DISPERSION OF SPIN WAVES IN METALS IN A STRONG MAGNETIC FIELD

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It is shown that at wavelengths larger than the Larmor radius of the conduction electrons, there occurs a strong renormalization of the spin-wave spectrum, equivalent to an increase of the exchange constant and change of its sign. At large magnon momenta, allowance for orbital motion of the electrons leads to no significant change of the spin-wave spectrum.

In consideration of the Bose branches of the energy spectrum of metals in a magnetic field, an important factor to be taken into account is orbital motion of the conduction electrons. In the phonon spectrum, for example, this leads to a whole series of resonance effects—geometrical resonance oscillations,[1] acoustical cyclotron resonance (ACR),[2] giant quantum oscillations,[3, 4] etc. It is shown below that allowance for the orbital motion of the conduction electrons in a ferromagnetic metal leads to a comparatively large renormalization of the spin-wave spectrum, independent of the magnetic field. This effect is essentially analogous to the renormalization of the phonon spectrum in a strong magnetic field.[5]

In consideration of the spin-wave spectrum, we shall start from the fact that there exist in a ferromagnetic metal two types of Fermi excitations: s- and d-electrons, characterized respectively by small and large densities of states. Exchange interaction leads to ordering of the spins of the conduction electrons. The Fermi surfaces corresponding to the two possible orientations of the spins of the s-electrons are, as a rule, separated by an amount of order $\sqrt{\Theta\varepsilon_F}$, where Θ is the Curie energy and ε_F is the Fermi energy of the conduction electrons. This same quantity determines the constant of interaction between the s-electrons and the spin waves.

On carrying out a calculation similar to that made in [6], we arrive at the equation that determines the spin-wave spectrum:

$$\omega = \gamma H + \alpha k^2 + \Pi(\omega, \mathbf{k}); \tag{1}$$

here ω and $\mathbf{k}$ are, respectively, the frequency and the wave vector of the spin wave, γ is the gyromagnetic ratio, H is the magnitude of the external magnetic field, and $\alpha \sim \Theta a^2$ is the exchange constant (a is the lattice constant).

The polarization operator $\Pi(\omega, \mathbf{k})$, according to [6], has the form

$$\Pi(\omega, \mathbf{k}) = g^2[GG-(GG)^\omega] = -i\frac{g^2}{(2\pi)^7}\int d\varepsilon\, d\mathbf{p}\, d\mathbf{p}' \cdot$$
$$\times [G_+(\varepsilon+\omega, \mathbf{p}+\mathbf{k}, \mathbf{p}')G_-(\varepsilon, \mathbf{p}'-\mathbf{k}, \mathbf{p}) - G_+(\varepsilon+\omega, \mathbf{p}, \mathbf{p}')G_-(\varepsilon, \mathbf{p}', \mathbf{p})]. \tag{2}$$

Here g is the electron-magnon coupling constant, $G_\pm$ are the Green's functions of the conduction electrons with oppositely oriented spins, and $(GG)^\omega$ is the limit of the integral of the product of the Green's functions as $\mathbf{k} \to 0$. In going over to the momentum representation, account was taken of the fact that in a magnetic field, the electronic Green's function does not depend solely on the difference of the spatial coordinates. On using the expression for the Green's function of an electron in a magnetic field and on integrating (2) over frequency and over the transverse components of momentum, we get

$$\Pi(\omega, \mathbf{k}) = -\frac{g^2}{(2\pi)^2}a_+a_-\frac{eH}{c}$$
$$\times \sum_{lm}\left\{|M_{lm}(\rho)|^2\int dp\frac{n(\varepsilon_l^+(p+q)) - n(\varepsilon_m^-(p))}{\omega - \varepsilon_l^+(p+q) + \varepsilon_m^-(p) + i\delta}\right.$$
$$\left. - \int dp \frac{n(\varepsilon_l^+(p)) - n(\varepsilon_l^-(p))}{\omega - \varepsilon_l^+(p) + \varepsilon_l^-(p) + i\delta}\delta_{lm}\right\}, \tag{3}$$

where $M_{lm}(\rho) = L_{\min(l,m)}^{|m-l|} e^{-\rho}\rho^{|m-l|}$ is the matrix element used in [4], $\rho = ck_\perp^2/2eH$, $q \equiv k_z$, $n(\varepsilon)$ is the Fermi distribution function at temperature zero, $a_\pm$ are renormalization constants of order unity,[7] and $\varepsilon_n^\pm(p)$ is the energy of an electron in a magnetic field:

$$\varepsilon_n^\pm(p) = \frac{v_\pm}{2p_\pm}\left[\frac{2eH}{c}\left(n+\frac{1}{2}\right) + p_z^2 - p_\pm^2\right]. \tag{4}$$

Here $v_\pm$ and $p_\pm$ are, respectively, the velocity and the momentum on the Fermi surface (for simplicity, we consider the dependence of the electron energy on momentum to be isotropic).

According to [6], the energy difference between electrons with oppositely oriented spins is

$$\varepsilon_n^+(p) - \varepsilon_n^-(p) \sim \sqrt{\Theta\varepsilon_F}. \tag{5}$$

Such a large "distance" between the Fermi surfaces precludes the existence of resonance effects similar to those that occur in the spectrum and precludes phonon damping.

1. We consider the possible singularities of the integral in the expression (3); we consider first the case q = 0. When q = 0, the denominator of the integrand in (3) can vanish when $m - l \sim \sqrt{\Theta\varepsilon_F}/\Omega$, where Ω is the cyclotron frequency. On the other hand, the matrix element M_{lm} behaves in the following fashion. When

$$[(l+m)\rho]^{1/2} > |m-l| \tag{6}$$

it is an oscillatory function, whereas in the contrary case the matrix element is exponentially small. From this it follows that cyclotron resonance in the spin-wave spectrum can occur only when $k \gtrsim \Delta = p_+ - p_-$. Beginning with $k = \Delta$, however, a collisionless, threshold-type extinction of spin waves, equal in order of magnitude to $\omega(\Theta/\varepsilon_F)^{1/2}$, becomes important. For this reason, no singularity of the ACR type is present in the magnon spectrum. In a similar manner, the great "distance" between the Fermi surfaces excludes the possibility of Pippard oscillations.

We consider next the case $2(l + m)\rho]^{1/2} \ll 1$, which corresponds practically to the condition

$$kR \ll 1, \tag{7}$$

where $R = cp_0/eH$ is the cyclotron radius. In this case, as follows from (6), it is necessary to retain in the sum (3) only the terms with $m - l = 1, 0, -1$. On the other hand, the summation over l may be replaced by integration. As a result we arrive at the following formula:

$$\Pi = -\frac{g^2}{(2\pi)^2}\frac{eH}{c}\int dl\left\{\sum_{\alpha=1,0,-1}|M_{l+\alpha,l}|^2\int dp\,\frac{n(\varepsilon^+_{l+\alpha}(p)) - n(\varepsilon_l^-(p))}{\omega - \varepsilon^+_{l+\alpha}(p) + \varepsilon_l^-(p)} - \int dp\,\frac{n(\varepsilon_l^+(p)) - n(\varepsilon_l^-(p))}{\omega - \varepsilon_l^+(p) + \varepsilon_l^-(p)}\right\}. \tag{8}$$

In the quasiclassical approximation, the matrix element has the form

$$M_{lm}^2(\rho) = J^2_{l-m}(\sqrt{2(l+m+1)\rho}), \tag{9}$$

where $J_n(x)$ is a Bessel function. On expanding $J_n(x)$ in (8) for small kR and retaining terms to and including the second order, we get after integration over p and l:

$$\Pi = -\frac{a_+a_-}{(2\pi)^2}\frac{g^2\Delta}{\Theta}k^2. \tag{10}$$

An estimate of the s – d exchange-interaction constant was obtained in [6]:

$$g^2 = \Theta\varepsilon_F\mu_0/M \sim \Theta\varepsilon_F/p_0^3, \tag{11}$$

where μ_0 is the Bohr magneton and M is the total magnetic moment of the metal. On using this estimate, we finally find

$$\Pi \approx -\sqrt{\Theta\varepsilon_F}k^2/p_0^2. \tag{12}$$

As is seen from a comparison of formulas (12) and (1), the renormalization of the spin-wave spectrum that results from the orbital motion of the electrons in a magnetic field is larger by a factor $(\varepsilon_F/\Theta)^{1/2}$ than the term αk^2 that is present in the absence of a magnetic field, and it has the opposite sign.

We recall that the expression (12) relates only to the initial part of the dispersion curve, limited by the condition (7). The figure represents schematically the dependence of ω on k for $k < \Delta$ (the mark on the axis of abscissas represents a value $kR \sim 1$). As is seen from the figure, the dispersion curve of the spin waves has a minimum, whose relative value is $\Delta\omega/\omega \sim \sqrt{\Theta/\varepsilon_F}\,\Omega/\varepsilon_F$.

Obviously one can speak of a correction to the spectrum only in a case in which the change $\Delta\omega$ in the spectrum is large in comparison with the damping Im ω. As has already been mentioned, at small magnon momenta, $k < p_+ - p_-$, damping caused by breaking up of a spin wave into an electron and a "hole" is absent. Another damping mechanism, connected with many-particle processes (for example, breaking up of a spin wave into a spin wave and an electron-hole pair or into several spin waves) leads, as estimates show, to a value of Im ω proportional at least to the square of the spectral change, $(\Delta\omega)^2$. Therefore the most important mechanism of spin-wave damping in metals at small momenta is apparently dissipation connected with the finite conductivity of the metal. Allowance for the finite conductivity can be made by solution of Maxwell's equations for a ferromagnetic metal whose magnetic permeability is determined by the microscopic treatment given above. Solution of Maxwell's equations leads to the following expression for the spin-wave spectrum and damping:

$$\omega = \gamma B - \sqrt{\Theta\varepsilon_F}\frac{k^2}{p_0^2} - \frac{c^2k^2}{\omega_0^2}2\pi\gamma M\omega\left(1 + i\frac{\nu}{\omega}\right), \tag{13}$$

where $\omega_0^2 = 4\pi Ne^2/m$ is the square of the plasma frequency of the metal, ν is the electron collision frequency, and $B = H + 4\pi M$.

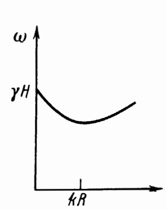
As is evident from formula (13), the change in the spectrum is large in comparison with the damping in magnetic fields that satisfy the condition

$$H > M\sqrt{\frac{\varepsilon_F}{\Theta}\frac{mc^2}{\sigma_0}} \tag{14}$$

where σ_0 is the static conductivity of the metal. For good metals ($N \sim 10^{23}$ cm^{-3}, $\tau \sim 10^{-9}$ sec, $M \sim 10^2$ to 10^3 G), this leads to the estimate $H > 10^3$ to 10^4 Oe.

We remark that interaction of spin waves with the electromagnetic field, in the strong-field range under consideration ($\omega\tau \gg 1$), causes an additional renormalization of the spin-wave spectrum, not connected with the Fermi-liquid interaction. As is evident from the expression (13), electromagnetic interaction leads to an additional amplification of the minimum on the dispersion curve $\omega(\mathbf{k})$.

The absolute value of the minimum ΔH is quite small, and in fields $H \sim 10^5$ Oe it is of the order of a few Oersteds. Nevertheless the minimum can be observed in sufficiently pure ferromagnetic metals with a path length of the order of a few millimeters, at low temperatures, in magnetic fields of order 10^4 to 10^5 Oe. An experiment that may be suitable for this purpose is one on reflection of an electromagnetic wave from a semi-infinite ferromagnetic metal. As was shown in reference [8], the existence of an extremum on the dispersion curve—a termination point of the spectrum—leads to the formation of a standing electromagnetic wave in the volume of the metal and, correspondingly, to complete reflection of the wave from the metal surface. Thus a minimum on the dispersion curve should manifest itself in the existence of a singularity of the surface impedance at the displaced ferromagnetic-resonance frequency.



2. We now consider the case in which the wave vector of the spin wave is parallel to the direction of the magnetic field. In this case $M_{nm}(\rho) = \delta_{nm}$, and the expression (3) takes the form

$$\Pi(\omega, \mathbf{k}) = -\frac{g^2}{(2\pi)^2} a_+ a_- \frac{eH}{c} \sum_{l=0}^{\infty} \left\{ \int dp \frac{n(\varepsilon_l^+(p+q)) - n(\varepsilon_l^-(p))}{\omega + \varepsilon_0 - kp/m + i\delta} - \int dp \frac{n(\varepsilon_l^+(p)) - n(\varepsilon_l^-(p))}{\omega + \varepsilon_0} \right\}. \quad (15)$$

For simplicity, we have taken the effective masses of electrons with oppositely directed spins to be equal, and we have designated by ε_0 the "distance" between the Fermi surfaces.

For small magnon momenta, the expression (15) for the polarization operator describes quantum oscillations in the spin-wave spectrum, of the de Haas–van Alphen type. On going over from a sum to an integral, with the aid of Poisson's formula, and carrying out the integration over p, we get

$$\Pi = -\frac{g^2}{(2\pi)^2} a_+ a_- \frac{(2\pi m)^{1/2} \Omega^{3/2}}{\varepsilon_0^2} k^2 \times \sum_{l=1}^{\infty} \frac{1}{(2\pi l)^{3/2}} \exp\left\{ 2\pi l i \left(\frac{\varepsilon_F + \varepsilon_0}{\Omega} + \frac{3\pi}{4} \right) \right\}. \quad (16)$$

As is seen by comparison of formulas (16) and (1), the relative amplitude of the oscillatory term is

$$\frac{\mathrm{Re}\,\Pi}{\alpha k^2} \sim \frac{\varepsilon_F}{\Theta} \left(\frac{\Omega}{\varepsilon_F} \right)^{3/2}. \quad (17)$$

We shall discuss the situation for large magnon momenta. When $k \gtrsim \Delta$, there are in the spin-wave spectrum singularities connected with the possibility of a spin wave's breaking up into an electron and a "hole" near the Fermi surface.[6] This singularity is analogous to a Kohn singularity in the phonon spectrum.[9] In a magnetic field, emission by a phonon of an electron-hole pair leads[3, 4] to giant quantum oscillations in the damping of phonons and to singularities in the phonon spectrum. In the spectrum and damping of magnons, however, giant oscillations are impossible, as before, because of the great distance between the Fermi surfaces. Already at values of the spin-wave momentum of order Δ, the term αk^2 becomes appreciably larger than the Zeeman term γH. Then the width of the interval in which the damping due to breaking up of a spin wave into an electron and a hole differs from zero is equal to $\omega/\Omega \sim \alpha k^2/\Omega \approx \Theta^2/\Omega\varepsilon_F \gg 1$; that is, it is large in comparison with the distance between neighboring peaks in the damping. As a result, all the "columns" in the damping overlap, and the total damping is the same as in the absence of a magnetic field.

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[1] A. B. Pippard, Phil. Mag. **2**, 1147 (1957).
[2] É. A. Kaner, Zh. Eksp. Teor. Fiz. **43**, 216 (1962) [Sov. Phys.-JETP **16**, 154 (1963)].
[3] V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **40**, 786 (1961) [Sov. Phys.-JETP **13**, 552 (1961)].
[4] A. Ya. Blank and É. A. Kaner, Zh. Eksp. Teor. Fiz. **50**, 1013 (1966) [Sov. Phys.-JETP **23**, 673 (1966)].
[5] I. O. Kulik, Zh. Eksp. Teor. Fiz. **47**, 107 (1964) [Sov. Phys.-JETP **20**, 73 (1965)].
[6] P. S. Kondratenko, Zh. Eksp. Teor. Fiz. **50**, 769 (1966) [Sov. Phys.-JETP **23**, 509 (1966)].
[7] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike [Methods of Quantum Field Theory in Statistical Physics], Fizmatgiz, 1962. [Translation: Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963.]
[8] A. Ya. Blank and M. I. Kaganov, Zh. Eksp. Teor. Fiz. **49**, 807 (1965) [Sov. Phys.-JETP **22**, 561 (1966)].
[9] W. Kohn, Phys. Rev. Lett. **2**, 393 (1959).

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