
$$\times\sum_{r_1,r_2,s}\delta^4[p_f+k_f-p_i-k_i+2k'(\varkappa_{2f}-\varkappa_{2i})-sk']$$
$$\times\{P_{r_1r_2s}(k_i,k_f)+P_{r_1r_2s}(-k_f,-k_i)\},\qquad(11)$$

$$P_{r_1r_2s}(k_i,k_f)=H^{(1)}_{r_1r_2s}(k_i,k_f)+H^{(2)}_{r_1r_2s}(k_i,f_f),\qquad(12)$$

$$H^{(1)}_{r_1r_2s}(k_i,k_f)=\sum_{\mu,\nu=1}^{4}g^{\mu\mu}g^{\nu\nu}\operatorname{Sp}\Big\{(\hat p_f+m)\sum_{r',s'}T^{(+)}_{r'}(f)$$
$$\times\frac{\gamma_\mu B_{r_1+s',\,r_1+r'+s}(p)\gamma_\nu|_{p=p_i+k_i+2\varkappa_{2i}k'+s'k'}}{D_{r_1}(k_i)}T^{(-)}_{s'}(i)(\hat p_i+m)$$
$$\times\sum_{\bar r',\bar s'}T^{(+)}_{\bar s'}(i)\frac{\gamma_\nu\overline{B}_{r_2+\bar s',\,r_2+\bar r'+s}(p)\gamma_\mu|_{p=p_i+k_i+2\varkappa_{2i}k'+s'k'}}{D^*_{r_2}(k_i)}T^{(-)}_{\bar r'}(f)\Big\},$$

$$H^{(2)}_{r_1r_2s}(k_i,k_f)=\sum_{\mu,\nu=1}^{4}g^{\mu\mu}g^{\nu\nu}\operatorname{Sp}\Big\{(\hat p_f+m)\sum_{r',s'}T^{(+)}_{r'}(f)$$
$$\times\frac{\gamma_\nu B_{r_1+s',\,r_1+r'+s}(p)\gamma_\mu|_{p=p_i-k_f+2\varkappa_{2i}k'+s'k'}}{D_{r_1}(-k_f)}T^{(-)}_{s'}(i)(\hat p_i+m)$$
$$\times\sum_{\bar s',\bar r'}T^{(+)}_{\bar s'}(i)\frac{\gamma_\nu\overline{B}_{r_2+\bar s',\,r_2+\bar r'+s}(p)\gamma_\mu|_{p=p_i+k_i+2\varkappa_{2i}k'+s'k'}}{D^*_{r_2}(k_i)}T^{(-)}_{\bar r'}(f)\Big\},$$

$$\overline{B}_{\mu\nu}(p)=\gamma_0B^+_{\mu\nu}(p)\gamma_0,\quad T^{(\pm)}_r(a)=L^{(0)}_r(a)\pm\frac{e}{2}\frac{\hat a\hat n}{np_a}L^{(1)}_r(a).$$

Here $\sum_{\sigma,\lambda}$ denotes the sum over the spin states of the electrons and the polarization states of the photons in the initial and final states. The functions $H^{(\alpha)}_{r_1r_2s}$ take the form

$$D_{r_1}(k_i)D^*_{r_2}(k_i)H^{(1)}_{r_1r_2s}=\sum_{\substack{l,k;\,mn\\(=0,1,2)}}R^{lk}_{r_2,\,r_2+s}R^{mn}_{r_1,\,r_1+s}f^{lkmn}_{r_1r_2},\qquad(13)$$

$$R^{i+k,\,n+j}_{r,\,r+s}(i,\bar p;f,\bar p)=\sum_{r',s'=-\infty}^{+\infty}L^{(n)}_{r'}(p_f)B^{ij}_{r+s',\,r+r'+s}(\bar p)L^{(k)}_{s'}(p_i).$$

Here $f^{lkmn}_{r_1r_2}$ are certain polynomials of fourth power in ea. The complete expressions for the functions $H^{(\alpha)}_{r_1r_2s}$ are very complicated. If we calculate the functions $f_{r_1r_2}$ only in the zeroth approximation in ea, we get

$$D_{r_1}(k_i)D^*_{r_2}(k_i)H^{(1)}_{r_1r_2s}(k_i,k_f)=32R^{00}_{r_1,\,r_1+s}(i,\bar p;f,\bar p)$$
$$\times R^{00}_{r_2,\,r_2+s}(i,\bar p;f,\bar p)\{-m^2(\bar pp_f)+(p_ik_i)(p_fk_i)+2m^2(p_ik_i)+2m^4\},$$
$$(\bar p=p_i+k_i)\qquad(14a)$$

$$D_{r_1}(-k_f)D^*_{r_2}(k_i)H^{(2)}_{r_1r_2s}(k_i,k_f)=16R^{00}_{r_1,\,r_1+s}(i,\bar{\bar p};f,\bar{\bar p})R^{00}_{r_2,\,r_2+s}(i,\bar p;f,\bar p)$$
$$\times\{m^2[p_fk_i-2p_ik_f+2p_ik_i-k_ik_f+m^2+p_ip_f-p_fk_f]$$
$$-2(p_ip_f)[k_ip_i-p_ik_f-k_ik_f]\},\quad\bar{\bar p}=p_i-k_f.\qquad(14b)$$

In order to carry out the integration with respect to $d\mathbf{p}_f$ in (11), we shall use the following expression (we make the following change in the integration variables:[1, 2] $p_f+2k'\kappa_{2f}=q$):

$$\Phi_s=\int d\mathbf{p}_f\frac{1}{2\varepsilon_f}\delta^4[p_f+k_f-p_i-k_i+2k'(\varkappa_{2f}-\varkappa_{2i})-sk']$$
$$=\int d^4q\,\delta(q^2-m^{*2})\Theta(qk')\delta^4[q+k_f-p_i-k_i+2k'\varkappa_{2i}-sk']$$
$$=\Theta(p_ik'+k_ik'-k_fk')\delta[2p_i(k_i-k_f)-2k_ik_f+2sk'(p_i+k_i-k_f]$$
$$+4\varkappa_{2i}k'(k_i-k_f)],\quad\Theta(x)=\begin{cases}1,&x>0\\0,&x<0\end{cases}.\qquad(15)$$

Let us consider the laboratory system of coordinates, in which the electron i is at rest ($\epsilon_{p_i}=m$, $\mathbf{p}_i=0$). In this system we obtain for the frequency of the scattered photon the formula

$$\omega_f=\Big\{s\omega+\omega_i\Big[1+(1-\cos\theta_i)\Big(\frac{s\omega}{m}-\frac{e^2a^2}{4m^2}\Big)\Big]\Big\}$$
$$\times\Big\{1+\frac{\omega_i}{m}(1-\cos\theta)+\Big(\frac{s\omega}{m}-\frac{e^2a^2}{4m^2}\Big)(1-\cos\theta_f)\Big\}^{-1},\qquad(16a)$$
$$-e^2a^2>0,\quad\cos\theta=\frac{\mathbf{k}_i\mathbf{k}_f}{\omega_i\omega_f},\quad\cos\theta_\alpha=\frac{\mathbf{n}\mathbf{k}_\alpha}{\omega_\alpha}\quad(\alpha=i,f).$$

When ea = 0 and s = 0, this formula goes over into the ordinary formula for the Compton effect, and when $\omega_i = 0$ it goes over into the formula obtained by Brown and Kibble[2] and by Gol'dman[4]

$$\omega_f=s\omega\Big/\Big[1+\Big(\frac{s\omega}{m}-\frac{e^2a^2}{4m^2}\Big)(1-\cos\theta_f)\Big].\qquad(16b)$$

The frequency of the scattered photon ω_f depends, according to (16a) on the intensity of the laser beam. As shown by Brown and Kibble,[2] this dependence is due to the change in the character of motion of the electron in the field of the laser beam compared with the free motion: the electron moving in the field of the beam acquires an additional mass $\Delta m^2=-e^2a^2/2$. The dependence of the frequency of the scattered quantum on the intensity of the beam has a simple classical interpretation (see [8]) as the Doppler shift resulting from the fact that the electron which is initially at rest acquires in the field of the plane wave a non-zero average velocity in the direction of beam propagation.

We shall henceforth confine ourselves to the nonrelativistic approximation:

$$\Big|\frac{e^2a^2}{m^2}\Big|\ll1,\quad\frac{\omega_i}{m},\frac{\omega_f}{m},\frac{\omega}{m}\ll1.\qquad(17)$$

In this approximation, when $\omega_i<\omega$ (formula (16a)), only processes with absorption of photons from the laser beam take place ($s\ge0$). When $\omega_i>\omega$, processes with emission of photons of frequency ω are also possible.

As expected, the probability $dw_{i\to f}$ (11) contains resonant singularities. At certain values of the 4-momenta, the denominators of the expressions for $P_{r_1r_2s}(k_ik_f)$ vanish. For example, let $D_r(k_i)=0$ when $r=r^*\neq0$. From this we obtain the following formula for the frequency of the incoming photon at resonance (in the laboratory system):

$$\omega_i=r^*\omega\Big/\Big[1-\Big(\frac{r^*\omega}{m}+\frac{e^2a^2}{4m^2}\Big)(1-\cos\theta_i)\Big],\quad r^*\geqslant1.\quad(18a)$$

From the condition for the vanishing of the second denominator we get ($D_r(-k_f)=0$)

$$\omega_f=-r^*\omega\Big/\Big[1-\Big(\frac{r^*\omega}{m}+\frac{e^2a^2}{4m^2}\Big)(1-\cos\theta_f)\Big],\quad r^*\leqslant-1.\ (18b)$$

Thus, the resonance condition for the Compton scattering has a very lucid form: resonance takes place when the energy of the scattered photon approximately coincides with the energy difference of any two virtual levels of the electron in the field of the plane wave (see formula (15) of I).

Using (15), we easily obtain an expression for the differential effective cross section of the Compton scattering in the laboratory system:

$$d\sigma = \frac{\alpha^2}{16m}\sum_{r_1 r_2 s}\frac{\omega_f}{\omega_i}\Theta(\omega m + k_i k' - k_f k')$$
$$\times\frac{P_{r_1 r_2 s}(k_i, k_f) + P_{r_1 r_2 s}(-k_f, -k_i)}{m + \omega_i(1-\cos\theta) + (s\omega - e^2a^2/4m)(1-\cos\theta_f)}\,d\Omega_f,$$
$$\alpha = \frac{e^2}{4\pi}, \quad \varepsilon_f' = m + \omega_i + s\omega - \omega_f - \frac{e^2a^2}{4m}\,\frac{nk_i - nk_f}{m + nk_i - nk_f}, \tag{19}$$

$d\Omega_f$ is the element of the solid angle in the direction of the vector $\mathbf{k}_f$.

In the absence of an external field (ea = 0) the functions $H^{(1)}_{r_1 r_2 s}$ and $H^{(2)}_{r_1 r_2 s}$ take the form

$$H^{(1)}_{r_1 r_2 s}(k_i, k_f)\,|_{ea=0} = \frac{32}{\beta_1^2}\left(1 - \frac{\beta_1}{2} - \frac{\beta_1\beta_2}{4}\right)\delta_{r_1 0}\delta_{r_2 0}\delta_{s0},$$
$$H^{(2)}_{r_1 r_2 s}(k_i, k_f)\,|_{ea=0} = -\frac{32}{\beta_1\beta_2}\left(1 - \frac{\beta_1}{4} - \frac{\beta_2}{4}\right)\delta_{r_1 0}\delta_{r_2 0}\delta_{s0},$$
$$\beta_1 = -2\frac{p_i k_i}{m^2}, \quad \beta_2 = 2\frac{p_i k_f}{m^2}. \tag{20}$$

With the aid of these relations, the cross section (19) in the absence of an external field can be reduced to the form

$$d\sigma|_{ea=0} = \frac{r_0^2}{2}\left(\frac{\omega_f}{\omega_i}\right)^2\left(\frac{\omega_f}{\omega_i} + \frac{\omega_i}{\omega_f} - \sin^2\theta\right)d\Omega_f, \quad r_0 = \frac{\alpha}{m}. \tag{21}$$

This is a well known formula for the cross section of ordinary Compton scattering (see, e.g., [10]).

We note that the cross section of the Compton scattering, calculated in [2,4] at zero external field, vanishes like an effect of first order in the interaction of the electron with the quantized electromagnetic field. In our case we consider a second-order effect which takes place also without a laser field.

3. RADIATIVE WIDTH OF A STATE WITH DEFINITE QUASI-ENERGY

As shown by Zel'dovich,[11] among the states of the system with a Hamiltonian that is periodic in time, we can separate states with a definite quasi-energy which play the same role as the stationary states with definite energy in the theory with a statistical Hamiltonian. These states comprise a full set in terms of which an arbitrary state of the system can be expanded. States with different values of the quasi-energy are orthogonal to one another. When the interaction with the quantized electromagnetic field is turned on, transitions occur from a state from one quasi-energy into a state with another value of the quasi-energy, i.e., a "spreading" of the wave packet takes place in the quasi-energy space. Formally, such a "spreading" can be taken into account by regarding the quasienergy ϵ_p as a complex quantity: $\epsilon_p = \epsilon_p' - i\Gamma_p$. The value of Γ_p is double the width (we shall call it the radiative width) of the state with quasi-energy ϵ_p. Its reciprocal $\tau_p = 1/\Gamma_p$ determines the lifetime of the state with definite quasi-energy.

In the case of the field of a plane wave, the states of the electron $\psi_p(z|A)$ are characterized by the value of the quasi-energy $\epsilon_p = \sqrt{\mathbf{p}^2 + m^2} + 2\omega\kappa_{2p}$. The states (4a), which belong to different values of the quasi-energy ϵ_p, are orthogonal to one another. In our problem, the spectrum of the quasi-energies is continuous. However, the spectrum of the virtual excitations of the system (formula (15) in I) turns out to be discrete. It is this discreteness which leads to a resonant behavior of the probabilities of the processes.

Allowance for the finite lifetime of the states (4a) makes it possible to obtain the value of the scattering cross section at the resonant points. To this end it is necessary to replace the quasi-energy ϵ_p in the denominators of (5) by $\epsilon_p - i\Gamma_p$.

The radiative width of the state with definite quasi-energy will be calculated in exactly the same way as for the stationary state.[10] The wave function of an electron interacting with a quantized electromagnetic field and with a classical field of a plane wave A satisfies the equation

$$(i\hat{\nabla}_1 - e\hat{A}_1 - m)\psi_p^\Gamma(1|A) = \int d^4z_2\mathscr{M}(1,2|A)\psi_p^\Gamma(2|A). \tag{22}$$

Here $\mathscr{M}(1, 2|A)$ is the electron mass operator. In perturbation theory of second order in the interaction with the quantized electromagnetic field, this operator takes the form

$$\mathscr{M}(1,2|A) = ie^2\sum_{n,m=1}^{4}\gamma^n G(1,2|A)\gamma^m D^{(0)}_{mn}(2,1),$$
$$D^{(0)}_{mn}(z_1 - z_2) = \delta_{mn}D^{(0)}(z_1 - z_2),$$
$$D^{(0)}(z_1 - z_2) = -\frac{1}{(2\pi)^4}\int d^4k\,\frac{\exp\{-ik(z_1 - z_2)\}}{k^2 + i\varepsilon}, \quad \varepsilon \to +0, \tag{23}$$

where $D^{(0)}(z_1 - z_2)$ is the free Green's function of the photon.

We seek the solution of (22) in the form (the chosen form of the solution is justified by the subsequent calculations)

$$\psi_p^\Gamma(z|A) = e^{-iE_p t}\psi_p(z|A), \tag{24}$$

E_p is the sought function and $\psi_p(z|A)$ is the solution of the Dirac equation (4a). Substitution of (24) in (22) yields

$$\gamma_0 E_p\psi_p(z|A) = \int d^4z_2\mathscr{M}(z, z_2|A)\exp\{-iE_p(t_2 - t)\}\psi_p(z_2|A). \tag{25}$$

We multiply both sides of (25) from the left by $\bar{\psi}_p(z|A)$ and integrate over the volume V (normalization volume); taking into account the orthonormality of the functions $\psi_p(z|A)$, we get

$$E_p = \int_{(V)} d\mathbf{r}\int d^4z_2\bar{\psi}_p(z|A)\mathscr{M}(z, z_2|A)\psi_p(z_2|A)\exp\{-iE_p(t_2 - t)\}. \tag{26}$$

The integral equation (26) can be solved by successive approximations. In the zeroth approximation $\psi_p^\Gamma(z|A) = \psi_p(z|A)$ and consequently $E_p^{(0)} = 0$. In the first approximation we obtain

$$E_p^{(1)} = \int_{(V)} d\mathbf{r}\int d^4z_2\bar{\psi}_p(z|A)\mathscr{M}(z, z_2|A)\psi_p(z_2|A). \tag{27}$$

Using (23) for the mass operator, we can transform the right side of the preceding relation into

$$\int_{(V)} d\mathbf{r}\int d^4z_2\bar{\psi}_p(z|A)\mathscr{M}(z, z_2|A)\psi_p(z_2|A) = -\frac{ie^2}{(2\pi)^4}\frac{1}{\varepsilon_p}\sum_{r=-\infty}^{+\infty}\int d^4k\frac{1}{k^2 + i\varepsilon'}$$
$$\times\frac{Q_{r,r}(p, p+k)}{(p + k + 2\omega n\varkappa_{2p} - r\omega n)^2 - m^{*2} + i\varepsilon''}, \tag{28}$$
$$\varepsilon', \varepsilon'' \to +0;$$

$$Q_{r,r}(p,p+k)=2(m^2-kp)B^{00}_{r,r}(p,p+k)$$
$$-2\left(nk\frac{eap}{np}-eak\right)B^{10}_{r,r}(p,p+k)-e^2a^2\left(1+\frac{np}{np+nk}\right)B^{20}_{r,r}(p,p+k)$$
$$+e^2a^2\left[2+nk\left(\frac{1}{np}-\frac{1}{np+nk}\right)\right]B^{11}_{r,r}(p,p+k); \qquad (29)$$
$$B^{\mu\nu}_{r,r}(p,p+k)=L_{-r}{}^{\mu}(\varkappa_p-\varkappa_{p+k})L_{-r}{}^{\nu}(\varkappa_p-\varkappa_{p+k}).$$

We separate the imaginary part of the function $E_p^{(1)}$ ($E_p^{(1)}=\mathrm{Re}\,E_p^{(1)}-i\Gamma_p^{(1)}$). For the radiative width $\Gamma_p^{(1)}$ we obtain the formula

$$\Gamma_p^{(1)}=-\frac{\alpha}{4\pi}\frac{1}{\varepsilon_p}\sum_{r=-\infty}^{+\infty}\int d\mathbf{k}\left\{\frac{1}{|\mathbf{k}|}Q_{r,r}(p,p+k)\right.$$
$$\times\delta\left[(p+k+2\omega n\varkappa_{2p}-r\omega n)^2-m^{*2}\right]|_{k_0=-|\mathbf{k}|}+\left[\frac{1}{[(\tilde{\mathbf{p}}+\mathbf{k}-r\omega\mathbf{n})^2+m^{*2}]^{1/2}}\right.$$
$$\left.\cdot\,Q_{r,r}(p,p+k)\,\delta(k_0^2-\mathbf{k}^2)]\,|_{k=r\omega-\tilde{p}_0-[(\tilde{\mathbf{p}}+\mathbf{k}-r\omega\mathbf{n})^2+m^{*2}]^{1/2}}\right\}, \qquad (30)$$
$$\tilde{p}=p+2\omega n\varkappa_{2p}.$$

In the derivation of this relation we used the results of Appendix 2, and also the formula

$$\frac{1}{x+i\varepsilon}=\mathrm{P}\frac{1}{x}-i\pi\delta(x),\qquad \varepsilon\to+0.$$

Expression (30) is calculated in the approximation

$$\frac{\omega}{m}\ll 1,\qquad \frac{|\mathbf{p}|}{m}\ll 1,\qquad \frac{e|\mathbf{a}|}{m}\ll 1. \qquad (31)$$

In this approximation, the second term in the curly brackets of (30) can be neglected ($p_0=\epsilon_p>0$). In addition, we use the approximate equalities ($J_r(\xi)$ are Bessel functions):

$$L_r^{(0)}(\varkappa_p-\varkappa_{p+k})\approx J_r(\xi),$$
$$L_r^{(1)}(\varkappa_p-\varkappa_{p+k})\approx {}^1/_2[J_{r+1}(\xi)+J_{r-1}(\xi)],$$
$$L_r^{(2)}(\varkappa_p-\varkappa_{p+k})\approx {}^1/_2J_r(\xi)+{}^1/_4[J_{r+2}(\xi)+J_{r-2}(\xi)], \qquad (32)$$
$$\xi=-eak/\omega(np).$$

Retaining in the expression for $\Gamma_p^{(1)}$ the largest terms, we obtain

$$\Gamma_p^{(1)}\approx-\frac{\alpha}{8\pi}\frac{1}{\varepsilon_p}\sum_{r=-1}^{-\infty}\int d|\mathbf{k}|\,d\Omega_{\mathbf{k}}\,|\mathbf{k}|\,\frac{1}{|\tilde{\varepsilon}_p+\tilde{\mathbf{p}}\mathbf{e}_{\mathbf{k}}|}.$$
$$\{2(m^2-kp)B^{00}_{r,r}(p,p+k)+2e^2a^2B^{11}_{r,r}(p,p+k)\}\,|_{k=-|\mathbf{k}|}\cdot$$
$$\cdot\,\delta\left[|\mathbf{k}|+\frac{r\omega n\tilde{p}}{\tilde{\varepsilon}_p+\tilde{\mathbf{p}}\mathbf{e}_{\mathbf{k}}}\right]\approx\frac{\alpha}{6}\,\omega\left(\frac{e|\mathbf{a}|}{m}\right)^2; \qquad (33)$$
$$\mathbf{e}_{\mathbf{k}}=\mathbf{k}/|\mathbf{k}|,\quad \tilde{\varepsilon}_p=\tilde{p}_0.$$

If the electric component of the laser field has an intensity $|E|\sim 10^7$ V/cm and the frequency is $\omega=3\times 10^{15}\ \mathrm{sec}^{-1}$, then the lifetime of the state with definite quasi-energy is

$$\tau_p=1/\Gamma_p\sim 6\cdot 10^{-6}\ \mathrm{sec} \qquad (34)$$

4. INTEGRAL CROSS SECTION OF THE RESONANT COMPTON SCATTERING

The radiative width of the state, calculated in the preceding section, will be used to estimate the integral Compton-scattering cross section.

We consider a case in which the resonance condition $D_r(-k_f)=0$ is satisfied at fixed values of the 4-vectors k_f and p_i. In order to calculate the probability at the resonance point, we shall make in the expression for $D_r(-k_f)$ the substitution $\epsilon_{p_i}\to\epsilon_i-i\Gamma$. Then in the vicinity of the resonant point $D_r(-k_f)$ will be replaced by $D_r(-k_f)-2i\epsilon_i\Gamma$. Corresponding to the separated resonance in the integral cross section is the expression

$$\sigma\approx\int d\Omega_{\mathbf{k}_f}\frac{F(\mathbf{k}_f)}{|D_r(-k_f)|^2+4m^2\Gamma^2},\quad d\Omega_{\mathbf{k}_f}=\sin\bar{\theta}\,d\bar{\theta}\,d\bar{\varphi}, \qquad (35)$$

Here $F(\mathbf{k}_f)$ is a certain function, the form of which can be readily established with the aid of relation (19). We choose a coordinate system in which the vector of the propagation direction of the laser beam is parallel to the z axis; in this system $\theta_f=\bar{\theta}$. Let the angle $\theta_f=\theta_f^*$ be the resonance angle. Owing to the smallness of the radiative width of the state Γ, the main contribution to the integral (35) is made by a small vicinity of the point $\theta_f=\theta_f^*$. Inasmuch as the function $F(\mathbf{k}_f)$ is smooth, it can be taken outside the integral sign at $\theta_f=\theta_f^*$. Since the function $F(\mathbf{k}_f)$ is smooth, it can be taken outside the integral sign at $\theta_f=\theta_f^*$. As a result we can rewrite (35) in the form

$$\sigma\approx\int_0^{2\pi}d\bar{\varphi}\,F(\mathbf{k}_f)\,|_{\theta_f=\theta_f^*}\int_{x^*-\Delta x}^{x^*+\Delta x}dx\,\frac{1}{|D_r(-k_f)|^2+4m^2\Gamma^2}$$
$$\approx\int_0^{2\pi}d\bar{\varphi}\,F(\mathbf{k}_f)\,|_{\theta_f=\theta_f^*}\frac{2\Delta x}{4m^2\Gamma^2},\quad x=\cos\theta_f,\quad x^*=\cos\theta_f^*. \qquad (36)$$

The quantity Δx (which we shall call the resonance width) is chosen to satisfy the condition that the numerical value of the function $[D_r(-k_f)+4m^2\Gamma^2]^{-1}$ at the points $x=x^*\pm\Delta x$ is half the resonant value $(4m^2\Gamma^2)^{-1}$. Using (16a) we get (the angles θ_i, φ_i, and φ_f are assumed fixed)

$$\Delta x=\frac{m}{s\omega+\omega_i}\frac{\Gamma}{d_{rs}},$$
$$d_{rs}=\left|-(r+s)\omega-\omega_i\cos\theta_i+\omega_i\frac{\cos\theta_f^*}{\sin\theta_f^*}\sin\theta_i\cos(\varphi_i-\varphi_f)\right|, \qquad (37)$$
$$r\leqslant-1,\quad \mathbf{k}_f=\mathbf{k}_f(\theta_f,\varphi_f),\quad \mathbf{k}_i=\mathbf{k}_i(\theta_i,\varphi_i).$$

In the derivation of this relation we expanded the function $D_r(-k_f)$ in powers of Δx and confined ourselves to the linear term. If for the chosen parameters (θ_i, θ_f^*, ω_i, ...) the quantity d_{rs} is zero or small compared with ω, then it is necessary to retain in the expansion of $D_r(-k_f)$ terms of higher order in Δx.

In estimating the total scattering cross section we shall use the approximate equations (14a). The largest contribution to the scattering cross section in the approximation (31) is made by the term with $|r_1|=|r_2|=1$, $s=0$; then

$$R_{r_1,r_1+s}(i,\bar{p};f,\bar{p})\,|_{r_1=-1,\,s=0}\approx\frac{1}{4}\left(\frac{e|\mathbf{a}|}{m}\right)^2\cos\bar{\varphi}_i\cos\bar{\varphi}_f,$$
$$\cos\bar{\varphi}_i=\frac{\mathbf{a}\mathbf{k}_i}{|\mathbf{a}|\omega_i},\quad \cos\bar{\varphi}_f=\frac{\mathbf{a}\mathbf{k}_f}{|\mathbf{a}|\omega_f}\quad(\omega_i\sim\omega_f\sim\omega). \qquad (38)$$

We take further the angles θ_f^* and θ_i near $\pi/2$ and $\pi/4$, respectively; then ($d_{-1,0}\approx 0.3\omega$)

$$\sigma\sim\frac{\pi}{\alpha}r_0^2\left(\frac{m}{\omega}\right)^3\left(\frac{e|\mathbf{a}|}{m}\right)^2\cos^2\bar{\varphi}_i. \qquad (39a)$$

At an electric field intensity $|\mathbf{E}|\approx 10^5$ V/cm and a frequency $\omega\sim 3\times 10^{15}\ \mathrm{sec}^{-1}$, we have

$$\sigma \sim 5\cdot 10^7 r_0^2 \quad (\text{for } \tilde{\varphi}_i = 0). \tag{39b}$$

To obtain a more accurate estimate for the total cross section it is necessary to calculate the corrections of the order ea and $(ea)^2$ in the functions $f_{r_1 r_2}$ (see (13)). These corrections, however, cannot change the order of magnitude of the cross section (39a), which is valid if the inequality $e|\mathbf{a}|/m \ll 1$ is satisfied. Therefore the estimate presented for the cross section of the Compton scattering is valid at frequencies $\omega \gg e|\mathbf{E}|/m$ (at $|\mathbf{E}| \sim 10^5$ V/cm and $\omega \gg 10^{10}$ sec^{-1}).

Thus, the calculation shows that at the presently attainable intensities of the laser radiation the cross section of the resonant Compton scattering is larger by several orders of magnitude than the cross section of the usual Compton effect. We emphasize that resonant Compton scattering should be observed in scattering of incoherent radiation by an electron moving in the field of a coherent light beam.

5. INTEGRAL CROSS SECTION OF RESONANT MOLLER SCATTERING

In I we gave a numerical value of the differential cross section of Moller scattering at the resonance point. The infinity at resonance was eliminated by replacing the free Green's function of the photon by a photon Green's function that takes into account the interaction with the field of the laser beam. No account was taken of the finite lifetime of the state with definite quasienergy. In this section we estimate the integral cross section of the Moller scattering with allowance for the radiation width of the state of the electron.

Let us consider one of the denominators of (10) of I, for example, $B_{s_1} = (\tilde{p}_i^1 - \tilde{p}_f^1 - s_1 k')^2$. The substitutions $\epsilon_i \to \epsilon_i - i\Gamma$ and $\epsilon_f^* \to \epsilon_f^* + i\Gamma$ transform this expression into

$$B_{s_1} + i4 s_1 k_0' \Gamma. \tag{40}$$

Recognizing that in the nonrelativistic approximation we have the approximate equality $\tilde{\theta}^1 \approx 2\tilde{\theta}$ (s = 0) we have, in succession (the notation is that of I)

$$\begin{aligned} B_{s_1} &= (\tilde{p}_i^1 - \tilde{p}_f^1)^2 - 2s_1 k'(\tilde{p}_i - \tilde{p}_f) \approx -2(\tilde{\mathbf{p}}_i^1)^2 + 2|\tilde{\mathbf{p}}_i^1||\tilde{\mathbf{p}}_f^1|\cos\tilde{\theta}^1 \\ &+ 2s_1\omega(\mathbf{n}\tilde{\mathbf{p}}_i - \mathbf{n}\tilde{\mathbf{p}}_f) \approx 2[2(\tilde{\mathbf{p}}_i^1)^2(\cos^2\tilde{\theta} - 1) \\ &+ s_1\omega(|\tilde{\mathbf{p}}_i|\cos\tilde{\varphi}_i - |\tilde{\mathbf{p}}_f|\cos\tilde{\varphi}_f)], \\ &\cos\tilde{\varphi}_i = \mathbf{n}\tilde{\mathbf{p}}_i/|\tilde{\mathbf{p}}_i|, \quad \cos\tilde{\varphi}_f = \mathbf{n}\tilde{\mathbf{p}}_f/|\tilde{\mathbf{p}}_f|. \end{aligned} \tag{41}$$

We put $x = \cos\tilde{\varphi}_f$. If resonance is observed when $x = x^* = \cos\tilde{\varphi}_f^*$, then for a small deviation from resonance, $x = x^* + \Delta x$, we get

$$\begin{aligned} B_{s_1}|_{x=x^*+\Delta x} &\approx \Delta x\{-2s_1\omega|\tilde{\mathbf{p}}_f| + 2(|\tilde{\mathbf{p}}_i|^2\cos\tilde{\theta} - s_1\omega|\tilde{\mathbf{p}}_i|\cos\tilde{\varphi}_f^*) \\ &\times \left[\cos\tilde{\varphi}_i - \frac{\cos\tilde{\varphi}_f^*}{\sin\tilde{\varphi}_f^*}\sin\tilde{\varphi}_i\cos(\tilde{\zeta}_i - \tilde{\zeta}_f)\right]\Bigg\}, \\ &\tilde{\mathbf{p}}_i = \tilde{\mathbf{p}}_i(\tilde{\varphi}_i, \tilde{\zeta}_i), \quad \tilde{\mathbf{p}}_f = \tilde{\mathbf{p}}_f(\tilde{\varphi}_f, \tilde{\zeta}_f), \\ &|\tilde{\mathbf{p}}_f| \approx |\tilde{\mathbf{p}}_i|\cos\tilde{\theta} \quad (\text{for } s = 0). \end{aligned} \tag{42}$$

The vector $\mathbf{n}$ is directed here along the z axis. To estimate the resonance width, we choose the angles $\tilde{\varphi}_f^*$ and $\tilde{\varphi}_i$ in the vicinity of $\pi/2$ and $\pi/4$, respectively. Then (42) yields

$$B_{s_1}|_{x=x^*+\Delta x} \approx 2|\tilde{\mathbf{p}}_f|\Delta x\{-s_1\omega + |\tilde{\mathbf{p}}_i|/\sqrt{2}\}. \tag{43}$$

From this we get for the resonance width Δx

$$\Delta x = 2s_1\Gamma/\delta|\tilde{\mathbf{p}}_f|, \quad \omega\delta = |s_1\omega - |\tilde{\mathbf{p}}_i|/\sqrt{2}|. \tag{44}$$

This expression is valid when $\delta \sim 1$. With the aid of a relation similar to (36) we obtain the following estimate for the integral cross section of the Moller scattering (see relation (18) of I):

$$\begin{aligned} \sigma|_{s_1=1} &\sim \frac{1}{\alpha} r_0^2 \left(\frac{m}{\omega}\right)^4 \left(\frac{e|\mathbf{a}|}{m}\right)^2 \quad \text{for } |\tilde{\mathbf{p}}_i| \gtrsim \omega, \ s = 0; \\ \sigma_{s_1=2}/\sigma_{s_1=1} &\sim (e|\mathbf{a}|/m)^4. \end{aligned} \tag{45}$$

at an electric field intensity $|\mathbf{E}| \sim 10^5$ V/cm and a frequency $\omega \sim 3 \times 10^{15}$ sec^{-1} we get

$$\sigma_{s_1=1} \sim 5\cdot 10^{12} r_0^2. \tag{46}$$

In conclusion, the author thanks Ya. B. Zel'dovich for interest in the work and a fruitful discussion and V. M. Buimistrov for numerous remarks during the discussion of the paper.

APPENDIX I

We present some of the relations used in the paper for the bispinors:

$$\begin{aligned} \bar{u}_{p\sigma_1}\hat{a}u_{p\sigma_2} &= \frac{ap}{\varepsilon_p}\delta_{\sigma_1\sigma_2}, \\ u^+_{p\sigma_1}\hat{n}^+\hat{a}^+\hat{a}\hat{n}u_{p\sigma_2} &= -\frac{2a^2 np}{\varepsilon_p}\delta_{\sigma_1\sigma_2} \\ (\hat{n}^2 = 0, &\quad \varepsilon_p > 0). \end{aligned} \tag{A1.1}$$

The bispinors satisfy the following orthogonality and normalization conditions:

$$\begin{aligned} u^+_{p\sigma_1}u_{p\sigma_2} &= \delta_{\sigma_1\sigma_2}, \\ \bar{u}_{p\sigma_1}u_{p\sigma_2} &= \frac{m}{\varepsilon_p}\delta_{\sigma_1\sigma_2}. \end{aligned} \tag{A1.2}$$

APPENDIX II

We calculate the integral ($\varepsilon', \varepsilon'' \to +0$)

$$X = \int_{-\infty}^{+\infty} dk_0 B^{\mu\nu}_{r,r}(p, p+k) \times \frac{1}{k^2 + i\varepsilon'} \frac{1}{(p + k + 2\varkappa_{2p}\omega n - r\omega n)^2 - m^{*2} + i\varepsilon''}. \tag{A2.1}$$

To this end, we represent the function $B^{\mu\nu}_{r,r}(p, p+k)$ (29) in the form

$$\begin{aligned} B^{\mu\nu}_{r,r}(p, p+k) &= \frac{1}{(2\pi)^2}\int_{-\pi}^{+\pi} d\varphi_1 d\varphi_2\, e^{ir(\varphi_1+\varphi_2)}\cos^\mu\varphi_1\cos^\nu\varphi_2 \\ &\times \exp i\left[\varkappa_{1p}(\sin\varphi_1 + \sin\varphi_2) + \varkappa_{2p}(\sin 2\varphi_1 + \sin 2\varphi_2) + \frac{\beta(k)}{np + nk}\right], \\ \beta(k) &= -\frac{ea(p+k)}{\omega}(\sin\varphi_1 + \sin\varphi_2) + \frac{e^2a^2}{8\omega}(\sin 2\varphi_1 + \sin 2\varphi_2). \end{aligned} \tag{A2.2}$$

Let us consider the expression (we close the integration contour in the upper half-plane)

$$\begin{aligned} X^{(n)} &= \int_{-\infty}^{+\infty} dk_0 \left[\frac{i\beta(k)}{np + nk + i\varepsilon}\right]^n \times \frac{1}{k^2 + i\varepsilon'}\frac{1}{(p + k + 2\omega n\varkappa_{2p} - r\omega n)^2 - m^{*2} + i\varepsilon''} \\ &= 2\pi i\left\{-\frac{1}{-2|\mathbf{k}|}\left[\frac{i\beta(k)}{np + nk + i\varepsilon}\right]^n\right. \end{aligned}$$

$$\times \frac{1}{(p+k+2\omega n\varkappa_{2p}-r\omega n)^2-m^{*2}+i\varepsilon''}\Bigg|_{k_0=-|\mathbf{k}|}$$
$$+\frac{1}{-2[(\tilde{\mathbf{p}}+\mathbf{k}-r\omega\mathbf{n})^2+m^{*2}]^{1/2}}\left[\frac{i\beta(k)}{np+nk+i\varepsilon}\right]^n$$
$$\times \frac{1}{k^2+i\varepsilon'}\Bigg|_{k_0=r\omega-\tilde{p}_0-[(\tilde{\mathbf{p}}+\mathbf{k}-r\omega\mathbf{n})^2+m^{*2}]^{1/2}}\Bigg\};\qquad (A2.3)$$
$$\varepsilon>0,\quad \tilde{p}=p+2\omega n\varkappa_{2p}.$$

Using (A2.2) and (A2.3), we get

$$X=\frac{1}{(2\pi)^2}\int_{-\pi}^{+\pi}d\varphi_1\,d\varphi_2\exp\{ir(\varphi_1+\varphi_2)\}\cos^{\mu}\varphi_1\cos^{\nu}\varphi_2 \qquad (A2.4)$$
$$\times\exp i[\varkappa_{1p}(\sin\varphi_1+\sin\varphi_2)+\varkappa_{2p}(\sin 2\varphi_1+\sin 2\varphi_2)]\lim_{\varepsilon\to+0}\sum_{n=0}^{\infty}\frac{1}{n!}X^{(n)}$$
$$=-\pi i\left\{\frac{1}{|\mathbf{k}|}\left[B^{\mu\nu}_{r,r}(p,p+k)\frac{1}{(p+k+2\omega n\varkappa_{2p}-r\omega n)^2-m^{*2}+i\varepsilon''}\right]\Bigg|_{k_0=-|\mathbf{k}|}+\right.$$
$$\left.+\left[\frac{1}{[(\tilde{\mathbf{p}}+\mathbf{k}-r\omega\mathbf{n})^2+m^{*2}]^{1/2}}\frac{B^{\mu\nu}_{r,r}(p,p+k)}{k^2+i\varepsilon'}\right]\Bigg|_{k_0=r\omega-\tilde{p}_0-[(\tilde{\mathbf{p}}+\mathbf{k}-r\omega\mathbf{n})^2+m^{*2}]^{1/2}}\right\}.$$

REFERENCES

[1] V. P. Oleĭnik, Zh. Eksp. Teor. Fiz. **52**, 1049 (1967) [Sov. Phys.-JETP **25**, 697 (1967)].

[2] L. S. Brown and T. W. B. Kobble, Phys. Rev. **133**, A705 (1964).

[3] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **46**, 776 (1964) [Sov. Phys.—JETP **19**, 529 (1964)]. N. B. Narozhnyĭ, A. I. Nikishov, and V. I. Ritus, ibid. **47**, 930 (1964) [**20**, 622 (1965)]. A. I. Nikishov and V. I. Ritus, ibid. **47**, 1130 (1964) [**20**, 757 (1965)].

[4] I. I. Gol'dman, ibid. **46**, 1412 (1964) [**19**, 954 (1964)]. I. I. Gol'dman, Voprosy fiziki élementarnykh chastits (Problems of Elementary-particle Physics), AN ArmSSR, 1964, p. 456.

[5] Z. Fried and J. S. Eberly, Phys. Rev. **136**, B871 (1964).

[6] V. M. Buimistrov and V. P. Oleĭnik, Fiz. Tekh. Poluprov. **1**, 85 (1967) [Sov. Phys.—Semicond. **1**, 65 (1967)].

[7] R. J. Glauber, Phys. Rev. **130**, 2529 (1963); **131**, 2766 (1963).

[8] T. W. B. Kibble, Phys. Rev. **138**, B740 (1965).

[9] L. M. Frantz, Phys. Rev. **139**, B1326 (1965).

[10] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika (Quantum Electrodynamics), Fizmatgiz, 1959 [Interscience, 1963].

[11] Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **51**, 1492 (1966) [Sov. Phys.—JETP **24**, 1006 (1967)].

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