PRODUCTION OF COHERENT LIGHT BY ATOMIC OR MOLECULAR BEAMS

V. S. LETOKHOV and B. D. PAVLIK

P. N. Lebedev Physics Institute, U.S.S.R. Academy of Sciences

Submitted April 7, 1967

Zh. Eksp. Teor. Fiz. 53, 1107–1115 (September, 1967)

The production of coherent light waves of stable frequency by means of an atomic (or molecular) beam is investigated theoretically. Inversion of the atomic-level population in the beam by a coherent beam is considered and the dynamics of generation and the frequency stability of a beam laser with coherent excitation is investigated. It is shown that such a laser may potentially be a source of light waves with a very stable frequency (up to 10^{-12}). The potentialities of beam lasers with incoherent excitation are considered.

INTRODUCTION

As is well known, beams of atoms or molecules are used in quantum generators operating in the radio band.[1, 2] The use of beams makes it possible, first, to obtain very narrow spectral lines, of width smaller by several orders of magnitude than the width of the resonator line, and, second, to sort the molecules by levels in order to create an inverted population. The first property is exceedingly important, since it has made it possible to develop highly stable frequency standards. The development of quantum generators for the optical band followed a different path. The inverted population of the optical levels of the atoms or molecules can be obtained by numerous incoherent pumping methods (using light, current, etc.), without the need for using beams,[1)] but at the cost of losing the narrowness of the spectral line of the active medium, and at the same time the stability of the generation frequency. In lasers, to the contrary, the spectral width of the active medium is as a rule broader than the resonator line and consequently the generation frequency is determined by the resonator frequency or, in other words, by the resonator dimensions. Therefore the frequency stability of lasers is worse by several orders of magnitude than the frequency stability of masers.

To construct lasers with a stable emission frequency (optical frequency standard) it is necessary to change the ratio of the spectral line widths of the active medium and the resonator. It was proposed in [4] to use for this purpose an atom beam[2)] excited by optical radiation as the active medium with a very narrow emission line. It was noted in [4] that when the beam is excited with coherent radiation it is possible to obtain a narrow spectral line, of width much smaller than the Doppler width of the beam and determined only by the travel time of the atoms through the exciting field. To obtain high frequency stability it was possible to use as the coherent pump radiation the amplified emission of the beam laser itself (Fig. 1).

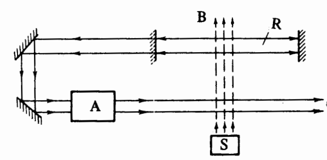

FIG. 1. Diagram of beam laser with coherent excitation: S—Beam source, B—beam of atoms or molecules, R—resonator, A—quantum amplifier, O—output.

In this paper we consider theoretically the problem of generating coherent light by beams of atoms (molecules). We consider the excitation of atoms in a beam by coherent radiation and investigate the dynamics of the generation and the stability of the frequency of a beam laser with coherent pumping (Fig. 1). We show that such a laser can serve potentially as a source of optical oscillations of very high frequency stability (to 10^{-12}). We consider also the possibilities of using beam lasers with incoherent excitation.

2. CURRENT BEAM EXCITATION

The idea of coherent excitation of atoms in a beam consists in the following. The atoms cross, parallel to the wave front, a ray of coherent light of frequency ω which coincides with the frequency ω_{21} of transition of the atoms to an excited long-lived state. Since there is no relaxation, the atom oscillates in the resonance field between two levels, with a frequency Ω proportional to the intensity of the optical field.[7] If the time τ_0 that the atoms stay in the ray coincides with the half-period of the oscillations π/Ω, then the initially unexcited atom becomes excited after passing through the light ray. Such an excitation method is analogous to the known nuclear-magnetic-resonance method of inverting the spin population by 180°.[8] In our case, the pulsed character of the excitation is connected with the finite time that the atoms stay in the ray, and not with the pulsed switching of the field. The short length of the light wave leads to a unique shape of the line of the beam after passing through the light ray.

[1)]A quantum generator for the submillimeter band, using a beam of molecules, was proposed by A. M. Prokhorov [3].

[2)]The use of forbidden optical transitions between lower levels of the atoms was discussed in [2]. In the infrared region, it is possible to use transitions between vibrational levels of the molecules. Particularly attractive are the transitions coinciding with the emission lines of large-gain lasers. For example, the 2947.906 cm^{-1} absorption line of CH_4 molecules coincides within 0.003 cm^{-1} with the λ=3.3913 μ line of the helium-neon laser [5], and the 2850.608 cm^{-1} absorption line of H_2CO molecules coincides within 0.09 cm^{-1} with the λ=3.5070 μ helium-xenon laser emission lines [6].

Let an atom moving with velocity $\mathbf{v}$ enter at the instant of time t_0 into a light wave $E_0 \exp[i(\mathbf{k}\cdot\mathbf{r} + \omega t)]$ at a point $\mathbf{r}_0$. The energy of the resonance interaction between the atom and the field is

$$\hbar V(t) = -p_{12}E_0 \exp\{i\mathbf{k}[\mathbf{r}_0 + \mathbf{v}(t - t_0)] + i\omega t\}, \quad (1)$$

where p_{12} is the matrix element of the dipole moment of the atom between states 1 and 2. Considering the resonance interaction during the time of flight of the atom through the wave $\tau = a/v$, it is easy to calculate the probability amplitudes of a_1 and a_2 of finding the atom at the levels 1 and 2[7] (a—diameter of the ray). In particular, the expression for the inversion is

$$|a_2|^2 - |a_1|^2 = -1 + (2p_{12}E_0/\hbar\Omega)^2(1 - \cos\Omega\tau), \quad (2)$$

$$\Omega = [(\omega - \omega_0 + \mathbf{kv})^2 + (2p_{12}E_0/\hbar)^2]^{1/2}, \quad (3)$$

where $\omega_0 = (E_2 - E_1)/\hbar$. The Doppler frequency shift $\mathbf{k}\cdot\mathbf{v}$ is connected with the fact that the atom velocity $\mathbf{v}$ is parallel to the wave front of the ray. If φ is the angle between the velocity $\mathbf{v}$ and the wave surface of the ray, then $\mathbf{k}\cdot\mathbf{r} = kv\varphi$ ($\varphi \ll 1$).

Let us consider first the case of resonant excitation ($\omega = \omega_0$) of an ideally collimated beam which experiences no Doppler broadening ($kv\varphi \ll 1/\tau$). The average population inversion per atom, Λ, of such a beam is determined by averaging (2) over the velocity distribution $w_0(v)$ of the atoms:

$$\Lambda = \langle |a_2|^2 - |a_1|^2 \rangle. \quad (4)$$

The velocity distribution in the beam is given by [9]

$$w_0(v) = \frac{9}{2}\frac{v^3}{v_0^4}\exp\left(-\frac{3}{2}\frac{v^2}{v_0^2}\right), \quad (5)$$

where $v_0 = \sqrt{3kT/M}$—most probable velocity of the atoms in the beam. It is obvious that the inversion Λ depends on the ratio γ of the average time of flight of the atom through the light ray, $\tau_0 = a/v_0$, to the total inversion time $\tau_i = \pi/\Omega = \pi\hbar/2p_{12}E_0$, and is maximal when $\gamma = \tau_0/\tau_i = 1$. The exact $\Lambda(\gamma)$ dependence, obtained by numerically averaging (2) over distribution (5), is shown in Fig. 2. The maximum possible inversion is $\Lambda_{max} \approx 0.5$.

Let us consider now the case of a beam of atoms with finite angular divergence φ_0. When $\varphi_0 > 2/kv_0\tau_0 = \lambda/\pi a$, the Doppler width of the beam line $\Delta\omega_D = \varphi_0 kv_0$ exceeds the line width $\Delta\omega_{tr} = 2/\tau_0$ due to the finite travel time of the atoms through the exciting ray, that is, the line has inhomogeneous broadening. It can be expected that the only inverted atoms will be those whose Doppler frequency displacement lies within the limits of the homogeneous width $\Delta\omega_{tr}$ relative to the field frequency ω. The dependence of the atom inversion Λ on the frequency $\omega' = \omega_0 + kv\varphi$ is determined by expression (2) averaged over the velocity distribution

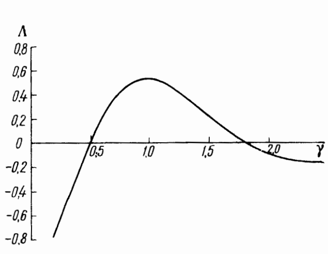

FIG. 2. Inversion of atoms in the beam after passage of the exciting ray vs. the parameter $\gamma = 2ap_{12}E_0/\pi\hbar v_0$.

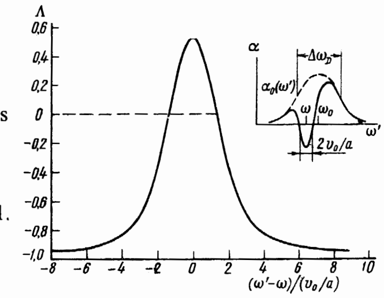

FIG. 3. Inversion of atoms in the beam after passage of exciting ray vs. detuning of transition frequency relative to the field frequency at $\gamma = 1$.

(5). Figure 3 shows the $\Lambda(\omega')$ dependence obtained by numerical averaging at the optimal $\gamma = 1$. In the upper part of Fig. 3 is shown the beam absorption line shape after passing through the ray: $\alpha(\omega') = \Lambda(\omega')\alpha_0(\omega')$, where $\alpha_0(\omega)$ is the initial beam absorption line shape, determined by the angular distribution of the transverse atom-velocity component $v_\perp = \mathbf{v}\cdot\mathbf{k}/k$. The maximum-gain frequency ω_m is determined by the maximum of the product $\Lambda(\omega')\alpha_0(\omega')$. If the field frequency ω does not coincide with the center of the absorption line ω_0, the frequency of the maximum gain ω_m is "pulled" towards the center of the line:

$$\omega_m = \omega_0 + \frac{\Delta\omega_{tr}}{\Delta\omega_D}(\omega - \omega_0). \quad (6)$$

An essential feature of the line shape of the excited beam is the narrowness of the negative-absorption line. The line width is extremely small, since it is determined only by the travel time through the exciting ray. We note that in the case of incoherent excitation of the beam atoms (for example, by optical or thermal pumping[10]) the negative-absorption line width coincides with the initial absorption line width $\Delta\omega_D$ inasmuch as the excitation probability does not depend on the direction of the atom velocity $\mathbf{v}$. When the real beam divergence is $\varphi_0 > 10^{-2}$ rad, we have $\Delta\omega_D \gg \Delta\omega_{tr}$. Therefore the highest potential frequency stability is possessed by a beam laser with coherent excitation. However, it is impossible to realize high stability if the source of the coherent radiation is another laser, since the frequency fluctuations of the latter will lead to a change in the frequency of the maximum gain of the beam. There is a way of getting around this difficulty[4] by using as the exciting radiation the coherent radiation from the beam laser itself, first amplified by means of an optical quantum amplifier. We investigate below the dynamics of the generation and the frequency stability of such a closed system.

3. DYNAMICS OF GENERATOR WITH COHERENT EXCITATION

The excited atom beam enters a closed cavity resonator, whose resonance range includes the beam negative-absorption line. In order for all the inversely populated atoms to interact with the field in the resonator, and to minimize the role of the absorbing atoms, the travel time through the ray in the resonator, T, is chosen equal to τ_0. The analysis can then be limited to the amplifying atoms, and their emission line can be regarded as homogeneously broadened with center at the frequency ω_m.

In this approximation, the interaction between the field in the cavity and the atom beam can be described by a system of three equations which relates the field intensity E, the polarization P, and the inverted-population density N of the atoms in the cavity:[11, 12]

$$\begin{gathered} \ddot{E}+\frac{\omega}{Q}\dot{E}+\omega_r^2E=-4\pi\ddot{P}, \\ \ddot{P}+\frac{2}{\tau_0}\dot{P}+\omega_m^2P=-2\frac{\omega p_{12}^2}{\hbar}NE, \\ \dot{N}+\frac{1}{\tau_0}[N-N_0(E)]=\frac{2}{\hbar\omega}E\dot{P}, \end{gathered} \tag{7}$$

where Q and ω_r are the figure and merit and the resonant frequency of the resonator mode, and $N_0(E)$ is the density of the inverted population of the atoms entering the resonator.[3)] Inasmuch as the beam is excited by coherent radiation from the generator itself, the initial inverted-population density N_0 depends on the amplitude of the field of the generated radiation $\mathscr{E}$:

$$N_0(t)=N_a\Lambda\left[\frac{2p_{12}}{\pi\hbar}\int_0^{\tau_0}k_0\mathscr{E}(t-\tau-t')\,dt'\right], \tag{8}$$

where N_a is the number of active particles, $\Lambda(\gamma)$ the excitation function shown in Fig. 2, τ the travel time of the atoms from the exciting ray to the resonator (delay time), and k_0 the ratio of the field amplitude in the resonator and in the exciting ray.

To investigate the system (7), (8) it is convenient to go over to dimensionless variables

$$\begin{gathered} t\to\omega_0 t, \quad E=\frac{\hbar}{p_{12}\tau_0}x, \quad P=p_{12}vN_a; \\ N=-\frac{N_a}{2}W, \quad N_0=-\frac{N_a}{2}W_0 \end{gathered} \tag{9}$$

and dimensionless symbols

$$\begin{gathered} \mu=\frac{2}{\tau_0\omega}, \quad \mu_1=\frac{1}{Q}\frac{\omega_r}{\omega}, \quad \delta=-4\pi\frac{p_{12}^2\tau_0}{\hbar}N_a, \\ \Delta=1-\varepsilon=1-\left(\frac{\omega_m}{\omega}\right)^2, \quad \Delta_1=1-\varepsilon_1=1-\left(\frac{\omega_p}{\omega}\right)^2. \end{gathered} \tag{10}$$

The solution of the system of oscillation equations will be sought in the form of slow motions:[13, 14]

$$x(t)=X(t)\cos[t+\varphi(t)], \quad v(t)=V(t)\cos[t+\psi(t)]. \tag{11}$$

Then the system (7) reduces to a system of five equations

$$\begin{gathered} \dot{X}=-\frac{\mu_1}{2}X+\frac{\delta}{2}V\sin\Phi, \quad \dot{\varphi}=-\frac{\Delta_1}{2}-\frac{\delta}{2}\frac{V}{X}\cos\Phi, \\ \dot{V}=-\frac{\mu}{2}V-\frac{\mu}{4}WX\sin\Phi, \quad \dot{\psi}=-\frac{\Delta}{2}-\frac{\mu}{2}W\frac{X}{V}\cos\Phi, \\ \dot{W}=-\frac{\mu}{2}W-\mu\Lambda[k\overline{X}(t-\tau)]+\mu XV\sin\Phi, \end{gathered} \tag{12}$$

where

$$\Phi=\psi-\varphi, \quad k=\frac{2}{\pi}k_0, \quad \overline{X}(t)=\frac{1}{\tau_0}\int_0^{\tau_0}X(t-t')\,dt'.$$

Assuming all the derivatives in (12) to be equal to zero, we obtain the stationary solutions. The stationary value of the phase difference Φ ($\tan\Phi=\mu/\Delta=-\mu_1/\Delta_1$) determines the generation frequency:

$$\omega=\omega_0+\frac{\Delta\omega_{tr}}{\Delta\omega_r}\left(1+\frac{\Delta\omega_{tr}}{\Delta\omega_r}+\frac{\Delta\omega_{tr}}{\Delta\omega_D}\right)^{-1}(\omega_r-\omega_0), \tag{13}$$

where $\Delta\omega_r=\omega/Q$ is the width of the resonator line. The stationary values of the field amplitude X are determined by the relation

$$\Lambda(kX)=\alpha X^2+\beta, \tag{14}$$

where $\alpha=\mu_1/\delta$ and $\beta=(\mu_1^2+\Delta_1^2)/\mu_1\delta$, with $\alpha\approx\beta$ when $|\omega-\omega_r|\ll\Delta\omega_r$.

A graphic solution of (14) shows (Fig. 4) that when the threshold is exceeded there exist two stationary values of the amplitude. For further analysis it is convenient to approximate $\Lambda(\gamma)$ in the region of interest to us ($0.5<\gamma<1.5$) by the analytic expression

$$\Lambda(\gamma)=\Lambda_{max}-2(\gamma-1)^2, \tag{15}$$

where $\Lambda_{max}=0.5$. Then, solving (14), we obtain the threshold value of the amplitude:

$$X_{thr}=2k/(\alpha_{thr}+2k^2).$$

Inasmuch as $X_{thr}>0$, the laser will have a hard self-excitation mode.[13] When threshold is exceeded, there are two stationary values of the amplitude:

$$X_{1,2}=\frac{2k\pm[4k^2-(2-\Lambda_{max}+\alpha)(\alpha+2k^2)]^{1/2}}{\alpha+2k^2}. \tag{16}$$

To ascertain which of the two states is realized in practice, let us investigate the solution of the system (12) for stability in the inner stationary states X_s ($s=1, 2$). We shall first simplify the system (12).

The resonator attenuation time Q/ω is much shorter than the relaxation time of the polarization and of the number of active beam particles, τ_0 ($\mu_1\gg\mu$). Consequently, the variation of the field can be regarded as "fast" compared with the "slow" variations of the polarization and of the inverted population.[14] In this case the field parameters respond instantaneously to the variations of V, W, and φ. Mathematically this means that the connection between the field amplitudes and the polarization becomes algebraic ($V=\alpha X$). The system (12) then reduces to a system of two equations:

$$\begin{gathered} \mu^{-1}\dot{X}=-X-\alpha^{-1}WX, \\ \mu^{-1}\dot{W}=-W+\alpha X^2-\Lambda[k\overline{X}(t-\tau)]. \end{gathered} \tag{17}$$

To investigate the stability, we linearize the system (17). Putting $X=X_s+\xi$, $W=W_s(\eta+1)$, we get for $|\xi|\ll X_s$ and $|\eta|\ll 1$

$$\begin{gathered} \mu^{-1}\dot{\xi}=\eta X_s, \\ \mu^{-1}\dot{\eta}=-\eta-2\xi X_s-\frac{4k}{\alpha}(kX_s-1)\overline{\xi}(t-\tau). \end{gathered} \tag{18}$$

Seeking the solution of the linearized system in the form $e^{\lambda t}$, we obtain the following characteristic equation for the determination of λ:

$$\lambda^2+\lambda+a+\frac{b}{2\lambda}(1-e^{-2\lambda})e^{-2\lambda\tau/\tau_0}=0, \tag{19}$$

where $a=2X_s^2$ and $b=4\alpha^{-1}kX_s(kX_s-1)$. The appearance of the quasipolynomial is connected with the delay

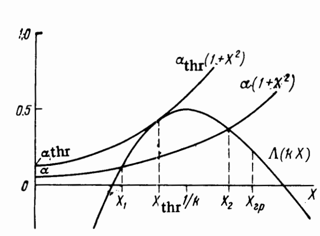

FIG. 4. Illustrating the determination of the stationary states of the field amplitudes X_1 and X_2.

[3)]The initial average polarization of the beam is equal to zero, owing to the averaging over the x phase of the exciting ray.

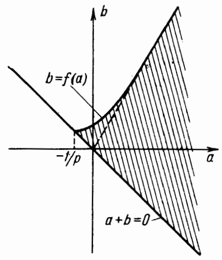

FIG. 5. Illustrating the determination of the stability region.

τ and the integral connection between the excitation and the field (8).

The best method for investigating the roots of the quasipolynomial (19) is the D-breakdown method.[15] D-breakdown consists in breaking down the space of the coefficients of the quasipolynomial (in our case the plane $\{a, b\}$) into separate regions by means of hypersurfaces, the points of which correspond to at least one zero on the imaginary axis (including $z = 0$). Then each region u_k of the D-breakdown can be assigned a number k, which is the number of zeroes λ_i with $\operatorname{Re} \lambda_i > 0$. The region of asymptotic stability of the solutions are the regions u_0 corresponding to the quasipolynomials which have not even one root with $\operatorname{Re} \lambda_i > 0$.

It can be shown that the region u_0 of the quasipolynomial (19) is the shaded area in Fig. 5, bounded by the line $a + b = 0$ and the curve $b = f(a)$, where the function $b = f(a)$ is specified parametrically:

$$b = \frac{y}{\sin y \sin py}, \quad a = y^2 - y \operatorname{ctg} py \quad (0 < y < \pi/p), \quad (20)$$

where $p = 1 + 2\tau/\tau_0$ is the delay parameter. Stable states correspond to those values of the field amplitude X_s, for which the coefficients a and b of the quasipolynomial (19) lie within the shaded region: $-a < b < f(a)$, that is,

$$-2X_s^2 < \frac{4}{\alpha} kX_s(kX_s - 1) < f(2X_s^2). \quad (21)$$

The left inequality of (21) gives the first stability condition

$$X_s > \frac{2k}{\alpha + 2k^2} = X_{\text{thr}}, \quad (22)$$

which shows that the smaller of the two stationary values of the amplitude, X_1, corresponds to unstable equilibrium (Fig. 4). The right-hand inequality of (21) imposes an upper limit ($X_s < X_{\text{lim}}$) on the region of the permissible values of the amplitude X_2. An approximate lower estimate of X_{lim} can be obtained by replacing the function f(a) by its asymptotic form $b = [(\pi/p)/\sin(\pi/p)]\,a$:

$$X_{\text{lim}} \leqslant \frac{1}{k}\left(1 - \frac{\alpha}{2k^2}\frac{\pi/p}{\sin(\pi/p)}\right)^{-1}. \quad (23)$$

It follows from (23) that when the delay time τ is decreased compared with the transit time τ_0, the value of X_{lim} increases and the stability region broadens. The estimate (23) can be easily made more precise in the region $a \ll 1$, where the curve f(a) lies much higher than the asymptote. This leads to an increase in the limiting value of the amplitude X_{lim}. We note that the obtained upper limit of the region of admissible values of the field amplitude is meaningful only when $X_{\text{lim}} < 3/2k$ since $X_2 < 3/2k$ (Fig. 4). Thus, the stable generation modes are those having a field amplitude X satisfying the condition

$$X_{\text{thr}} < X < \min\{X_{\text{lim}} 3/2k\}. \quad (24)$$

4. INCOHERENT EXCITATION OF BEAMS

In the case of incoherent excitation of the beam atoms, the width of the negative-absorption spectral line is determined by the Doppler broadening. The ratio of the homogeneous width of the beam $\Delta\omega_{\text{tr}}$ determined by the transit time of the beam to the resonator, to the inhomogeneous (Doppler) width $\Delta\omega_D$ is

$$\rho = \frac{\Delta\omega_{\text{tr}}}{\Delta\omega_D} = \frac{1}{\varphi_0}\frac{\lambda}{\pi a} \quad (25)$$

where φ_0 is the beam divergence and a the resonator width (Sec. 2). In the optical range $\lambda/a = 10^{-3} - 10^{-4}$, and therefore, even for a well collimated beam ($\varphi_0 \approx 10^{-2}$ rad), we have $\rho \ll 1$, that is, the line is broadened essentially homogeneously. An increase in the line width obviously leads to a corresponding lowering of the frequency stability.

It is known that in lasers with inhomogeneous amplification line a "dip" appears at the generation frequency. ω[16, 17]. In a beam laser, the width of the resonator line is relatively large ($\Delta\omega_r \gg \Delta\omega_D$), and therefore the occurrence of the "dip" can lead to lasing at neighboring frequencies. Consequently, when the excitation level of such a beam laser is increased, so that the depth of the "dip" increases, the single-frequency generation regime in the vicinity of the line center may become unstable.

This can be proved by using Lamb's expression for the generation frequency ω of a laser with a Doppler-broadened line,[17] modifying this expression for the case $\Delta\omega_r > \Delta\omega_D$:

$$\frac{\omega - \omega_r}{\omega_0 - \omega} = \frac{\Delta\omega_r}{\Delta\omega_D}\left\{\eta - \frac{\eta - 1}{4}\frac{\Delta\omega_D}{\Delta\omega_{\text{tr}}}\left[1 + \frac{1}{2}\left(\frac{\omega - \omega_0}{\Delta\omega_{\text{tr}}}\right)^2\right]^{-1}\right\}, \quad (26)$$

where η is the coefficient of the excess of excitation above threshold ($\eta = 1$—threshold) and $\Delta\omega_D \gg \Delta\omega_{\text{tr}}$. We represent Eq. (26) in the form

$$x\left(1 + \eta\frac{\Delta\omega_r}{\Delta\omega_D}\right) - x_0 = \frac{\Delta\omega_r}{\Delta\omega_{\text{tr}}}\frac{\eta - 1}{2}\frac{x}{2 + x^2}, \quad (27)$$

where $x = (\omega - \omega_0)/\Delta\omega_{\text{tr}}$ and $x_0 = (\omega_r - \omega_0)/\Delta\omega_{\text{tr}}$. Examining the plots of the left and right sides of (27), it is easy to verify that the condition for the existence of a single root x in the vicinity of the center of the line is the requirement that the derivative of the left side be larger than the derivative of the right side at the point $x = 0$. This condition can be written in the form

$$\eta - 1 < 4\frac{\Delta\omega_{\text{tr}}}{\Delta\omega_D}\left(1 + \frac{\Delta\omega_D}{\Delta\omega_r}\right). \quad (28)$$

Consequently, when $\Delta\omega_r \gg \Delta\omega_D \gg \Delta\omega_{\text{tr}}$ the single-frequency regime exists only at a slight excess of excitation above threshold. Therefore the generation of coherent light of stable frequency by means of beams of atoms or molecules with incoherent excitation is apparently a very complicated problem.

5. FREQUENCY STABILITY

The main advantages of a laser using beams of atoms or molecules is the high stability of the frequency of the generated radiation. In the case of coherent excitation of the beams, the stability of the generation frequency is determined by the expression (13). A stabilizing factor $\Delta\omega_r/\Delta\omega_{tr} \approx 10^{-3}$ is quite realistic ($\Delta\omega_{tr} \approx 10^5$ sec^{-1}, $\Delta\omega_r \approx 10^8$ sec^{-1}). When the resonator frequency ω_r has a stability on the order of 10^{-8}—10^{-9}, it is possible to obtain a generation-frequency stability on the order of 10^{-11}—10^{-12}. The resonator frequency can be stabilized with the required accuracy by means of relatively narrow lines of superradiance of gases or by means of a generation frequency of a gas laser, stabilized accurate to 10^{-9}.[18, 19] We note that beam lasers are also subject to a radiation-frequency shift due to a decrease in the inverted population of the beam on traveling through the field in the resonator. The resultant inhomogeneous transverse distribution of the gain leads to a shift of the natural frequency of the resonator.[20] An estimate shows that this effect necessitates a correction on the order of 10^{-9} in the resonator frequency. The degree of transverse inhomogeneity depends on the field amplitude. This effect therefore leads to a dependence of the generation frequency on the field amplitude in the twelfth significant figure when $\Delta\omega_r/\Delta\omega_{tr} \approx 10^3$. The remaining effects (instability of the optical path of the exciting ray, instability of the gain and frequency of the quantum-amplifier, etc.) make an even smaller contribution to the instability of the generation frequency.

In conclusion, the authors are deeply grateful to Academician N. G. Basov for a discussion of the work and valuable remarks, and to A. T. Matachun for help with the numerical calculations.

---

[1] N. G. Basov and A. M. Prokhorov, Zh. Eksp. Teor. Fiz. **27**, 431 (1954); ibid. **28**, 249 (1955) [Sov. Phys.-JETP **1**, 184 (1955)]. J. P. Gordon, H. J. Zeiger, and C. H. Townes, Phys. Rev. **95**, 284 (1954) and **99**, 1264 (1955).

[2] H. M. Goldenberg, D. Kleppner, and N. F. Ramsey, Phys. Rev. Lett. **5**, 361 (1960); Phys. Rev. **126**, 603 (1962).

[3] A. M. Prokhorov, Zh. Eksp. Teor. Fiz. **34**, 1658 (1958) [Sov. Phys.-JETP **7**, 1140 (1958)].

[4] N. G. Basov and V. S. Letokhov, ZhETF Pis. Red. **2**, 6 (1965) [JETP Lett. **2**, 3 (1965)].

[5] B. N. Edwards and D. E. Burch, JOSA **55**, 174 (1965); H. J. Gerritsen, Proc. Puerto Rico Conference on Physics of Quantum Electronics, N. Y. 1966, p. 581.

[6] K. Sakurai, K. Shimoda, and H. Takami, J. Phys. Soc. Japan **21**, 1838 (1966).

[7] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Fizmatgiz, 1963 [Quantum Mechanics, Addison-Wesley, 1958].

[8] E. L. Hahn, Phys. Rev. **80**, 580 (1950).

[9] N. Ramsey, Molecular Beams, Oxford, 1956 (Russian translation, IIL, 1960).

[10] J. R. Singer and I. Gorog, Bull. Amer. Phys. Soc. **7**, 14 (1962); N. G. Basov, A. N. Oraevskiĭ, and V. A. Shcheglov, ZhETF Pis. Red. **4**, 61 (1966) [JETP Lett. **4**, 41 (1966)].

[11] V. M. Faĭn, Zh. Eksp. Teor. Fiz. **33**, 945 (1957) [Sov. Phys.-JETP **6**, 726 (1958)].

[12] A. N. Oraevskiĭ, Radiotekhnika i élektronika **4**, 718 (1959).

[13] A. A. Andronov, A. A. Vitt, and S. E. Khaĭkin, Teoriya kolebaniĭ (Theory of Oscillations), Fizmatgiz, 1959.

[14] Kh. Yu. Khaldre and R. V. Khokhlov, Vuzov Radiofizika **1**, 60 (1958).

[15] L. E. El'sgol'ts, Vvedenie v teoriyu differentsial'nykh uravneniĭ s otklonyayushchimsya argumentom (Introduction to the Theory of Differential Equations with Variable Argument), Nauka, 1964.

[16] W. R. Bennett, Jr., Phys. Rev. **126**, 580 (1962).

[17] W. E. Lamb, Phys. Rev. **134**, A1429 (1964).

[18] K. Shimoda and A. Javan, J. Appl. Phys. **36**, 718 (1965).

[19] A. D. White, IEEE J. Quantum Electronics **1**, 349 (1965).

[20] A. F. Suchkov, Zh. Eksp. Teor. Fiz. **49**, 1495 (1965) [Sov. Phys.-JETP **22**, 1026 (1966)].

Translated by J. G. Adashko
125